Women in Photonics (WiP)
School on Photonic Metamaterials
Paris, April 13-18, 2008

Transmission lines (TL) and TL-based Metamaterials – introduction and review of the field

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Introduction

Metamaterials are artificial structures that can be designed to exhibit specific electromagnetic properties not commonly found in nature.

Metamaterials are characterized by simultaneously negative permittivity (ε **< 0**) and permeability (μ **< 0**). Metamaterials are also referred to left-handed (LH) materials.

The Magazine ***"Science"*** named LH materials as one of the top scientific breakthroughs of 2003 **["Breakthrough of the year: The runners-up," *Science*, vol. 302, No.5653, pp. 2039–2045, 2003.]**

Description of electromagnetic waves

Maxwell equations:

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \times \vec{H} = \frac{\partial \vec{D}}{\partial t} + \vec{J}$$

$$\nabla \cdot \vec{D} = \rho$$

$$\nabla \cdot \vec{B} = 0$$

$$\vec{D} = \varepsilon \vec{E}$$

$$\vec{B} = \mu \vec{H}$$

($\vec{J} \rightarrow 0$, $\rho \rightarrow 0$)

E - Electric field [Volt/meter; V/m]
H - Magnetic field [Ampere/meter; A/m]
B - Magnetic induction [Tesla; T]
D - Electric displacement [Coulomb/m^2 ; C/m^2]
J - Electric current density [Ampere/m^2 ; A/m^2]
ρ - Electric charge density [Coulomb/m^3; C/m^3]
ε – Dielectric permittivity [Farad/m; F/m]
μ – Magnetic permeability [Henry/m; H/m]

Plane EM wave description (1)

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

Maxwell equations:

$$\varepsilon \frac{\partial E_x}{\partial t} = -\frac{\partial H_y}{\partial z} \quad (1)$$

$$-\mu \frac{\partial H_y}{\partial t} = \frac{\partial E_x}{\partial z} \quad (2)$$

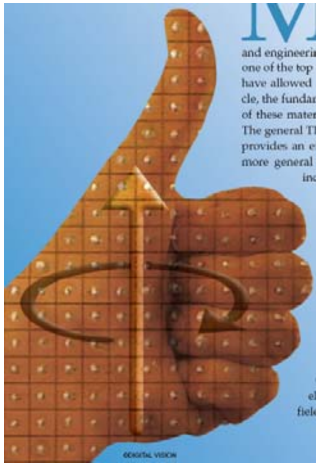
Let $\vec{E} = E_0 \cdot \vec{e}_x \cdot \cos(\omega t - kz)$ $\quad k = \omega \sqrt{\varepsilon} \sqrt{\mu}$

or $\vec{E} = E_0 \cdot \vec{e}_x \cdot e^{i(\omega t - kz)}$ $\quad \vec{H} = H_0 \cdot \vec{e}_y \cdot e^{i(\omega t - kz)}$

Using $\frac{\partial}{\partial t} = i\omega$ $\quad \frac{\partial}{\partial z} = -ik$ find from (2): (next slide)

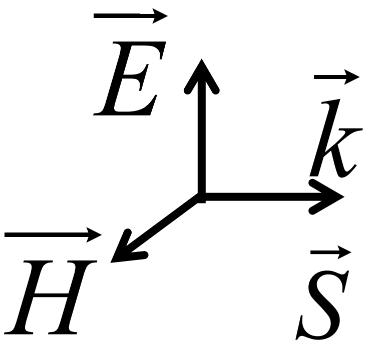
Plane EM wave description (2)

$$\omega\mu\vec{H} = \vec{k}\times\vec{E}; \quad -\omega\varepsilon\vec{E} = \vec{k}\times\vec{H}$$



$$\vec{k} = k\cdot\vec{e}_z$$

$$\varepsilon > 0 \quad \mu > 0$$



$$k > 0$$

RHM

Pointing vector

$$\vec{S} = \vec{E}\times\vec{H} = \frac{\left|\vec{E}\right|^2}{\omega\mu}\cdot\vec{k} = \frac{\left|\vec{H}\right|^2}{\omega\varepsilon}\cdot\vec{k}$$

Forward wave

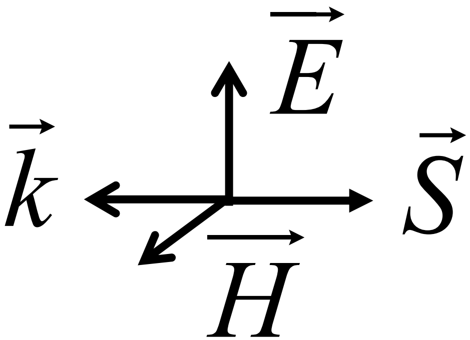
Plane EM wave description (3)

$$\varepsilon < 0 \quad \mu < 0$$

$$-\omega\mu\vec{H} = \vec{k} \times \vec{E}; \quad \omega\varepsilon\vec{E} = \vec{k} \times \vec{H}$$

$$k < 0$$

$$LHM$$

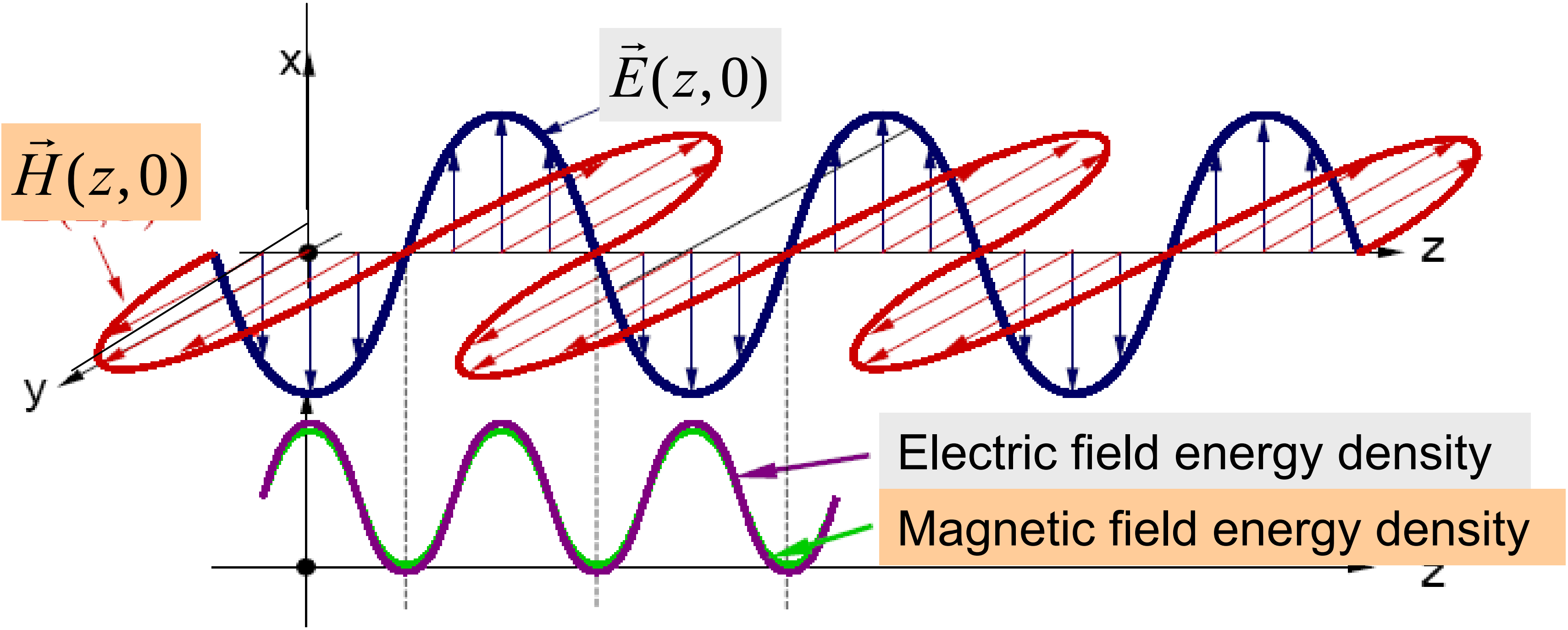


Backward wave

Pointing vector

$$\vec{S} = \vec{E} \times \vec{H} = -\frac{\left|\vec{E}\right|^2}{\omega\mu} \cdot \vec{k} = -\frac{\left|\vec{H}\right|^2}{\omega\varepsilon} \cdot \vec{k}$$

RH plane wave



Characteristics of the plane EM wave

Characteristic (wave) impedance:

$$Z_0 = \frac{E_x(z,t)}{H_y(z,t)} = \sqrt{\frac{\mu}{\varepsilon}}$$

Wave number (propagation constant):

$$k = \omega\sqrt{\varepsilon}\sqrt{\mu}$$

Phase velocity is defined as the velocity of a propagation of the constant phase surface $(\omega t - kz = const)$

$$V_{ph} = \frac{\omega}{k} = \frac{1}{\sqrt{\varepsilon}\cdot\sqrt{\mu}}$$

For RHM $\varepsilon > 0, \mu > 0 \rightarrow V_{ph} > 0$

For LHM $\varepsilon < 0, \mu < 0 \rightarrow V_{ph} < 0$

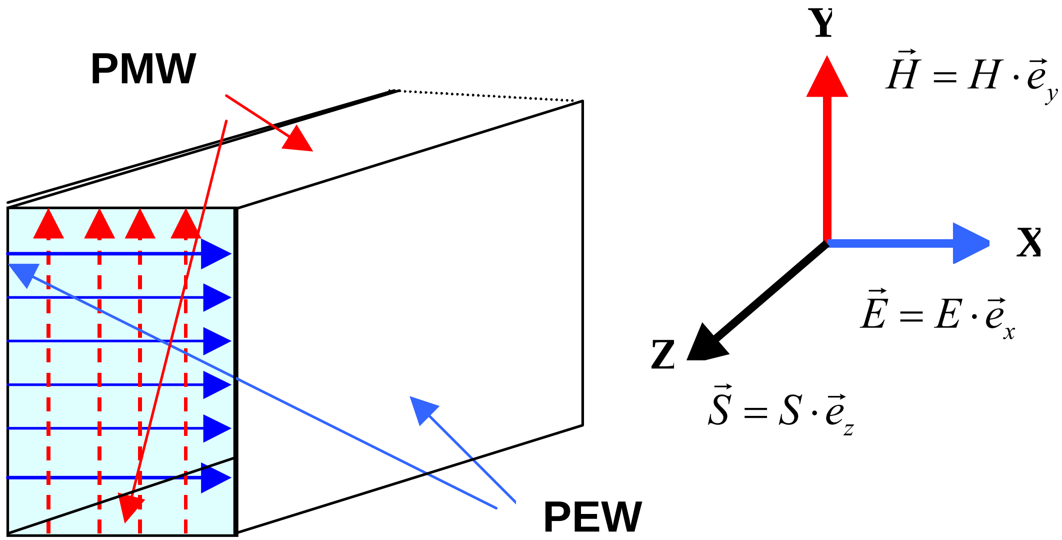
Forward wave propagates in RHM, backward wave propagates in LHM

Group velocity is defined as the velocity of a propagation of the surface of the group of waves with the constant amplitude

$$V_g = \left(\frac{dk}{d\omega}\right)^{-1}$$

For RHM and LHM $V_g > 0$

Plane EM wave in a perfect waveguide



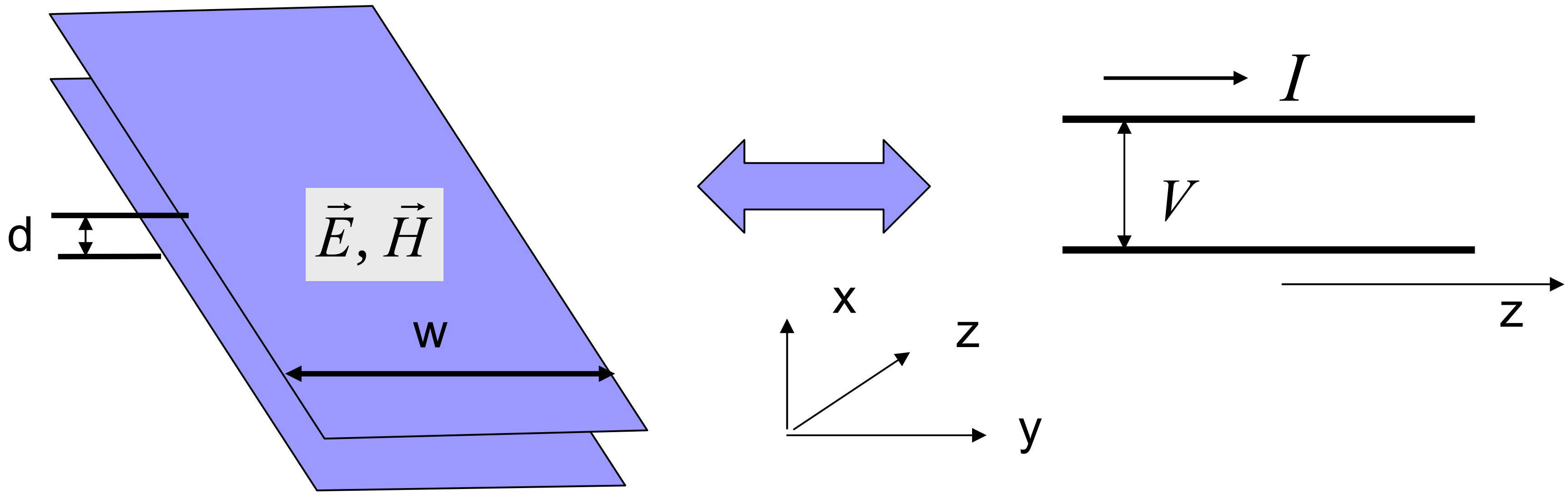
RHM

The plane wave propagates in the perfect waveguide

a) filled with the medium with $\varepsilon > 0$ and $\mu > 0$ as in line with the right-handed rule

b) filled with the medium with $\varepsilon < 0$ and $\mu < 0$ as in line with the left-handed rule

Parallel plate transmission line



$$\vec{E} = E_x(z)\cdot\vec{e}_x$$

$$V(z) = \int_0^d E_x dx = E_x \cdot d$$

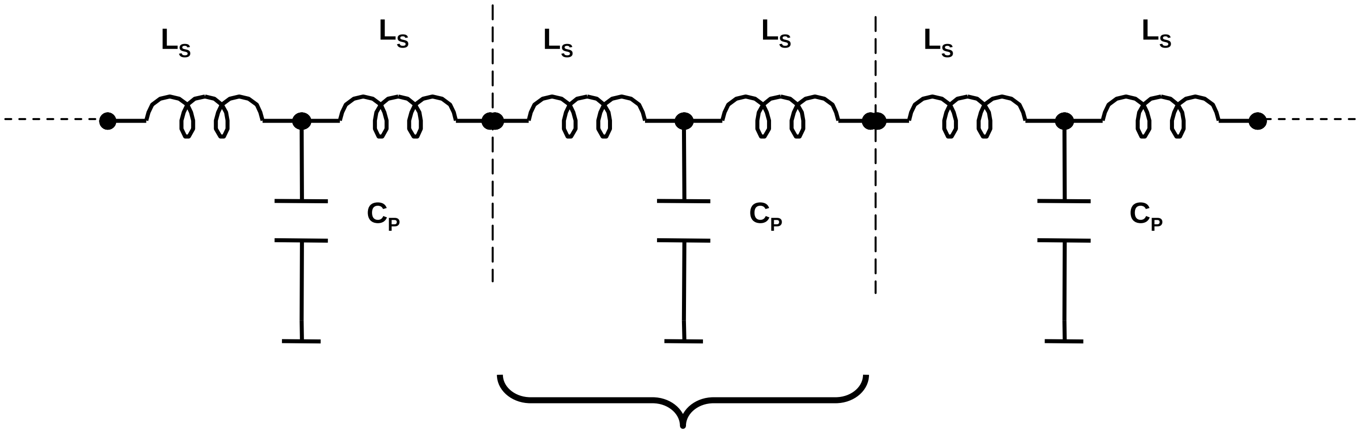
$$\vec{H} = H_y(z)\cdot\vec{e}_y$$

$$I(z) = \int_0^w \vec{e}_z \cdot\left(\vec{e}_x \times \vec{H}\right) dy = H_y \cdot w$$

Transmission line concept

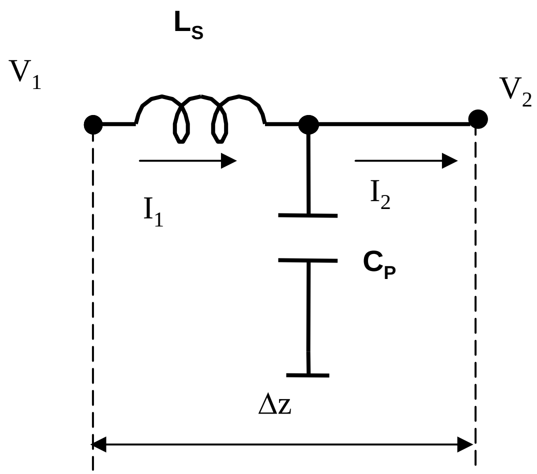
- The parallel plate transmission line with TEM mode described by E and H components can be equivalently replaced by a transmission line with ***V*** (voltage) and ***I*** (current) components.
- In the case of backward wave the equivalent transmission line is defined as left-handed transmission line (LH TL) and is considered as **one-dimensional metamaterial**.
- In the case of forward wave the equivalent transmission line is defined as right-handed transmission line (RH TL)

How to realize an equivalent transmission line?



Single cell of RH TL

Telegraph equations for RH TL (1)



$$L_s = L_1 \cdot \Delta z \qquad C_p = C_1 \cdot \Delta z$$

L_1 is inductance per unit length [H/m]

C_1 is capacitance per unit length [F/m]

$$V_1 - V_2 = -\Delta V = L_1 \cdot \Delta z \cdot \frac{\partial I}{\partial t} \qquad \frac{\Delta I}{\Delta z} \rightarrow \frac{\partial I}{\partial z}$$

$$I_1 - I_2 = -\Delta I = \frac{\partial q}{\partial t} = C_1 \cdot \Delta z \cdot \frac{\partial V}{\partial t} \qquad \frac{\Delta V}{\Delta z} \rightarrow \frac{\partial V}{\partial z}$$

$$\Delta z \rightarrow 0$$

Telegraph equations for RH TL (2)

$$\frac{\partial V}{\partial z} = -L_1 \frac{\partial I}{\partial t} = -i\omega L_1 I$$

$$\frac{\partial I}{\partial z} = -C_1 \frac{\partial V}{\partial t} = -i\omega C_1 V$$

They are followed by the wave equations:

$$\frac{\partial^2 I}{\partial z^2} = L_1 C_1 \cdot \frac{\partial^2 I}{\partial t^2} = -\omega^2 L_1 C_1 I$$

$$\frac{\partial^2 V}{\partial z^2} = L_1 C_1 \cdot \frac{\partial^2 V}{\partial t^2} = -\omega^2 L_1 C_1 V$$

RH TL characteristics

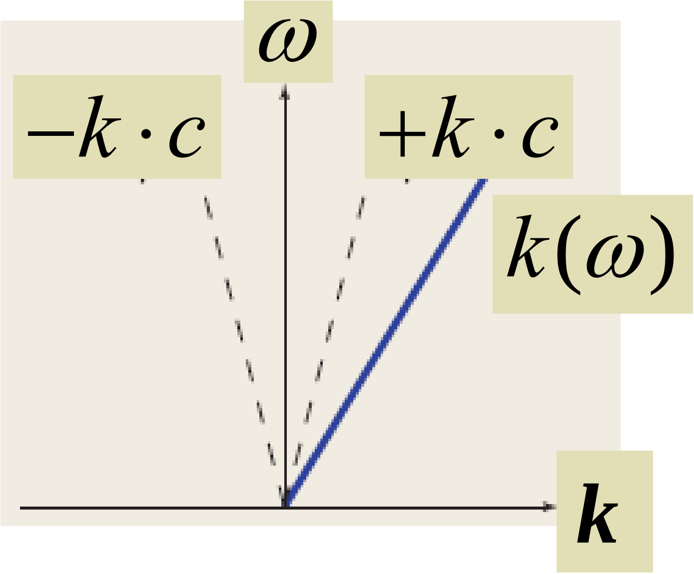
$$k = \omega\sqrt{L_1 C_1} > 0$$

$$V_{ph} = \frac{\omega}{k} = \frac{1}{\sqrt{L_1 C_1}} > 0$$

$$Z_0 = \sqrt{\frac{L_1}{C_1}}$$

$$V_g = \left(\frac{\partial k}{\partial \omega}\right)^{-1} = V_{ph} = \frac{1}{\sqrt{L_1 C_1}}$$

Dispersion diagram for the perfect RH TL

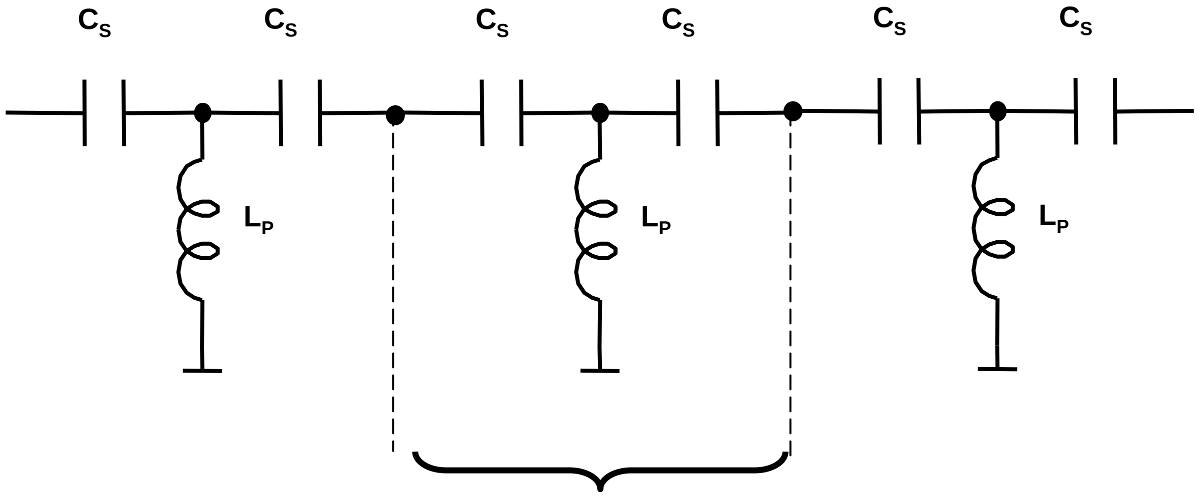


How to realize an artificial LH transmission line?

Let us use the duality principle:

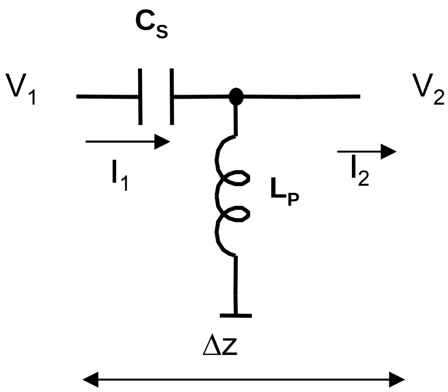
$$L \Rightarrow C \quad series \Rightarrow shunt$$

$$C \Rightarrow L \quad shunt \Rightarrow series$$



Single cell of LH TL

Telegraph equations for LH TL (1)



$$L_p^{-1} = \left(\frac{1}{L}\right)_1 \cdot \Delta z \qquad C_s^{-1} = \left(\frac{1}{C}\right)_1 \cdot \Delta z$$

$\left(\frac{1}{L}\right)_1$ is inverse inductance per unit length [$H^{-1}m^{-1}$]

$\left(\frac{1}{C}\right)_1$ is inverse capacitance per unit length [$F^{-1}m^{-1}$]

Telegraph equations

$$\frac{\partial V}{\partial z} = -\frac{1}{i\omega} \cdot \left(\frac{1}{C}\right)_1 I$$

$$\frac{\partial I}{\partial z} = -\frac{1}{i\omega} \cdot \left(\frac{1}{L}\right)_1 V$$

Wave equations

$$\frac{\partial^2 I}{\partial z^2} = -\frac{1}{\omega^2}\left(\frac{1}{L}\right)_1 \left(\frac{1}{C}\right)_1 I$$

$$\frac{\partial^2 V}{\partial z^2} = -\frac{1}{\omega^2}\left(\frac{1}{L}\right)_1 \left(\frac{1}{C}\right)_1 V$$

LH TL characteristics

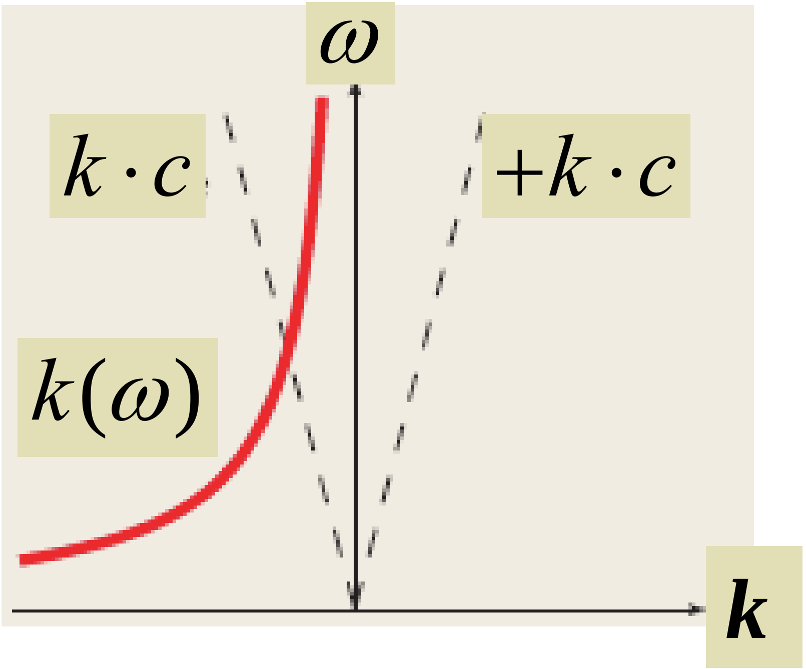
$$k = -\frac{1}{\omega}\sqrt{\left(\frac{1}{L}\right)_1 \cdot \left(\frac{1}{C}\right)_1} < 0$$

$$V_{ph} = \frac{\omega}{k} = -\omega^2 \frac{1}{\sqrt{\left(\frac{1}{L}\right)_1 \cdot \left(\frac{1}{C}\right)_1}} < 0$$

$$Z_0 = \sqrt{\frac{\left(\frac{1}{C}\right)_1}{\left(\frac{1}{L}\right)_1}}$$

$$V_g = \left(\frac{\partial k}{\partial \omega}\right)^{-1} = \omega^2 \frac{1}{\sqrt{\left(\frac{1}{L}\right)_1 \cdot \left(\frac{1}{C}\right)_1}} > 0$$

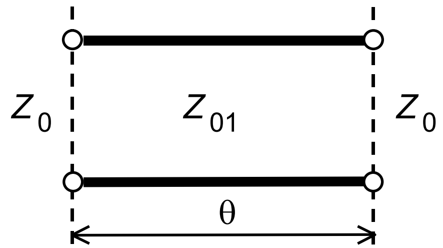
Dispersion diagram for the perfect LH TL



Remarks

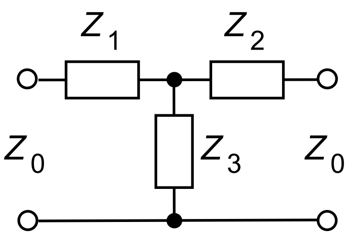
- In the LH TL, a backward wave propagates with the phase and the group velocities being in opposite directions
- The dispersion in the LHTL is characterised by the propagation constant *k* being inversely proportional to the frequency ω
- The dispersion in the RH TL is characterised by the linear dependence of the propagation constant *k* on the frequency ω

Equivalent presentation of a single cell of RH and LH TL

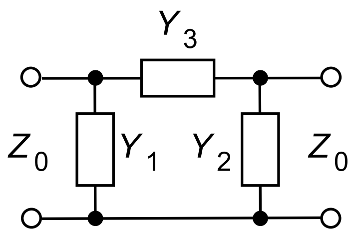


$$\theta = k \cdot l$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}_{TRL} = \begin{bmatrix} \cos\theta & jZ_{01}\sin\theta \\ j\sin\theta / Z_{01} & \cos\theta \end{bmatrix}$$



$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}_{T} = \begin{bmatrix} 1+\dfrac{Z_1}{Z_3} & Z_1+Z_2+\dfrac{Z_1Z_2}{Z_3} \\ \dfrac{1}{Z_3} & 1+\dfrac{Z_2}{Z_3} \end{bmatrix}$$

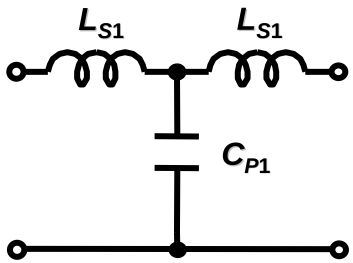


$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}_{\Pi} = \begin{bmatrix} 1+\dfrac{Y_2}{Y_3} & \dfrac{1}{Y_3} \\ Y_1+Y_2+\dfrac{Y_1Y_2}{Y_3} & 1+\dfrac{Y_1}{Y_3} \end{bmatrix}$$

Lumped-Element Equivalents of a TL Section

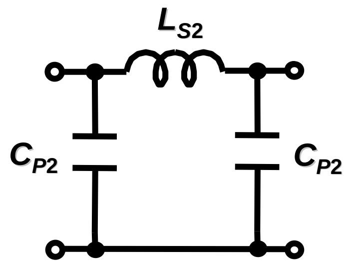
| θ | ≤ 90°

RH TL



$$L_{S1} = \frac{Z_{01}\tan(\theta/2)}{\omega}$$

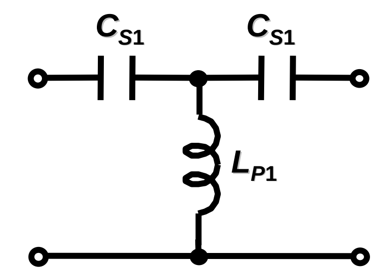
$$C_{P1} = \frac{\sin(\theta)}{\omega Z_{01}}$$



$$L_{S2} = \frac{Z_{01}\sin(\theta)}{\omega}$$

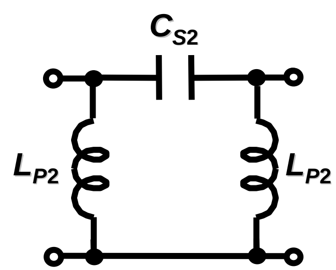
$$C_{P2} = \frac{\tan(\theta/2)}{\omega Z_{01}}$$

LH TL



$$C_{S1} = \frac{1}{\omega Z_{01}\tan(\theta/2)}$$

$$L_{P1} = \frac{Z_{01}}{\omega\sin(\theta)}$$

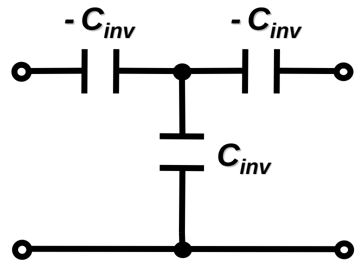


$$C_{S2} = \frac{1}{\omega Z_{01}\sin(\theta)}$$

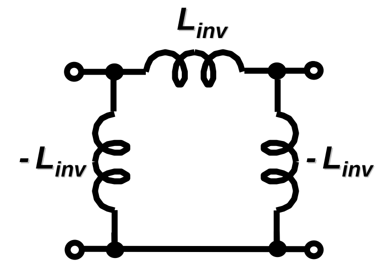
$$L_{P2} = \frac{Z_{01}}{\omega\tan(\theta/2)}$$

| θ | = 90°

RH TL

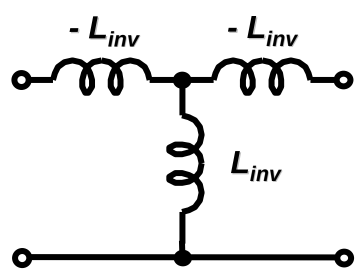


$$C_{inv} = \frac{1}{\omega Z_{01}}$$

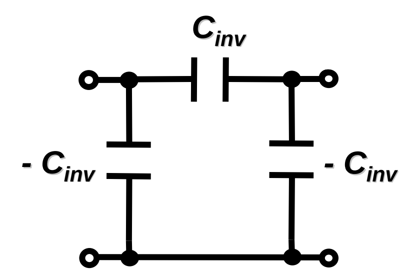


$$L_{inv} = \frac{Z_{01}}{\omega}$$

LH TL

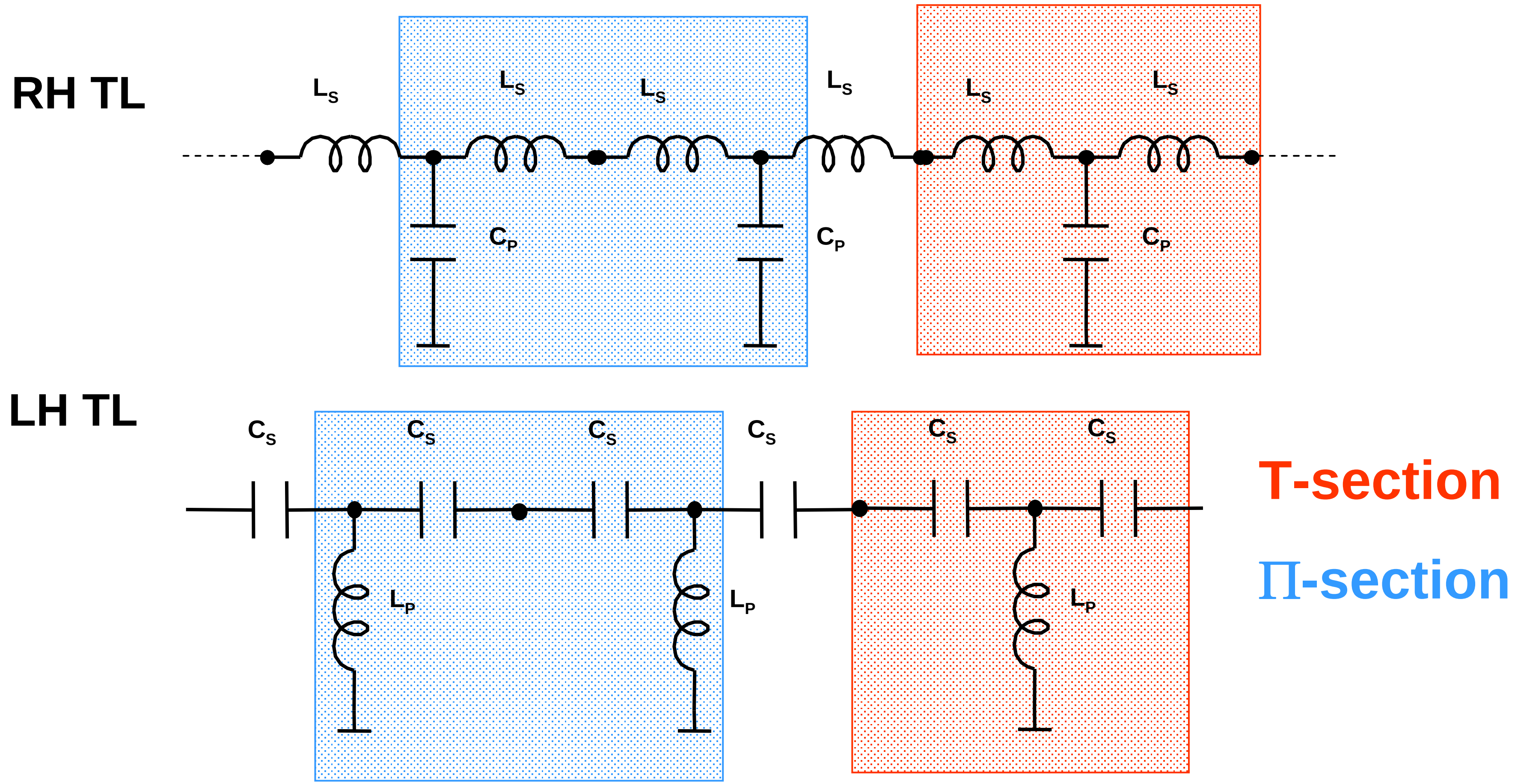


$$L_{inv} = \frac{Z_{01}}{\omega}$$

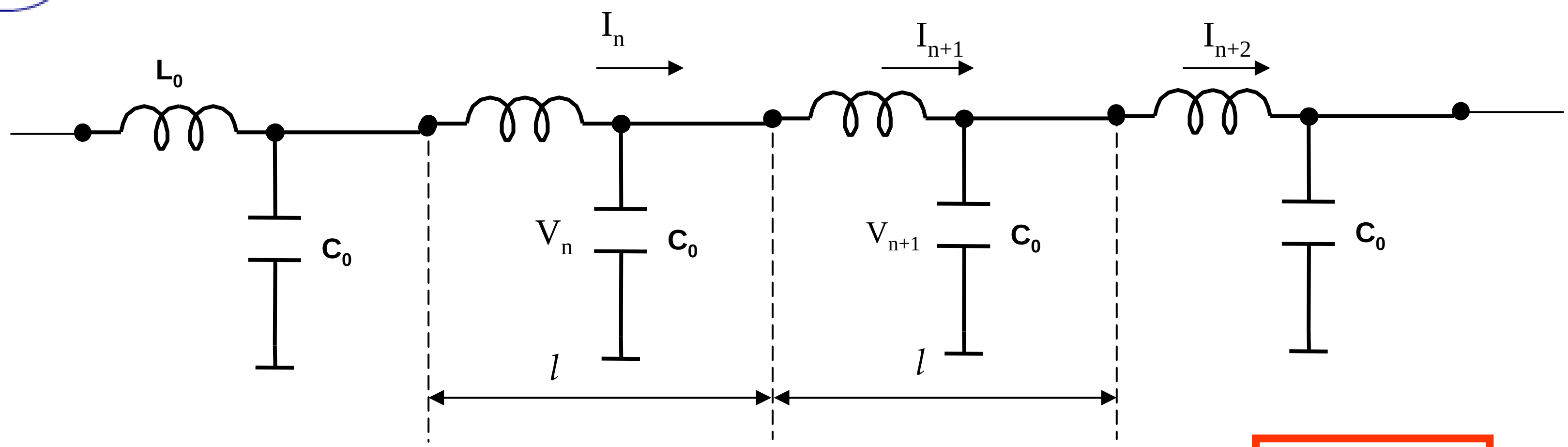


$$C_{inv} = \frac{1}{\omega Z_{01}}$$

Combination of RH and LH TL formed as cascaded single cells



Lumped element RH transmission lines



RH TL

Kirchhoff voltage and current low:

$$V_n - V_{n+1} = i\omega L_0 I_{n+1} \quad (1)$$

$$I_n - I_{n+1} = i\omega C_0 V_n \quad (2)$$

Identifying

$$kz = knl = n\theta \quad \text{for} \quad \theta = kl$$

One finds

$$V_n = V_+ e^{-in\theta} \quad (3)$$

$$I_n = I_+ e^{-in\theta} \quad (4)$$

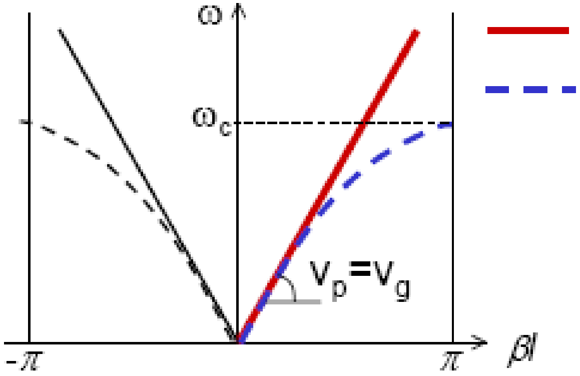
Dispersion characteristic for the lumped element RH TL

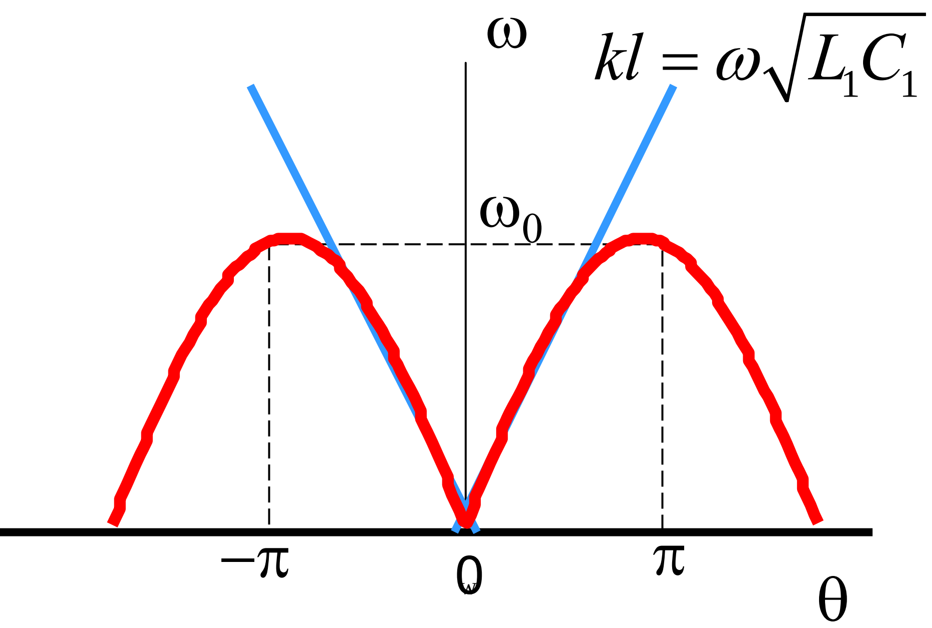
Substituting (3) and (4) in (1) and (2) and multiplying (1) and (2) we obtain the dispersion equation:

$$\sin^2\left(\frac{\theta}{2}\right) = \frac{1}{4}\omega^2 L_0 C_0 = \frac{\omega^2}{\omega_0^2} \quad (5)$$

$$\omega_0 = \frac{2}{\sqrt{L_0 C_0}} \quad (6)$$

Dispersion characteristic of the lumped element RH TL





$$\omega_0 = \frac{2}{\sqrt{L_0 C_0}}$$

ω_0 is the cut-off frequency

The main features of the lumped element RH TL

- In the low-frequency limit $(\omega << \omega_0)$

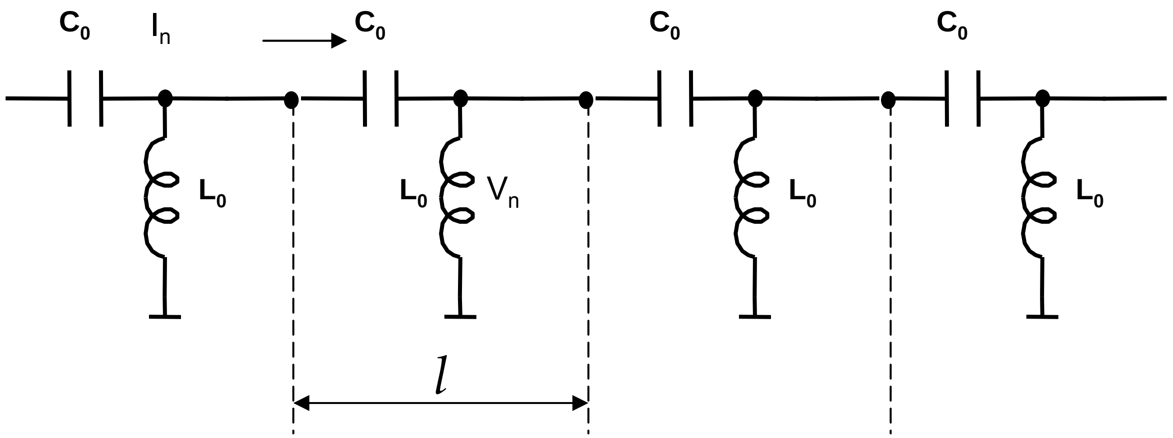
$$\frac{\omega}{\omega_0} = \pm \sin\left(\frac{\theta}{2}\right) \approx \pm \frac{\theta}{2} \qquad k = \frac{2\omega}{l\omega_0} = \omega\sqrt{L_1 C_1}$$

and the artificial TL behaves as an infinitely long RH TL with the linear dispersion low

- When $\omega > \omega_0$, θ becomes a complex number and the wave is attenuated: the higher frequency, the more is the attenuation

•Thus the lumped element RH TL behaves as the low-pass transmission line

Lumped element LH transmission lines



LH TL

Using the same procedure as for the RH TL we obtain the dispersion equation:

$$\sin^2\left(\frac{\theta}{2}\right)=\frac{1}{4}\frac{1}{\omega^2 L_0 C_0}=\frac{\omega_0^2}{\omega^2} \quad (7)$$

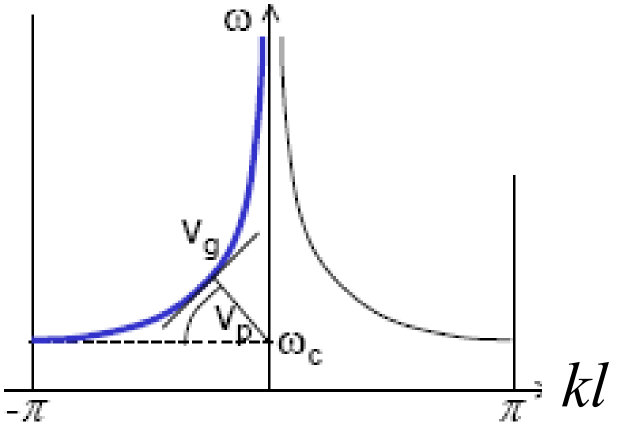
$$\omega_0=\frac{1}{2\sqrt{L_0 C_0}} \quad (8)$$

The main features of the lumped element LH TL

- In the high-frequency limit $(\omega >> \omega_0)$

$$\frac{\omega_0}{\omega} = \pm \sin\left(\frac{\theta}{2}\right) \approx \pm\frac{\theta}{2} \qquad k = \frac{2\omega_0}{l\omega} = \frac{1}{\omega}\cdot\frac{-1}{\sqrt{L_1 C_1}}$$

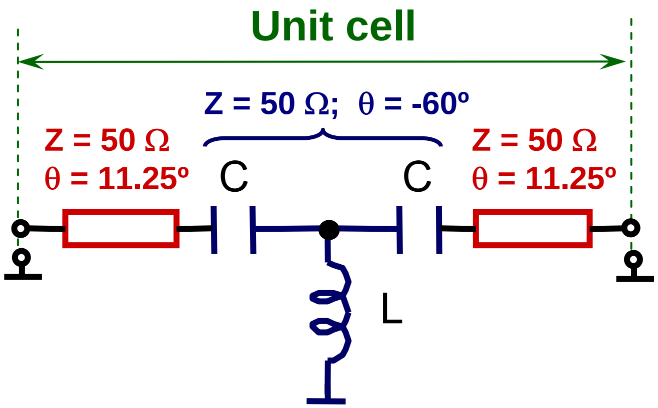
and the artificial TL behaves as an infinitely long LH TL with nonlinear dispersion low:



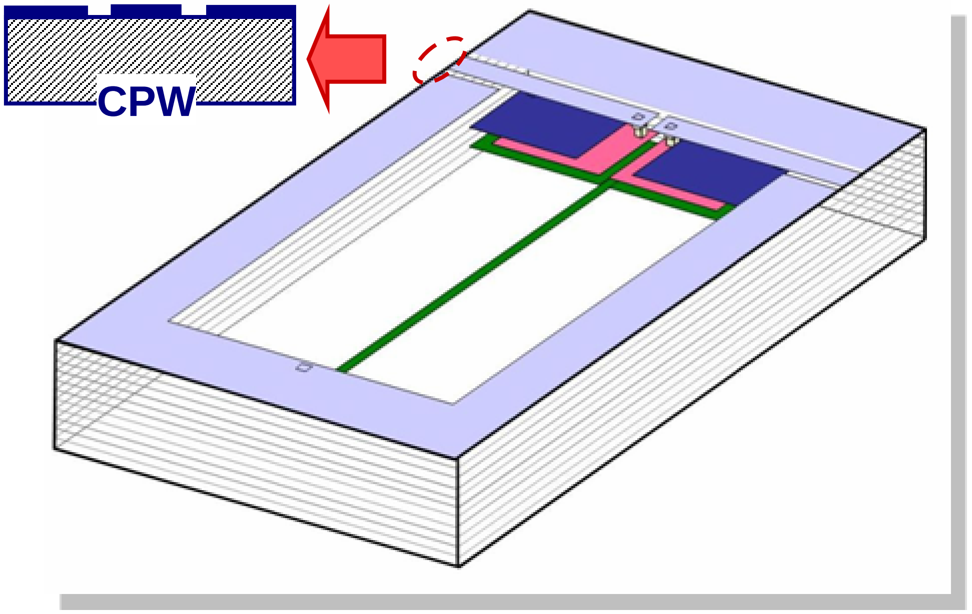
• When $\omega < \omega_0$, θ becomes a complex number and the wave is attenuated: the lower frequency, the more is the attenuation

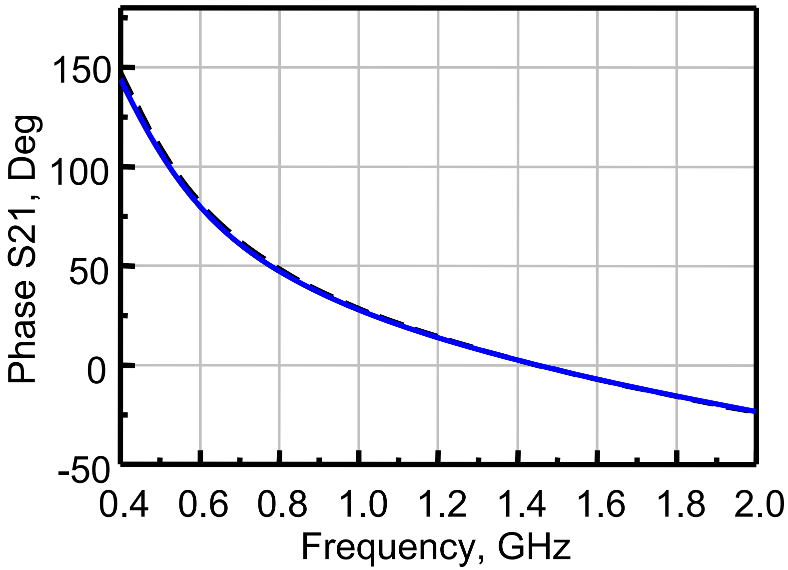
•Thus the lumped element LH TL behaves as the high-pass transmission line

CPW/LH transmission line section

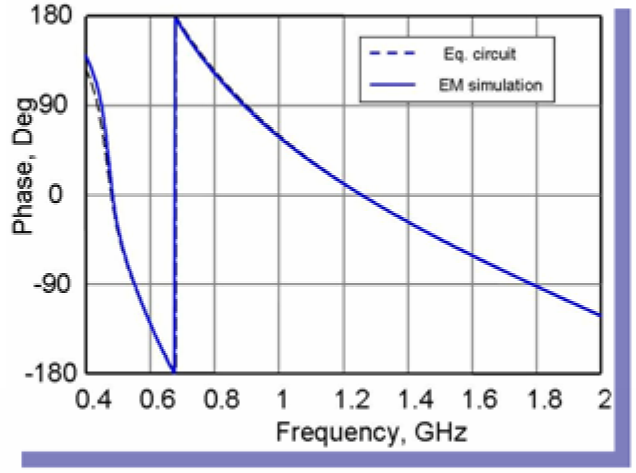


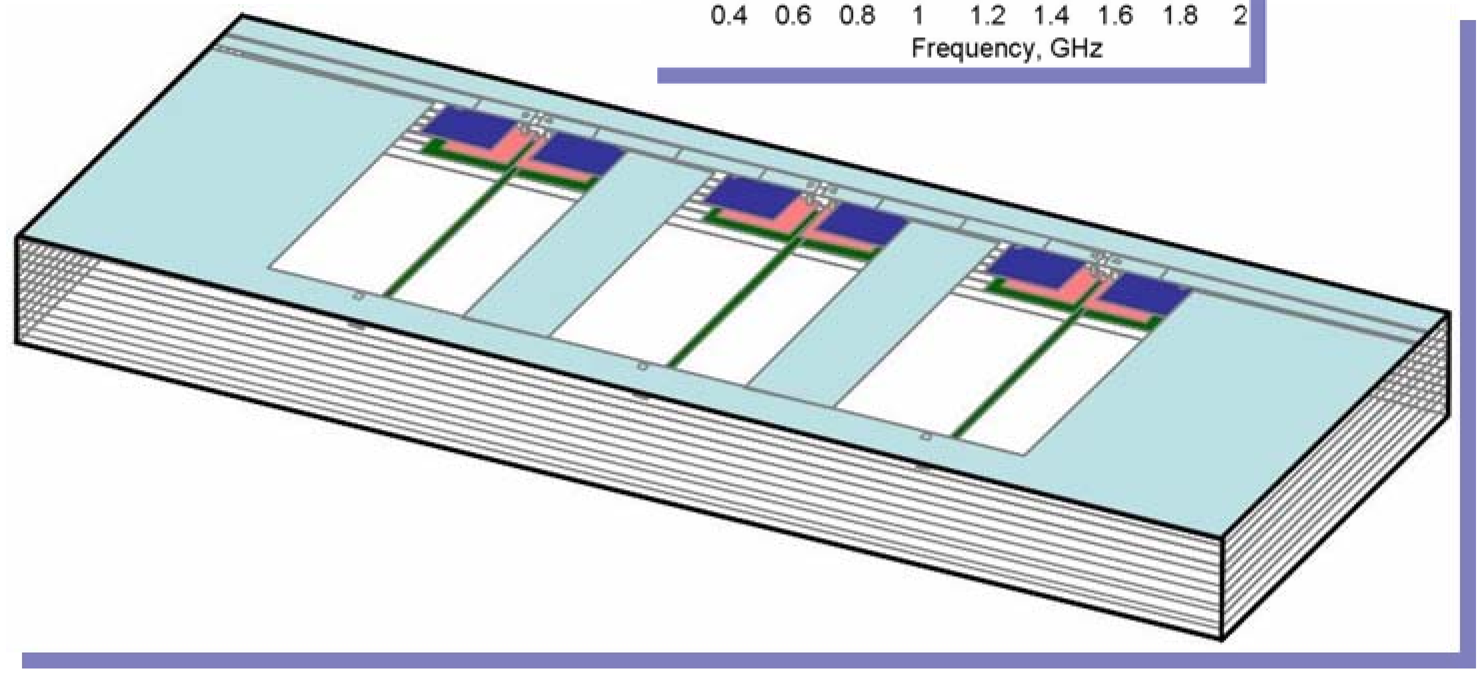
$$\omega < \omega_0$$



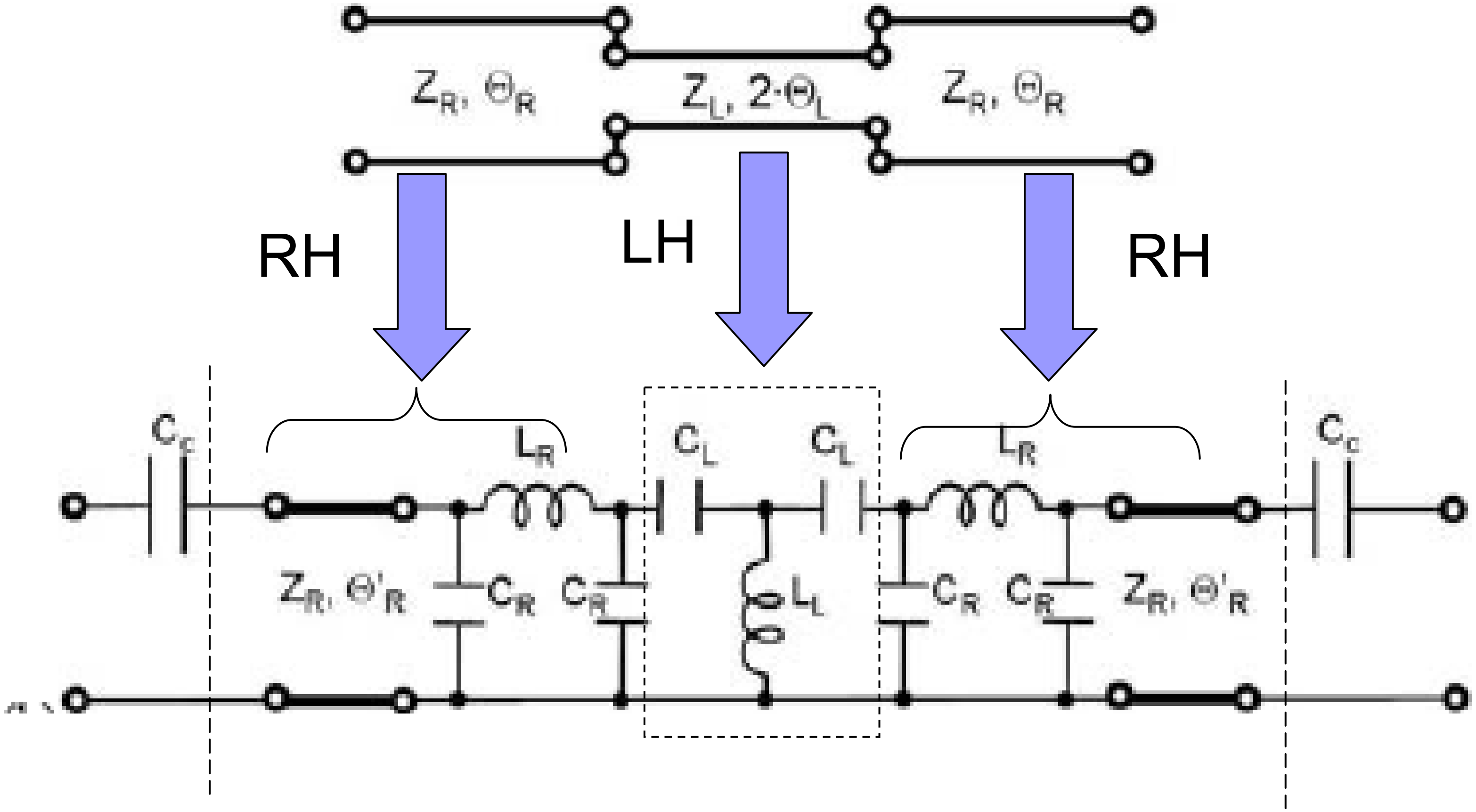


RH / LH TL section (three unit cells)

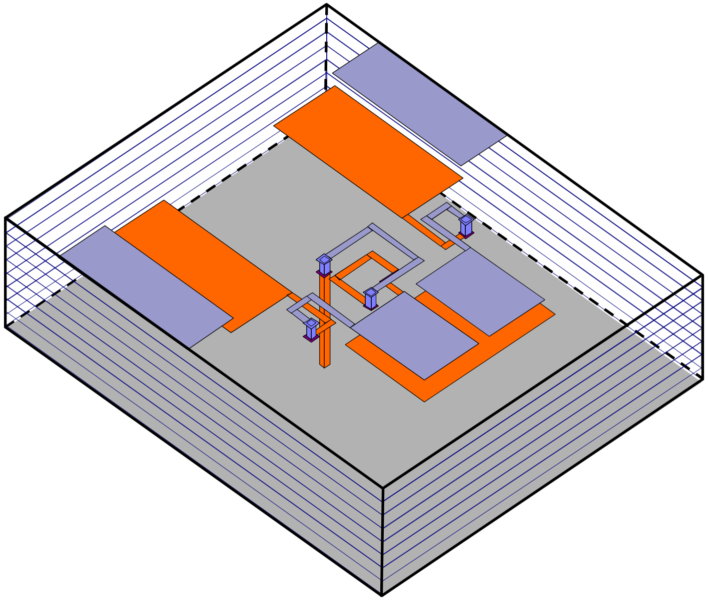


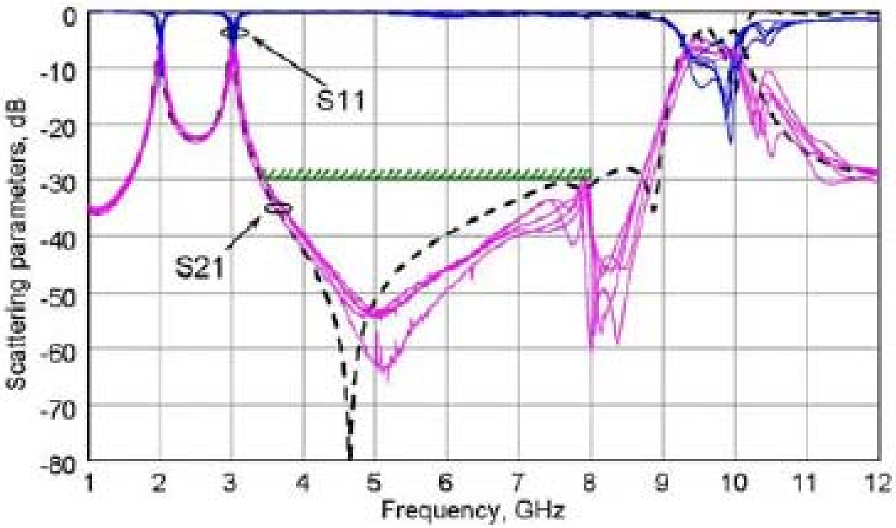


Example of a combination of RH/LH TLs

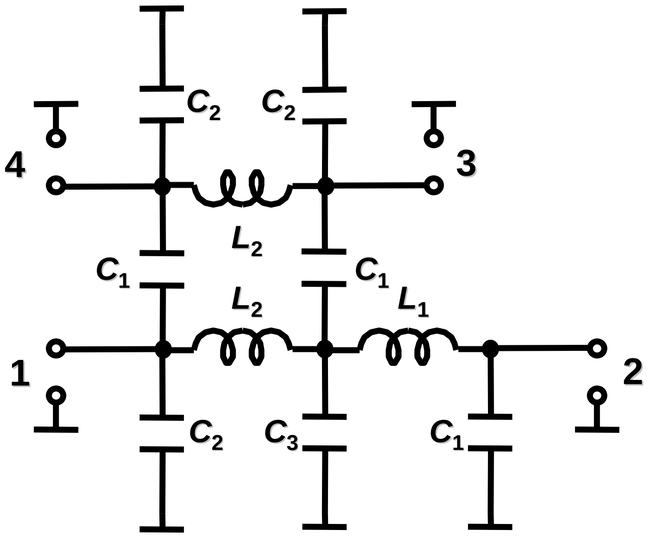


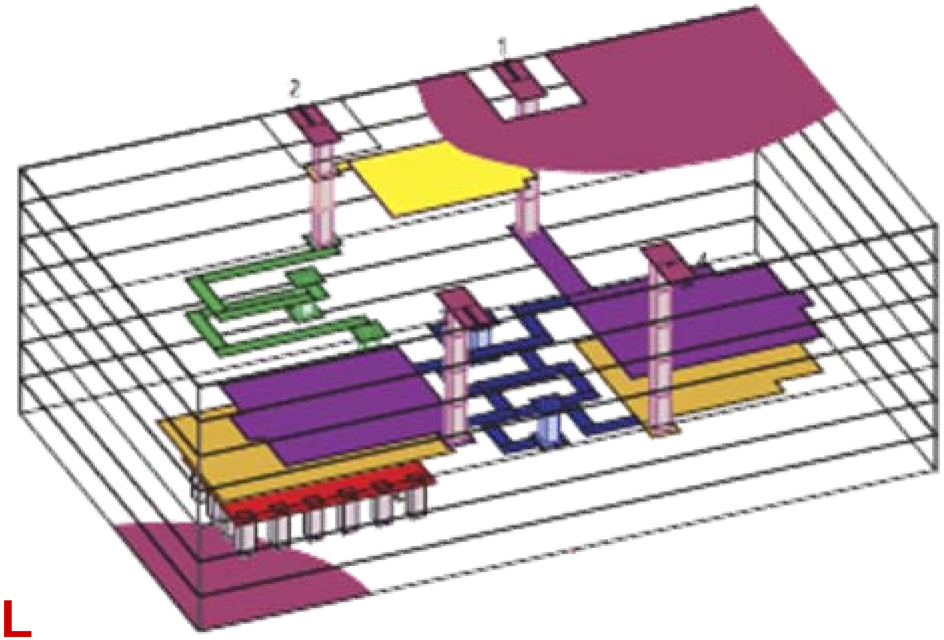
RH/LH TL section based resonator (dual-band)



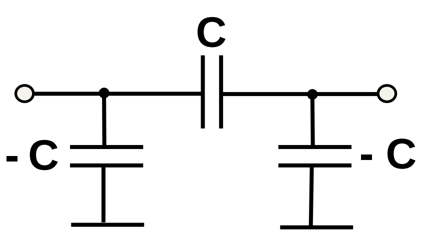


3-dB directional coupler on a combination of RH/LH TL sections

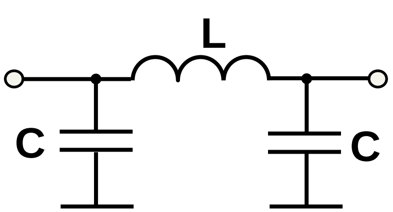




LHTL



RHTL



Conclusion

- Artificial LH TL demonstrates a propagation of backward waves with the opposite directions of the phase velocity and the group velocity.
  The LH TL is one -dimensional metamaterial
- In the LHTL, the propagation constant k is inversely proportional to the frequency ω
- There is a possibility to build more sophisticated TL using combination of LH and RH TL