Nonlinear Optical Effects in Phc and Metamaterials

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Outline

Introduction :Nl Optics

Nonlinear 1-D Photonic Crystals: enhancement of quadratic interaction.

- Fields localization & Phase matching
- Relevance of fields'overlap on frequency conversion efficiency.

Metallo-Dielectrics

Nanostructured metals

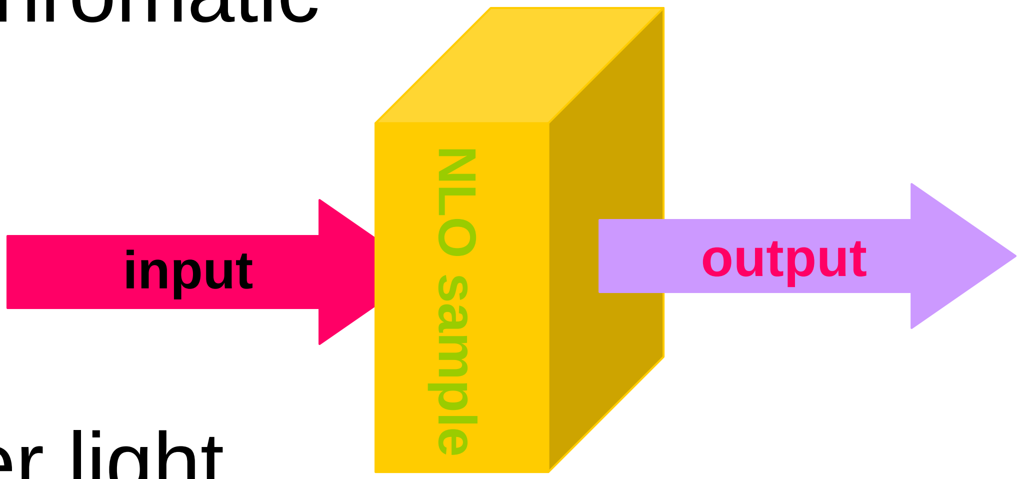
Generation of Entangled two-photon state

Conclusions

Introduction

Question:

Is it possible to change the color of a monochromatic light?

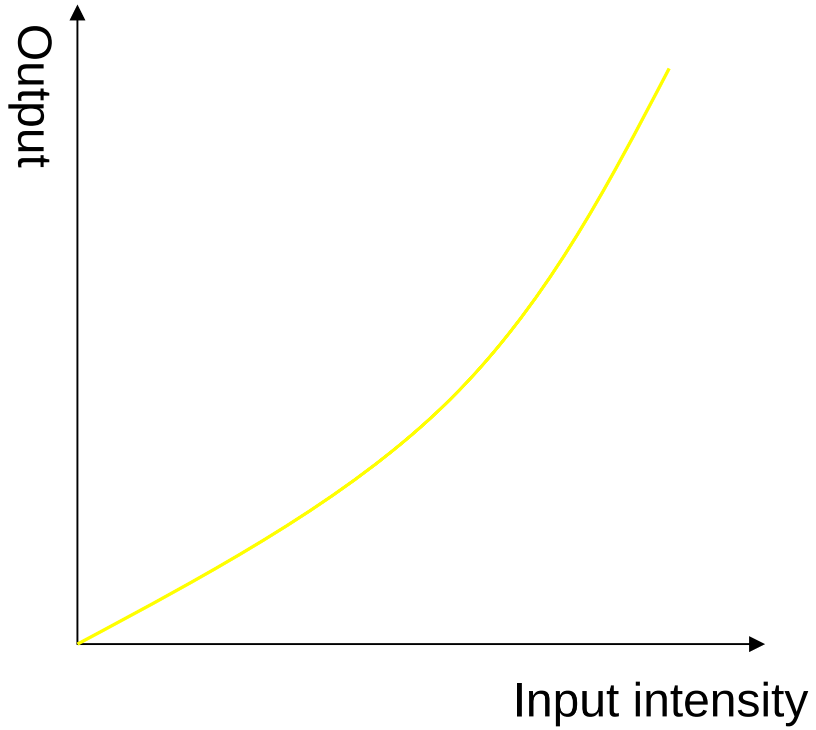


Answer:

Not without a laser light

2. The Essence of Nonlinear Optics

When the intensity of the incident light to a material system increases the response of medium is no longer linear



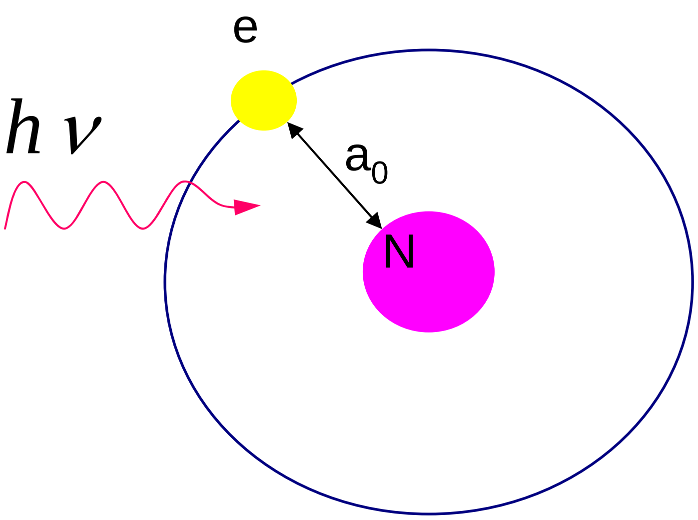
How does optical nonlinearity appear

The strength of the electric field of the light wave should be in the range of atomic fields

$$E_{at} = e / a_0^2$$

$$a_0 = \hbar^2 / me^2$$

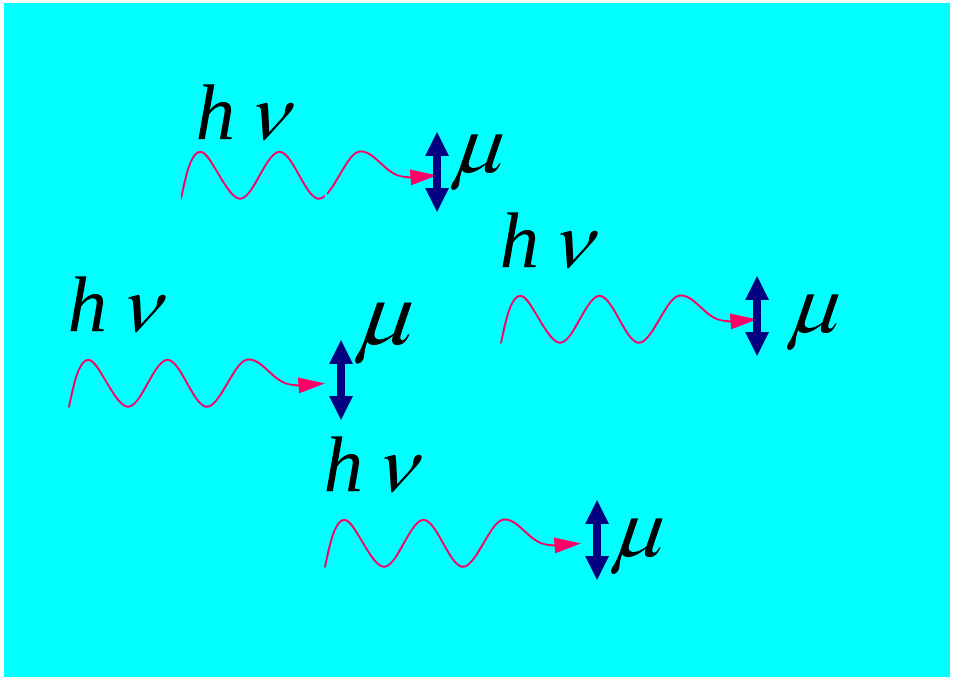
$$E_{at} \approx 2 \times 10^{=7} \text{esu}$$



Response of an optical Medium

The response of an optical medium to the incident electro magnetic field is the induced dipole moments inside the medium

$$\vec{P}(t) = \hat{\alpha}\vec{E}(t)$$



Introduction to non linear optics

When the strength of the applied field is comparable to the atomic field strength, the linear approximation is no longer valid. By performing a power expansion of P it is possible to consider the **nonlinear** terms in the polarization function:

$$\vec{P}(t) = \alpha\vec{E}(t) + \beta\vec{E}^2(t) + \gamma\vec{E}^3(t) + \ldots = \vec{P}^{(1)}(t) + \vec{P}^{(2)}(t) + \vec{P}^{(3)}(t) + \ldots$$

Lasers made available highly coherent radiation that could be concentrated and focused to give extremely high local intensities that can reach 10^{18} W/cm^2.

Discovery of second harmonic generation by Franken et al. in 1961 is considered the beginning of the field of nonlinear optics. A rich stream of new phenomena soon followed. Nonlinear optics plays an important role in telecommunications and future computer technologies. The relatively long interaction lengths and small cross sections available in waveguides and fibers means that low energy optical pulses can achieve sufficiently high peak intensities to put in evidence non linear effects also in many transparent optical materials with weak nonlinearities.

Nonlinear Polarization

- Permanent Polarization

$$P_i^0$$

- First order polarization:

$$P_i^1 = \chi_{ij}^{(1)} E_j$$

- Second order Polarization

$$P_i^2 = \chi_{ijk}^{(2)} E_j E_k$$

- Third Order Polarization

$$P_i^3 = \chi_{ijkl}^{(3)} E_j E_k E_l$$

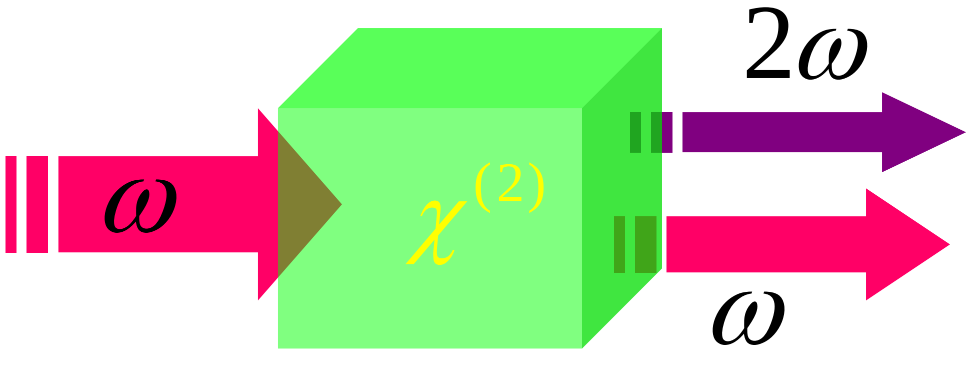
Nonlinear Optical Interactions

- The E-field of a laser beam

$$\tilde{E}(t) = Ee^{-i\omega t} + \text{C.C.}$$

- 2nd order nonlinear polarization

$$\tilde{P}^{(2)}(t) = 2\chi^{(2)}EE^{*} + (\chi^{(2)}E^{2}e^{-2i\omega t} + \text{C.C.})$$



2^{nd} Order Nonlinearities

- The incident optical field

$$\tilde{E}(t) = E_1 e^{-i\omega_1 t} + E_2 e^{-i\omega_2 t} + C.C.$$

- Nonlinear polarization contains the following terms

$$P(2\omega_1) = \chi^{(2)} E_1^2 \quad \text{(SHG)}$$

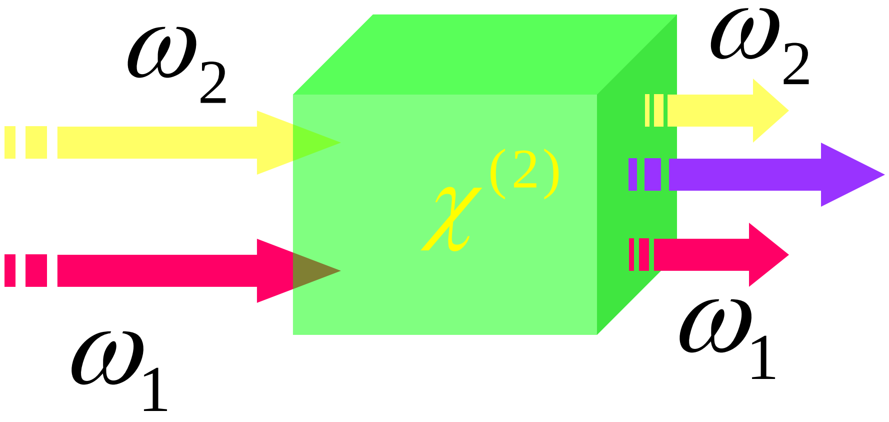
$$P(2\omega_2) = \chi^{(2)} E_2^2 \quad \text{(SHG)}$$

$$P(\omega_1 + \omega_2) = 2\chi^{(2)} E_1 E_2 \quad \text{(SFG)}$$

$$P(\omega_1 - \omega_2) = 2\chi^{(2)} E_1 E_2^* \quad \text{(DFG)}$$

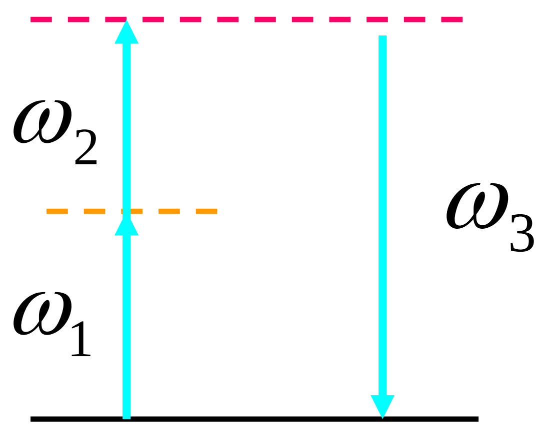
$$P(0) = 2\chi^{(2)} (E_1 E_1^* + E_2 E_2^*) \quad \text{(OR)}$$

Sum Frequency Generation

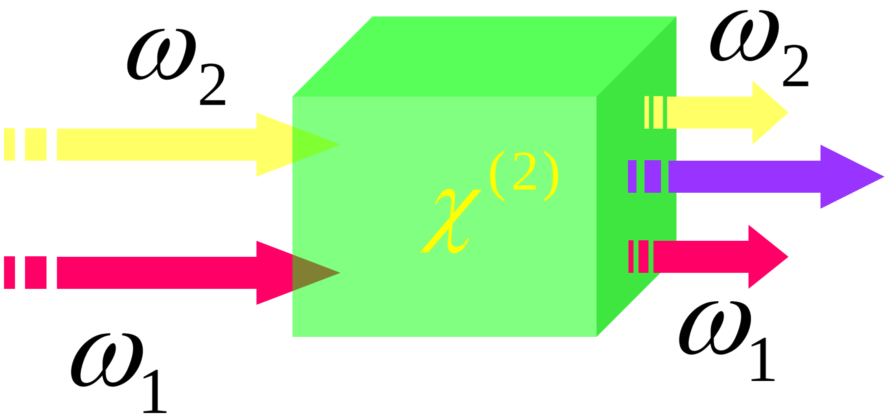


$$\omega_3 = \omega_1 + \omega_2$$

Application:
Tunable radiation in the UV Spectral region.

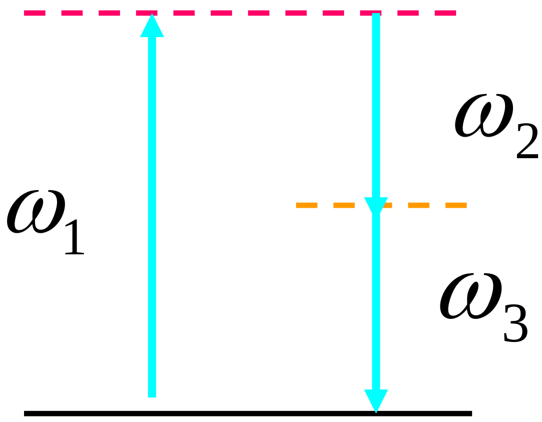


Difference Frequency Generation



$$\omega_3 = \omega_1 - \omega_2$$

Application:
The low frequency photon, ω_2 amplifies in the presence of high frequency beam ω_1. This is known as parametric amplification.



The Non linear Optical Susceptibility Tensor

Linear susceptibility tensor:

$$\vec{P}^{(1)} = \varepsilon_0 \hat{\chi}^{(1)} \vec{E}$$

For anisotropic media $\hat{\chi}^{(1)}$ is a second rank tensor with 9 components (3x3 matrix).

For isotropic media

$$\hat{\chi}^{(1)} = \chi^{(1)} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \longrightarrow \vec{P}^{(1)} = \varepsilon_0 \chi^{(1)} \vec{E}$$

Nonlinear susceptibility tensor:

$$P_i^{(2)} = \varepsilon_0 \sum_{j,k=x,y,z} \hat{\chi}_{ijk}^{(2)} E_j E_k; \quad i = x, y, z$$

$\hat{\chi}^{(2)}$ Third rank tensor (27 components) [m/V]

$$P_i^{(3)} = \varepsilon_0 \sum_{j,k,l=x,y,z} \hat{\chi}_{ijkl}^{(3)} E_j E_k E_l; \quad i = x, y, z$$

$\hat{\chi}^{(3)}$ Fourth rank tensor (81 components) [m^2/V^2]

The Non linear Optical Susceptibility Tensor

Only non centrosymmetric crystals can posses a non-vanishing second order nonlinear susceptibility tensor. In a centrosymmetric crystal, for every point (x, y, z) in the unit cell there is an indistinguishable point (-x, -y, -z). Thus a reversal of the sign of E_j and E_k must cause a reversal in the sign of $P_i^{(2)}$:

$$P_i^{(2)}\left(-E_j,-E_k\right)=\varepsilon_0\sum_{j,k=x,y,z}\hat{\chi}_{ijk}^{(2)}(-E_j)(-E_k)=-P_i^{(2)}\left(E_j,E_k\right)$$

Possible only if all the elements of the nonlinear susceptibility tensor are zero.

Second order nonlinear effects

$$\nabla^2 \vec{E} = \mu_0 \sigma \frac{\partial}{\partial t}\vec{E} + \mu_0 \varepsilon_0 \varepsilon_r \frac{\partial^2}{\partial t^2}\vec{E} + \mu_0 \varepsilon_0 \frac{\partial^2}{\partial t^2}\left(\hat{\chi}^{(2)} : \vec{E}\vec{E}\right),$$

In our one dimensional model we have:

$$\nabla^2 \vec{E}(z,t) = \frac{\partial^2}{\partial z^2}\left(\vec{E}_{(\omega_1)}(z,t) + \vec{E}_{(\omega_2)}(z,t) + \vec{E}_{(\omega_3)}(z,t)\right),$$

$$E_i^{(\omega_1)}(z,t) = \frac{1}{2}\left[E_{1i}(z)e^{i(\omega_1 t - k_1 z)} + c.c.\right]$$

$$E_k^{(\omega_2)}(z,t) = \frac{1}{2}\left[E_{2k}(z)e^{i(\omega_2 t - k_2 z)} + c.c.\right]$$

$$E_j^{(\omega_3)}(z,t) = \frac{1}{2}\left[E_{3j}(z)e^{i(\omega_3 t - k_3 z)} + c.c.\right]$$

Being *i,j,k* the Cartesian coordinates and can each take on values x and y.

Performing the second order derivative and assuming that the variation of the complex field envelopes with z are small enough so that:

$$\frac{d}{dz}E_{1i}(z)k_1 >> \frac{d^2}{dz^2}E_{1i}(z),$$

Slow varying envelope approximation
SVEA

Second order nonlinear effects

The valuation of the Laplacian within the SVEAT approximation gives:

$$\nabla^2 \vec{E} = \frac{d^2}{dz^2}\left[\vec{E}^{(\omega_1)}(z,t) + \vec{E}^{(\omega_2)}(z,t) + \vec{E}^{(\omega_3)}(z,t)\right] \cong -\frac{1}{2}\left\{\begin{array}{l} \left[k_1^2 E_{1i}(z) + 2ik_1 \dfrac{dE_{1i}(z)}{dz}\right] e^{i(\omega_1 t - k_1 z)} + \\ + \left[k_2^2 E_{2k}(z) + 2ik_2 \dfrac{dE_{2k}(z)}{dz}\right] e^{i(\omega_2 t - k_2 z)} + \\ + \left[k_3^2 E_{3j}(z) + 2ik_3 \dfrac{dE_{3j}(z)}{dz}\right] e^{i(\omega_3 t - k_3 z)} \end{array}\right\}$$

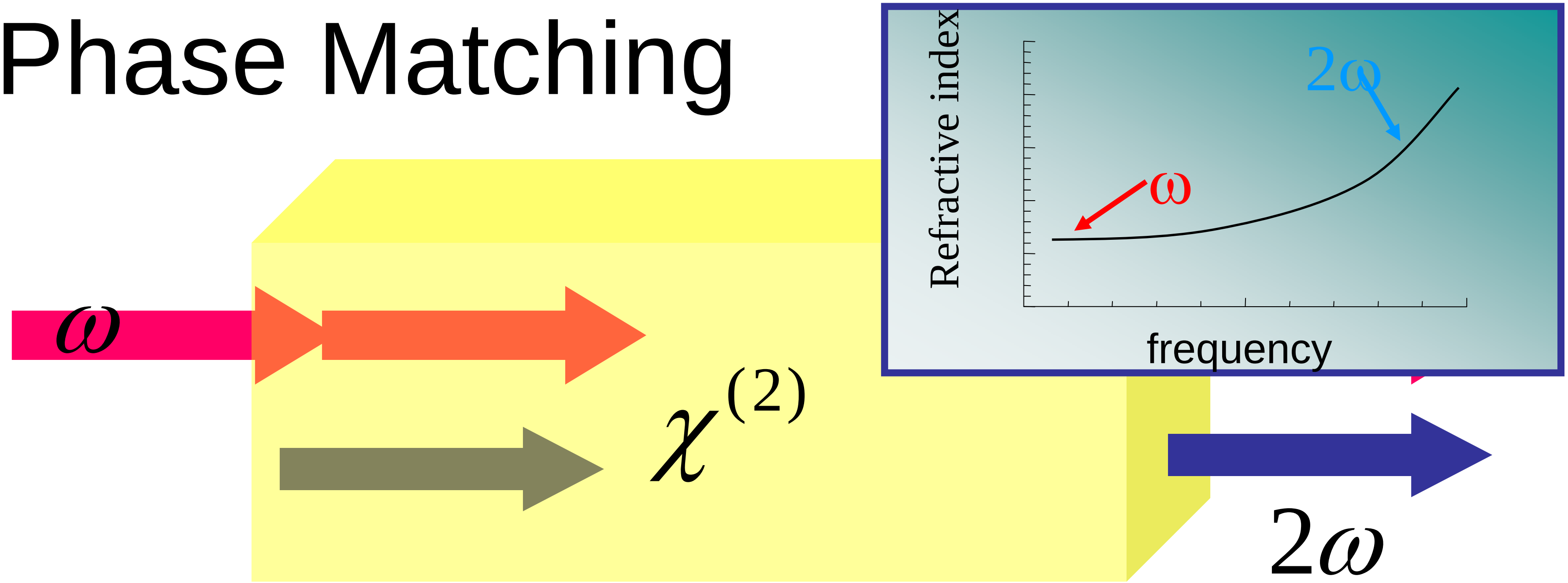
Finally, with some algebra:

$$\frac{dE_{1i}}{dz} = -\frac{\sigma_1}{2}\sqrt{\frac{\mu_0}{\varepsilon_0 \varepsilon_{r1}}} E_{1i} - \frac{i\omega_1}{2}\sqrt{\frac{\mu_0 \varepsilon_0}{\varepsilon_{r1}}}\left(\sum_{j,k=x,y} \hat{\chi}'^{(2)}_{ijk} E_{3j} E^*_{2k}\right) e^{-i(k_3 - k_2 - k_1)z}$$

$$\frac{dE^*_{2k}}{dz} = -\frac{\sigma_2}{2}\sqrt{\frac{\mu_0}{\varepsilon_0 \varepsilon_{r2}}} E^*_{2k} + \frac{i\omega_2}{2}\sqrt{\frac{\mu_0 \varepsilon_0}{\varepsilon_{r2}}}\left(\sum_{i,j=x,y} \hat{\chi}'^{(2)}_{kij} E_{1i} E^*_{3j}\right) e^{-i(k_1 - k_3 + k_2)z}$$

$$\frac{dE_{3j}}{dz} = -\frac{\sigma_3}{2}\sqrt{\frac{\mu_0}{\varepsilon_0 \varepsilon_{r3}}} E_{3j} - \frac{i\omega_3}{2}\sqrt{\frac{\mu_0 \varepsilon_0}{\varepsilon_{r3}}}\left(\sum_{i,k=x,y} \hat{\chi}'^{(2)}_{jik} E_{1i} E_{2k}\right) e^{-i(k_1 + k_2 - k_3)z}$$

Phase Matching



- Since the optical (NLO) media are dispersive,
  The fundamental and the harmonic signals have different propagation speeds inside the media.

- The harmonic signals generated at different points interfere destructively with each other.

Second order nonlinear effects

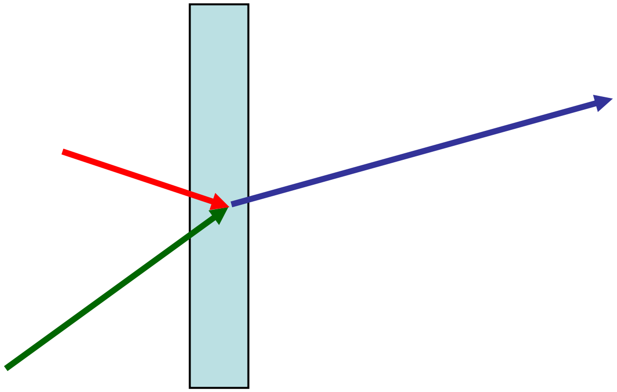
NOTE: In order to have efficient energy transfer among the fields, a long interaction length is required, the phase mismatch should be as close as possible to zero. In other words:

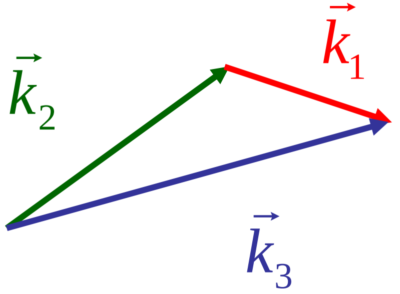
$$\mathbf{k_3=k_2+k_1}$$

In a more general case, the condition to be fulfilled is:

$$\vec{k}_3 = \vec{k}_1 + \vec{k}_2$$

It is called phase matching condition and it can be interpreted as a momentum conservation requirement. Example: Two pump fields at frequencies w1 and w2 can generate a sum frequency (ω_3) field. The wavevector of the generated field will fulfill:





Optical Second harmonic Generation

To obtain an expression for the second harmonic power output, considering A linearly polarized pump with non vanishing electric field component E_{1x} we use the relation:

$$\frac{P^{(2\omega)}}{Area} = \frac{1}{2}\varepsilon_0 c\sqrt{\varepsilon_{r3}}E_{3j}E_{3j}^*$$

If we consider the SH generated linearly polarized with non vanishing field component E_{3y}we can write an expression for the conversion efficiency:

$$\eta = \frac{P^{(2\omega)}}{P^{(\omega)}} = \frac{1}{2}\frac{\omega^2 L^2 \left(\chi_{yxx}'^{(2)}\right)^2}{\varepsilon_0 c^3 n_{2\omega} n_{\omega}^2}\frac{P^{(\omega)}}{Area}\left[\frac{\sin(\Delta kL/2)}{\Delta kL/2}\right]^2;$$

The conversion efficiency is linearly growing with the pump intensity. This means that the generated second harmonic intensity with respect to the pump intensity follows a quadratic law. The conversion efficiency increases with the squared length of the nonlinear medium

Optical Second harmonic Generation

First experimental report on second harmonic generation was performed by Franken, Hill, Peters and Weinreich in 1961. (Fig: A. Yariv, Quantum electronics)

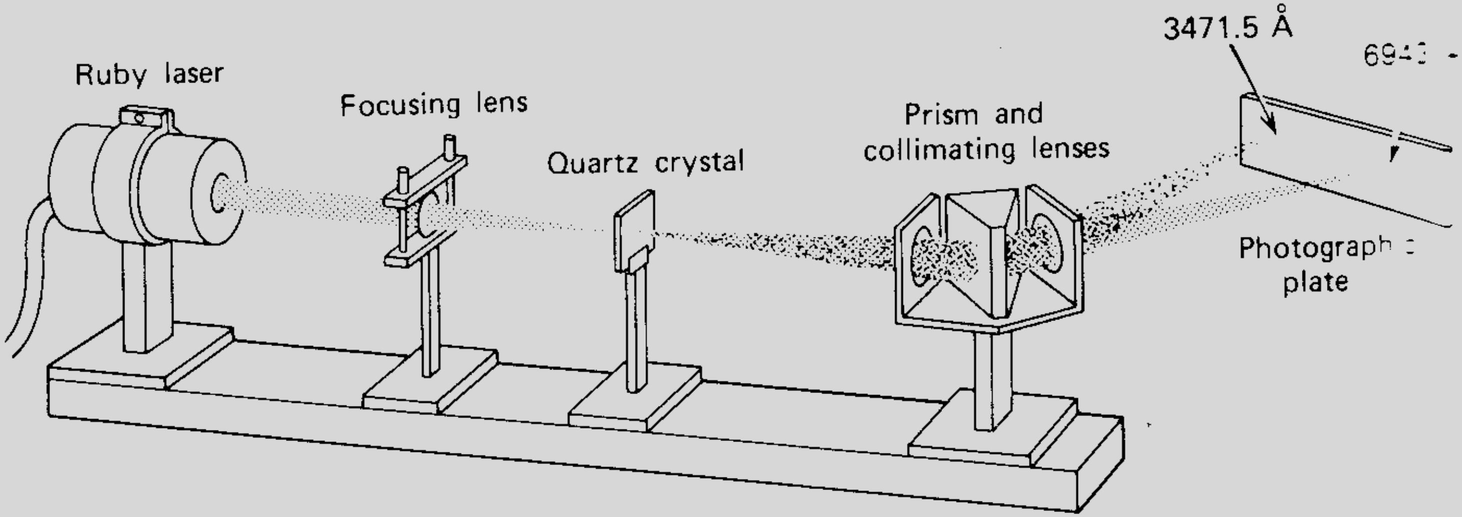
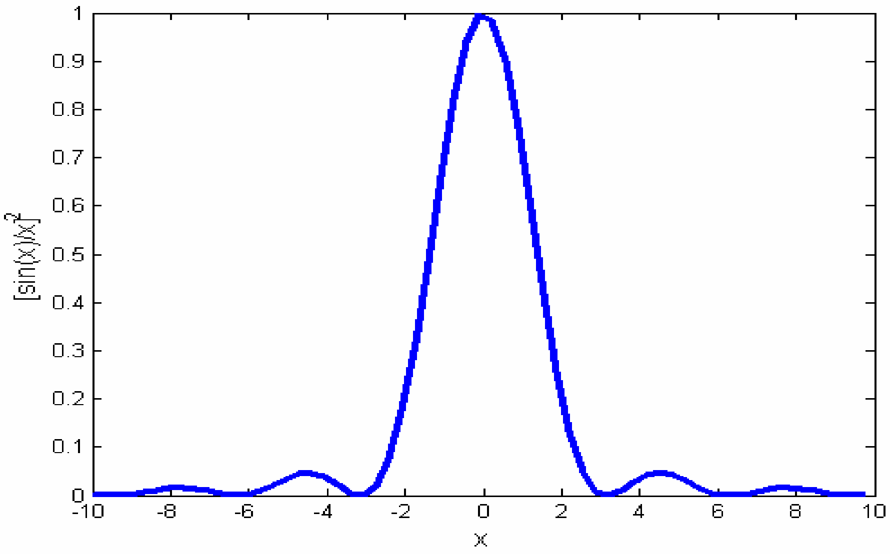


FIGURE 16.4 Arrangement used in first experimental demonstration of second-harmonic

SHG can be studied as the limiting case of the three frequency interaction where two of the frequencies ω_1 and ω_2 are equal and $\omega_3=2\omega_1$.
We assume as first approximation that the amount of power lost by the input beam is negligible so that $dE_{1i}/dz=0$ and the medium is transparent at ω_3 ($\sigma=0$)

Phase matching

We note that the conversion efficiency is crucially determined by the sinc function:



Maximum conversion efficiency is achieved for $\Delta kL/2=0$,

$\mathbf{\Delta k=k_{2\omega}-2k_{\omega}=0}$ is called phase matching condition and it is fulfilled as long as The refractive indices at FF and SH frequencies are the same: $\mathbf{n_{2\omega}=n_{\omega}}$.

If such condition is fulfilled, the FF and SH fields propagate with the same phase velocities

Phase matching

In common materials the refractive index is an increasing function of the frequency thus we have $n_\omega < n_{2\omega}$, and **Δk>0**.

During the non linear process there is an exchange of energy between FF and SH field and the SH production is zero for any propagation length that satisfy the law:

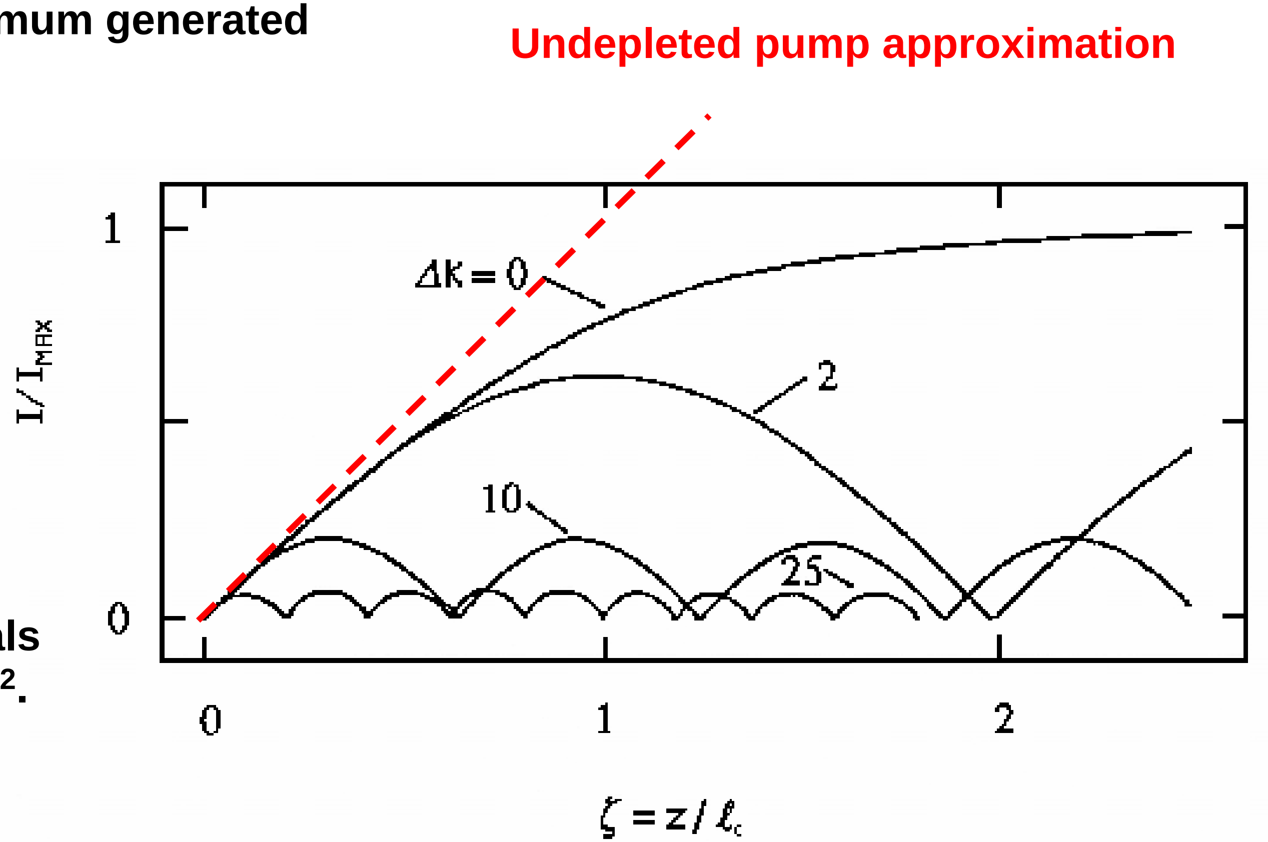
$\Delta kL/2 = m\pi$, where m is an integer.

For a mismatched interaction, the length of non linear material that produce the maximum generated second harmonic field can be calculated by:

$$\frac{\Delta k l_c}{2} = \frac{\pi}{2};$$

$$l_c = \frac{\pi}{\Delta k} = \frac{\lambda_0}{4\left(n_{2\omega} - n_\omega\right)}$$

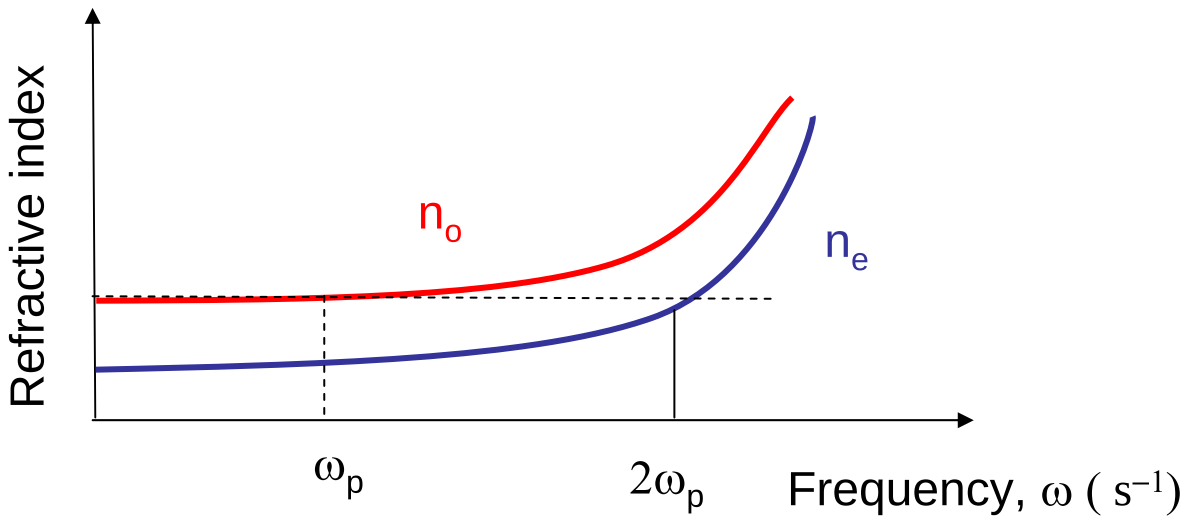
Typically in nonlinear materials Δn is of the order of $10^{-1}$to 10^{-2}. Coherence lengths are only a few numbers of wavelenghts.



Phase matching techniques

A widely used technique takes advantage of the natural birefringence of anisotropic crystals.

Under certain circumstances it is possible to use the different refractive indices for the ordinary wave and the extraordinary wave. For example, a typical behaviour of dispersion of the refractive indices of a negative ($n_e<n_o$) uniaxial crystal is:



If $n_e(2\omega_p)<n_o(\omega_p)$, it exists an angle θ_m at which $n_e^{2\omega}(\theta_m)=n_o^{\omega}$. It can be calculated by:

$$\frac{1}{n_e^2(\theta)} = \frac{\cos^2\theta}{n_o^2} + \frac{\sin^2\theta}{n_e^2}$$

Quasi Phase matching

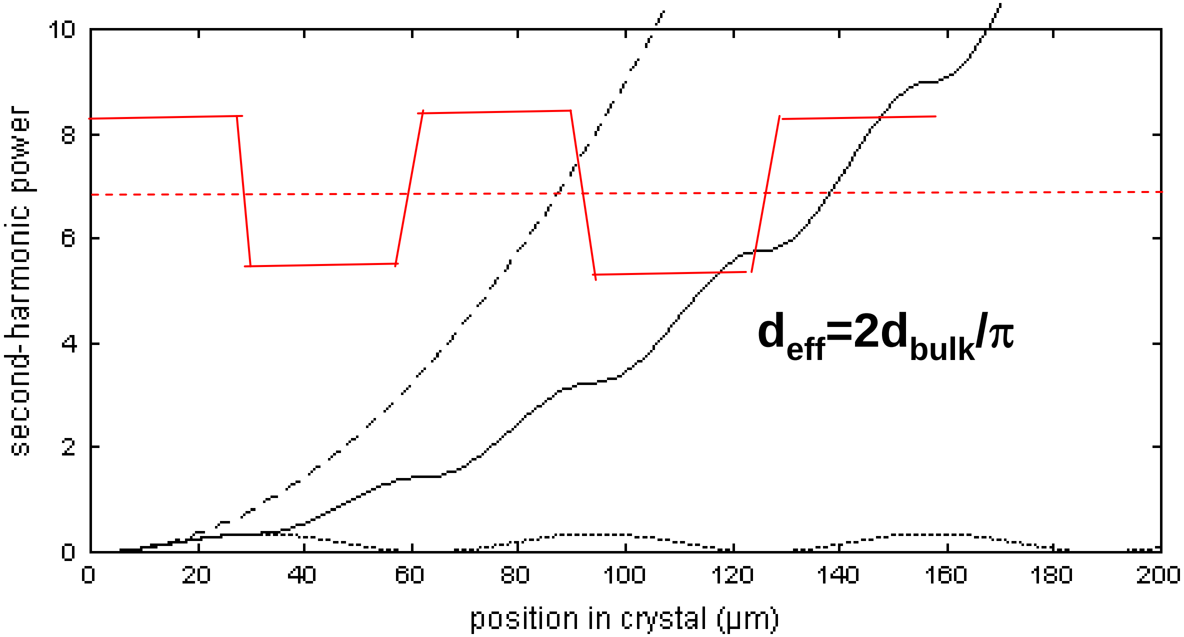
Periodic modulation of the non linear coefficients tensor elements responsible for the interaction. It can be shown that the phase matching condition becomes:

$$\Delta k = 2\pi m/\Lambda$$

Where m is an integer and Λ is the period of the nonlinearity.

EXAMPLE: If the sign of the non linear interaction is reversed at every coherence length d(z) is a periodic function of period $2l_c = 2\pi/\Delta k$. QPM is achieved for m=1.

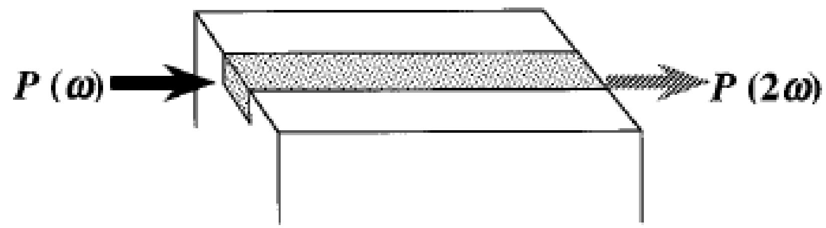
Periodically poled $LiNbO_3$
Periodical reversal of electric field in order to induce a permanent periodically modulated electric polarization



Phase matching techniques

Considering dielectric waveguides, the modal dispersion can be used to achieve phase matching. The phase matching relation for second harmonic generation becomes:

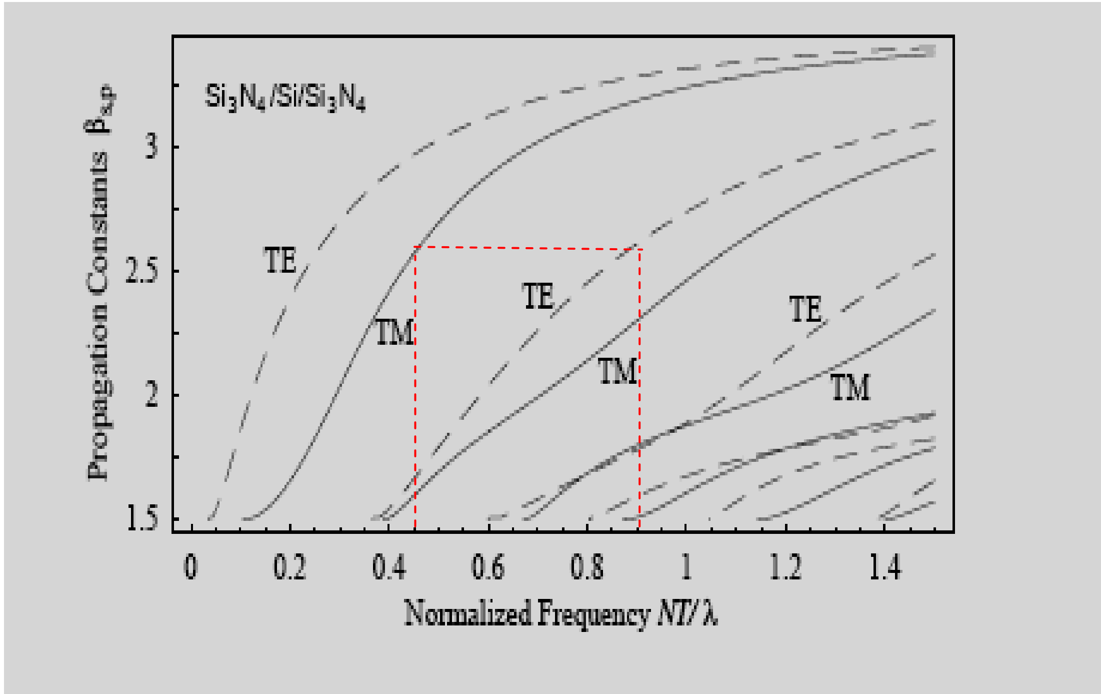
$$\tilde{\beta}_{2\omega} = \tilde{\beta}_{\omega}$$



This condition can be easily fulfilled by considering modes for the pump and for the SH of different order or polarization.

Nevertheless the non linear process is governed by how the fields overlap: usually overlaps between fields belonging to different orders are not efficient.

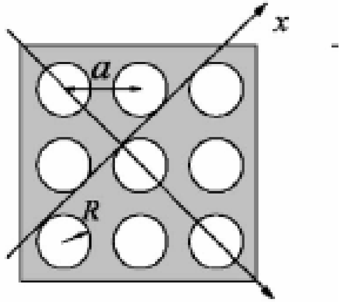
$$\eta \propto \left[\int_{-\infty}^{+\infty} \hat{\chi}^{(2)} : \left(\vec{E}_m^{\omega}(x) \right)^2 \cdot \left(\vec{E}_n^{2\omega}(x) \right)^* dx \right]^2$$



Photonic Crystals

In a photonic crystal both the linear and the nonlinear susceptibility functions can be periodically modulated. Modulation of the linear susceptibility is responsible for peculiar properties of the linear dispersion curves of these structures. Fields are characterized by a Bloch wave vector and an periodic function over the unit cell. Thus the quasi momentum conservation for second harmonic generation is:

$$\vec{k}_{2\omega} - \vec{k}_{\omega} - \vec{G} = \vec{0}$$



(a)

Where **G** is a reciprocal lattice vector. Usually unit cells are of the order of the wavelength or less unlike in the QPM regime where domains are inverted every coherence length.

Three wave mixing

Summary

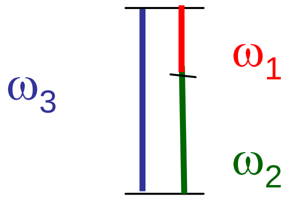
The basic equations govern the interaction of three fields due to second order nonlinearity:

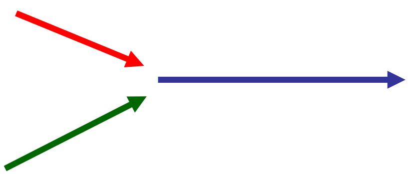
$$\frac{dE_1}{dz} = -\frac{i\omega_1}{2}\sqrt{\frac{\mu_0\varepsilon_0}{\varepsilon_{r1}}}\hat{\chi}'^{(2)} E_3 E_2^* e^{-i\Delta kz}$$

$$\frac{dE_2^*}{dz} = +\frac{i\omega_2}{2}\sqrt{\frac{\mu_0\varepsilon_0}{\varepsilon_{r2}}}\hat{\chi}'^{(2)} E_1 E_3^* e^{i\Delta kz}$$

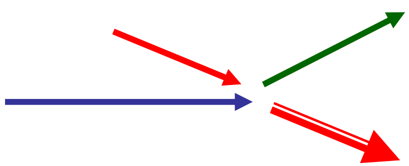
$$\frac{dE_3}{dz} = -\frac{i\omega_3}{2}\sqrt{\frac{\mu_0\varepsilon_0}{\varepsilon_{r3}}}\hat{\chi}'^{(2)} E_1 E_2 e^{-i\Delta kz}$$

Different processes can happen depending on the initial condition:

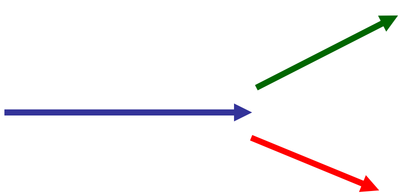




Sum frequency generation
Frequency up-conversion



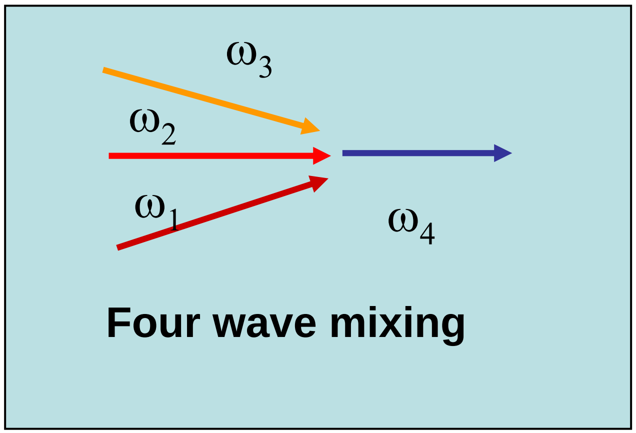
Parametric amplification
Difference frequency generation
Stimulated parametric down conversion

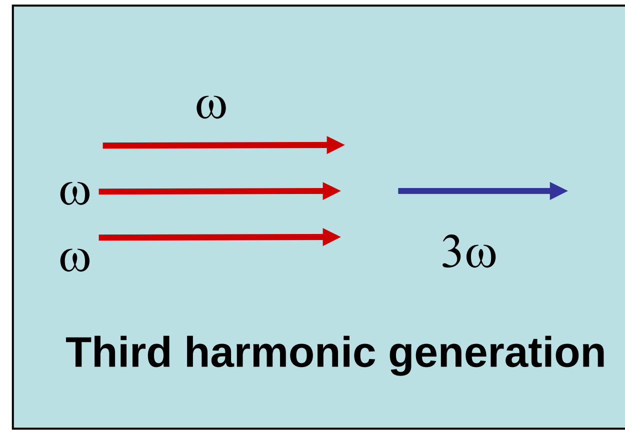


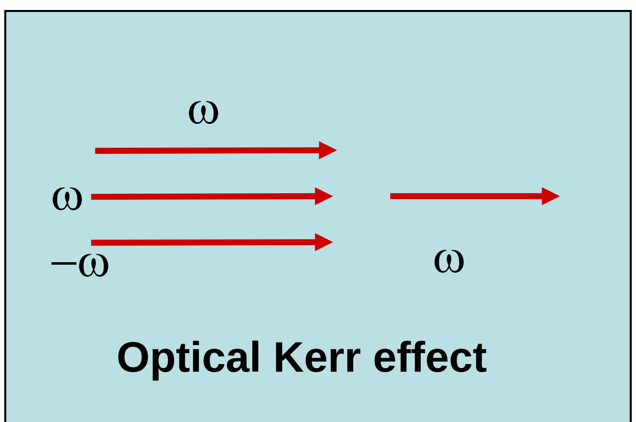
Spontaneous parametric down conversion
Spontaneous parametric fluorescence

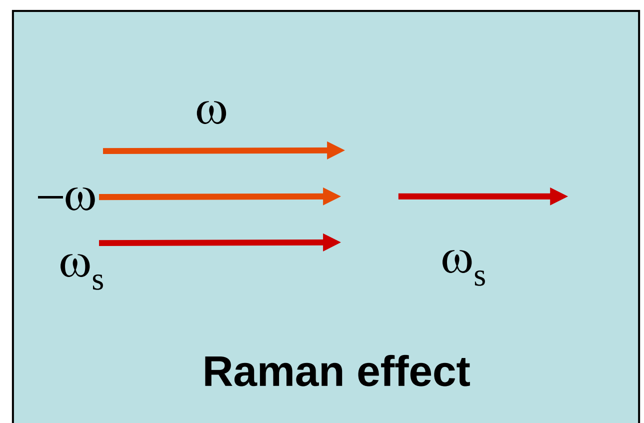
Third order optical nonlinearities

Third order nonlinear polarization allow coupling of fields at different frequencies, Here we present an sketched overview of the most relevant effects that will be analyzed later:









Intensity dependence of the refractive index

The refractive index of many materials depends on the intensity of the incident optical field. If we consider an optical field of the form:

$$\vec{E}(t) = \frac{1}{2}\left[E(\omega)e^{-i\omega t} + c.c\right]\hat{x}$$

Time averaged intensity is defined:

$$I = \frac{1}{2}c\varepsilon_0 n_0 \left|E(\omega)\right|^2$$

The nonlinear refractive index is defined:

$$\tilde{n}\left(\omega, \left|E(\omega)\right|^2\right) = n_0(\omega) + n_2\left|E(\omega)\right|^2$$

For instantaneous response:

$$\vec{P}_{NL}(t) = \varepsilon_0 \hat{\chi}^{(3)} \vdots \vec{E}(t)\vec{E}(t)\vec{E}(t)$$

Intensity dependence of the refractive index

Intensity dependence of the refractive index has drastic effects both on the spatial properties of optical beams and on temporal (spectral) properties of ultrashort pulses.

Self phase modulation (SPM)

Phase shift experienced by short pulse propagating through a nonlinear refractive index

- spectral broadening;
- Optical solitons in anomalous dispersion regimes of fibres.

Cross phase modulation (XPM)

Phase shift of a field induced by a co-propagating intense field at different wavelength.

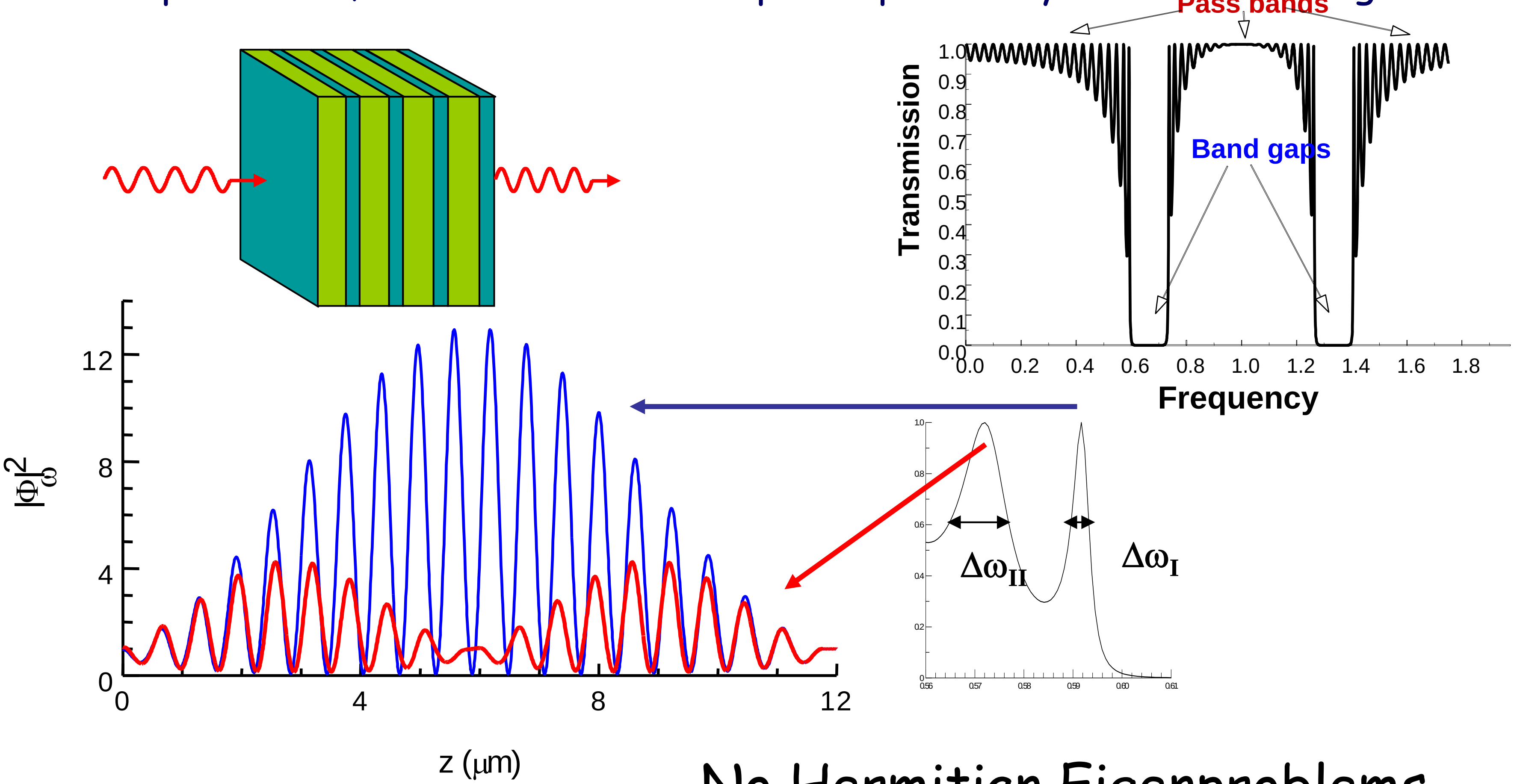
Self focusing and self defocusing of optical beams

Depending on the sign on the nonlinearity, the central (more intense) portion of a Beam experiences higher (lower) index of refraction with respect to the outer edges.An effective lens is created.

Photonic Crystals

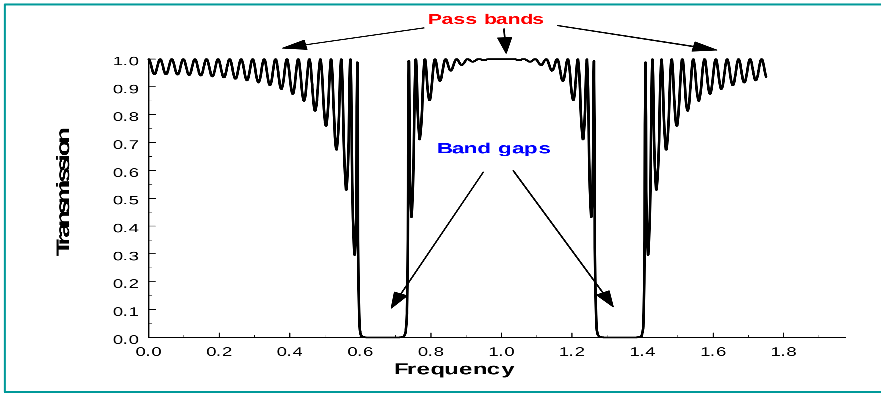
1D PCh - Band Edge Effects

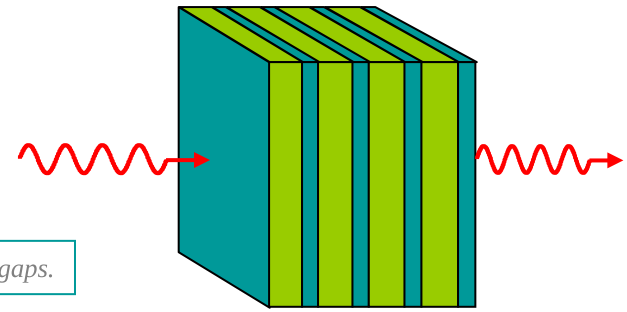
Finite size 1D photonic crystals transmission bands exhibit resonance peaks due to the boundary conditions with the external (homogenous) world. In particular, resonances are sharper in proximity of the band edge.



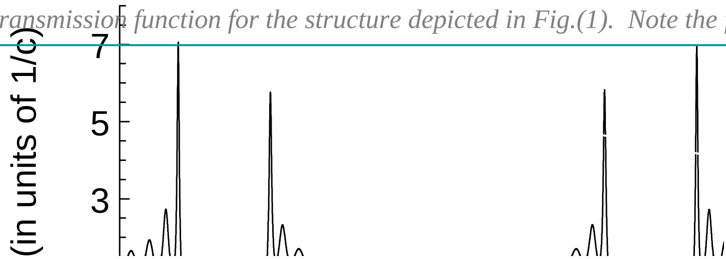
No Hermitian Eigenproblems

No Hermitian Eigenproblems





:Transmission function for the structure depicted in Fig.(1). Note the frequency pass bands and band gaps.



$$t(\omega) = x(\omega) + i\,y(\omega) = \sqrt{T}\,e^{i\phi_t}$$

$$\phi_t = \tan^{-1}(y/x) \pm m\pi$$

$$\phi_t = k(\omega)D = \frac{\omega}{c} n_{eff}(\omega)D$$

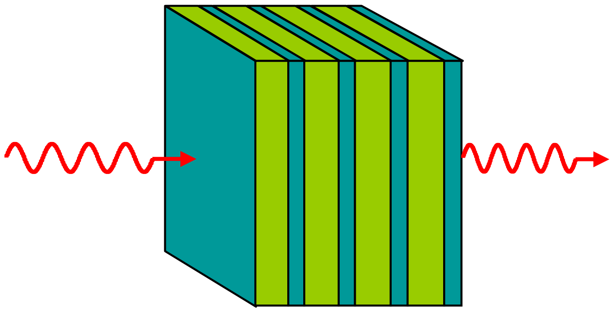
Density of modes :

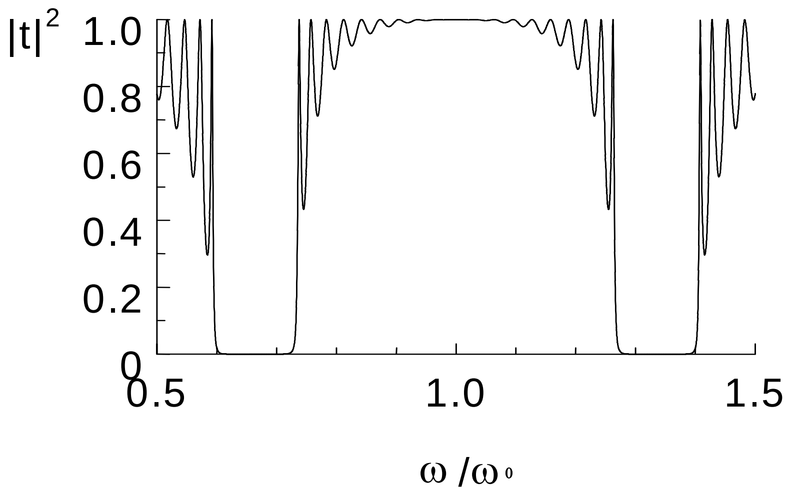
dφ/dw = (1/D)dk/d w

DOM = dk/dw=

(1/D)(y'x-x'y)/(x^2+y^2)

1D PCh - Band Edge Effects - Group velocity





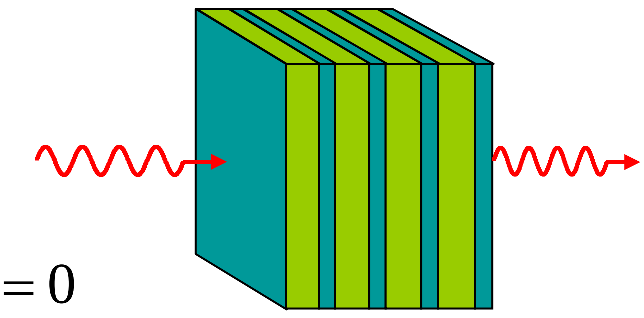
$$t(\omega) = x(\omega) + i\,y(\omega) = \sqrt{T}\,e^{i\phi_t}$$

$$\phi_t = \tan^{-1}(y/x) \pm m\pi$$

$$\phi_t = k(\omega)D = \frac{\omega}{c} n_{eff}(\omega) D$$

Vg= dw/dk=1/DOM

1D PCh - Band Edge Effects - Energy velocity



$$\frac{d^2 E_\omega}{dz^2} + \frac{\omega^2 \varepsilon_\omega(z)}{c^2} E_\omega = 0$$

The boundary conditions at the input *(z=0)* and output *(z=L)* surfaces are:
$E^I_w + E^r_w = E_w(0)$; $E^t_w = E_w(L)\exp[-i(w/c)L)]$;
$i(w/c)(E^I_w - E^r_w) = dE_w(0)/dz$;
$i(w/c)E^t_w = (dE_w(L)/dz)\exp[-i(w/c)L)$.

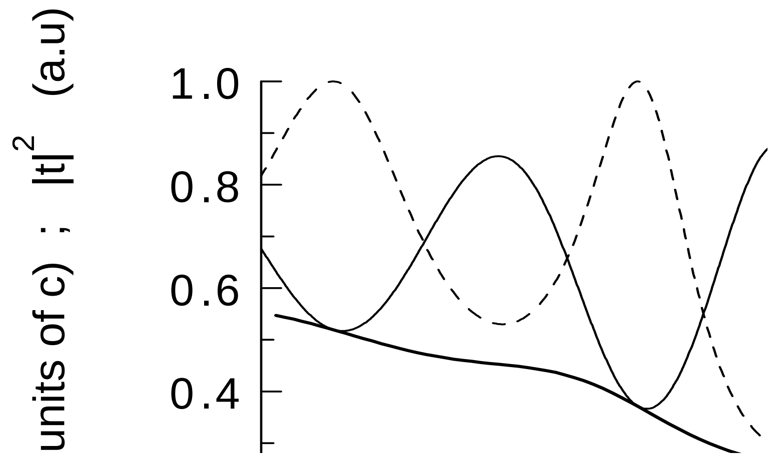
$\Phi_\omega(z) = E_\omega(z)/E^I_\omega$; $t_\omega = (E^t_\omega \exp[i(\omega/c)L]/E^I_\omega$;

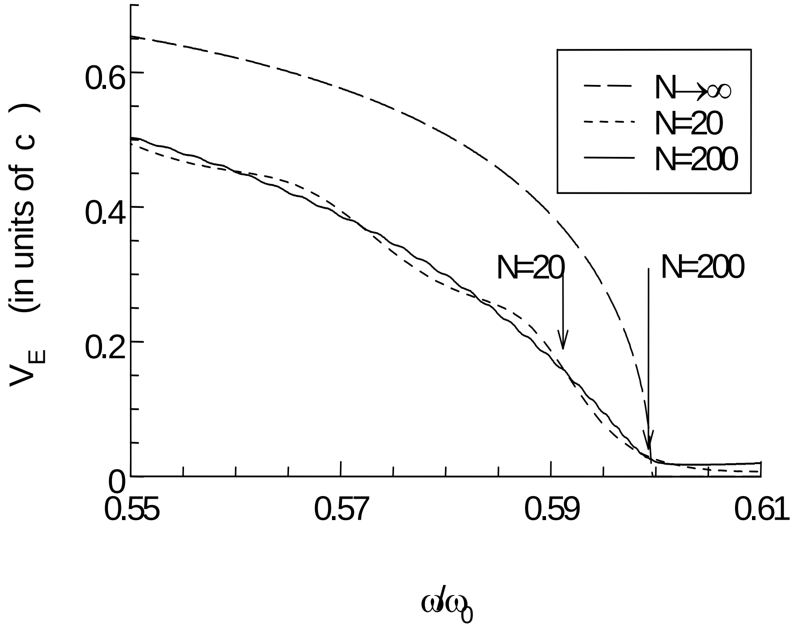
No Hermitian Eigenproblems

The energy velocity is defined as the ratio of the Poynting vector integrated over the volume to the average total energy stored in the same volume

$$V_E^{(\omega)} = \frac{\mathrm{Re}\left[-ic^2 \int_0^L \Phi^*_\omega \frac{d\Phi_\omega}{dz} dz\right]}{\omega \int_0^L \varepsilon_\omega(z) |\Phi_\omega|^2 dz}$$

$$V_E^{(\omega)} = \left|t_\omega\right|^2 V_g^{(\omega)}$$



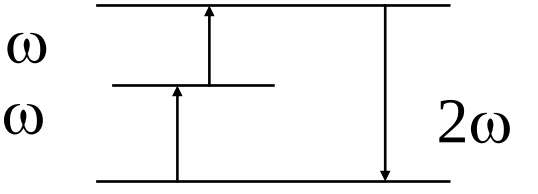


G.D'Aguanno et al

Nonlinear Interaction

$$P = \varepsilon_0 \chi E$$
$$= \varepsilon_0 \chi^{(1)} E + \varepsilon_0 \chi^{(2)} E^2 + \varepsilon_0 \chi^{(3)} E^3 + \cdots$$

-$\chi^{(2)}$ – Nonlinear frequency Conversion- SHG

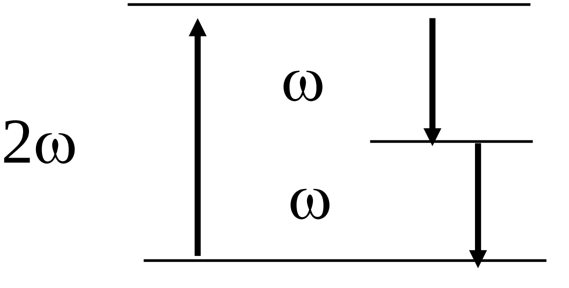


Pnl(2ω) $\rightarrow$ $\chi^{(2)} E^2(\omega)$

-Down Conversion

In Bulk:

PM and χ(2)



Nonlinear quadratic Interaction in PhC

- $\chi^{(2)}$, interface , bulk (in each layer)
- P.M. (equal PHASE velocity),

  For SH $2K_1 = K_2$

  $n(w) = n(2\,w)$

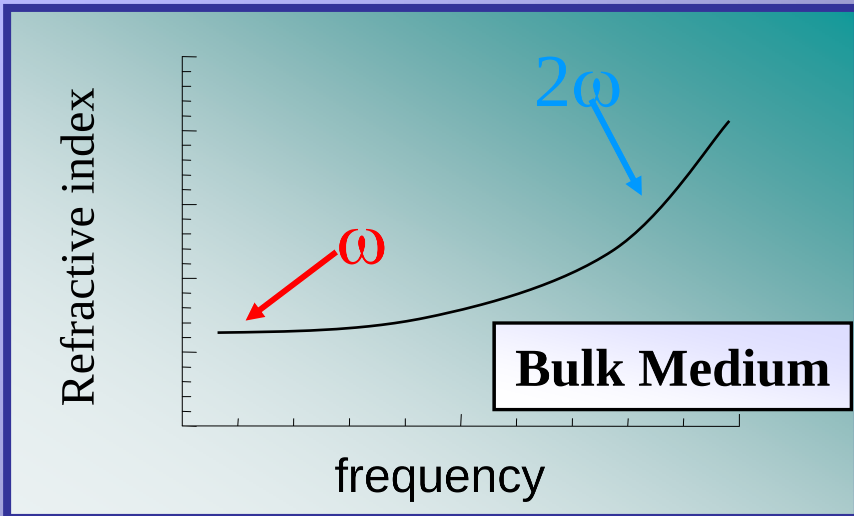
  thanks to the geometrical dispersion
- Field enhancement due to *field localization*

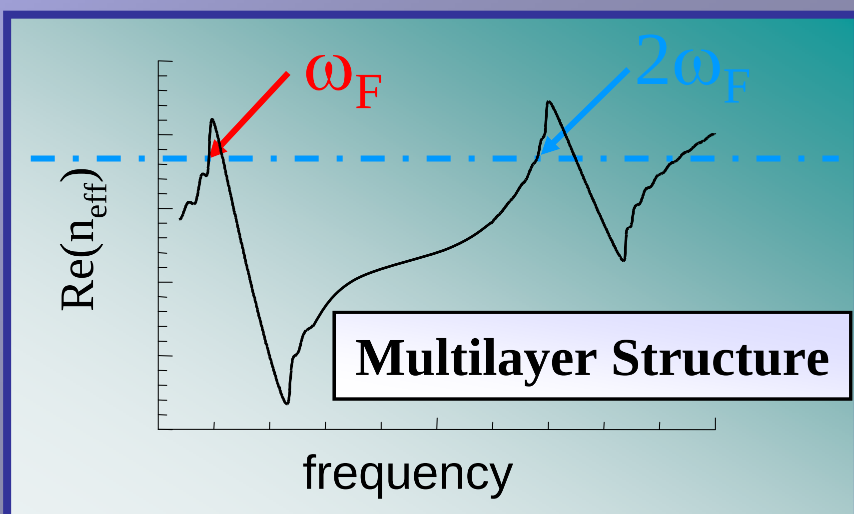
  (band edge and when T=1)
- Large conversion efficiency

M.Scalora et al 1997

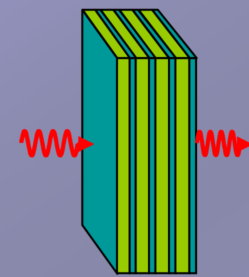
Phase Matching

Effective refractive index





$$t = x + iy \equiv \sqrt{T}\, e^{i\varphi_t} \equiv e^{i\Phi}$$



$$\varphi_t = tg^{-1}\left(\frac{y}{x}\right) \pm m\pi$$

$$\Phi = \varphi_t - i\ln\sqrt{T} \equiv \hat{k}_{eff} D \equiv \frac{\omega}{c}\hat{n}_{eff} D$$

$$\hat{n}_{eff} = \frac{c}{\omega D}\left[\varphi_t - \frac{i}{2}\ln(x^2 + y^2)\right]$$

*Centini et al, Phys. Rev. E 60, 4891-4898 (1999)

But

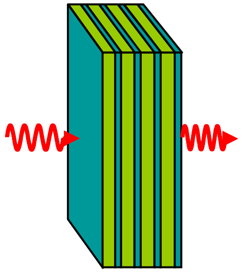
Field overlap more important than P.M !!!!

Conversion Efficiency

The expression of the conversion efficiency ($\eta = I_{SH} / I_{pump}$)
in the non-depleted pump regime is similar to the bulk case

$$\eta^{(+,-)} = \frac{8\pi^2 \left| d_{eff}^{(+,-)} \right|^2 L^2 I_{pump}}{\varepsilon_0 c \lambda^2 n_{eff}^{(\omega,p)} n_{eff}^{(\omega,s)} n_{eff}^{(2\omega,p)}}$$

$$\tilde{d}_{eff}^{(+,-)} = \frac{1}{L} \int_0^D \chi^{(2)}(z) \Phi_{\omega}^{2(+)}(z) \Phi_{2\omega}^{*(+,-)}(z) dz$$



d_{eff} contains both information on PM conditions and fields overlap

***G. D'Aguanno, et al. J. Opt. Soc. Am. B 19, 2111 (2002)**

Phase Matching

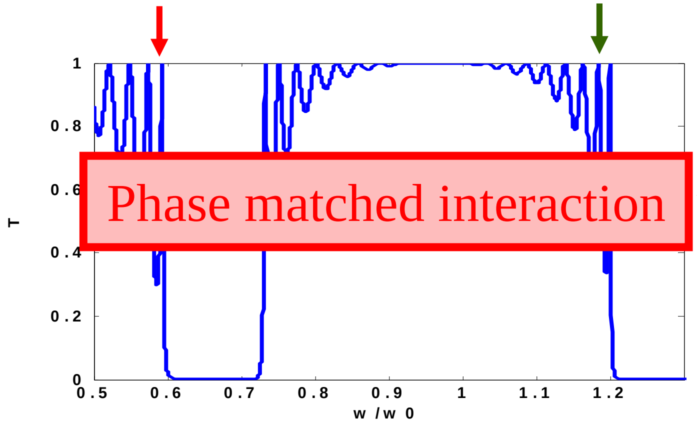
Fields' Overlap

$$\tilde{d}_{eff}^{(+,-)} = \frac{1}{L}\int_0^D \chi^{(2)}(z)\Phi_{\omega}^{2(+)}(z)\Phi_{2\omega}^{*(+,-)}(z)dz$$

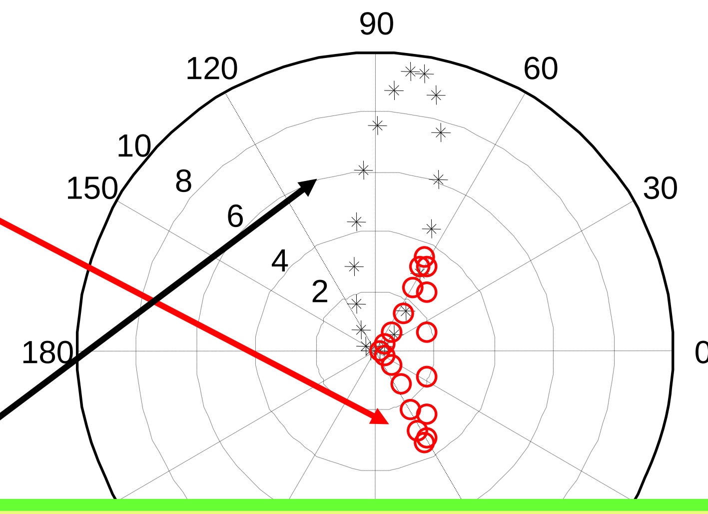
$$\eta^{(+,-)} = \frac{8\pi^2 \left|d_{eff}^{(+,-)}\right|^2 L^2 I_{pump}}{\varepsilon_0 c \lambda^2 n_{eff}^{(\omega,p)} n_{eff}^{(\omega,s)} n_{eff}^{(2\omega,p)}}$$

NL layers $=\lambda_0/4$
Linear layers $= \lambda_0/2$

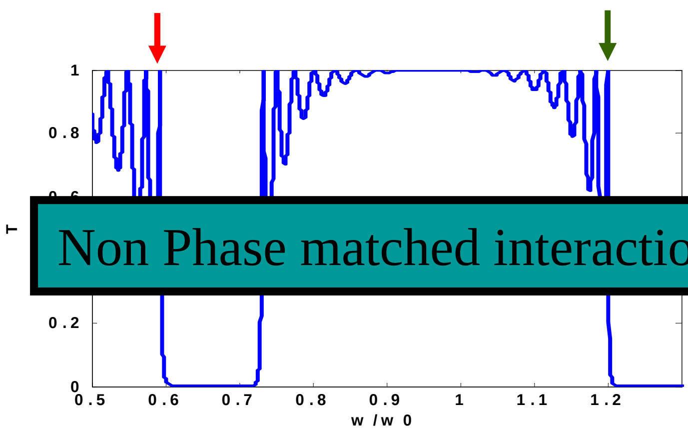
n1=1; n2(ω_F)=1.428; n2($2\omega_F$)=1.616.



Phase matched interaction



n1=1; n2(ω_F)=1.428; n2($2\omega_F$)=1.676.



Non Phase matched interaction

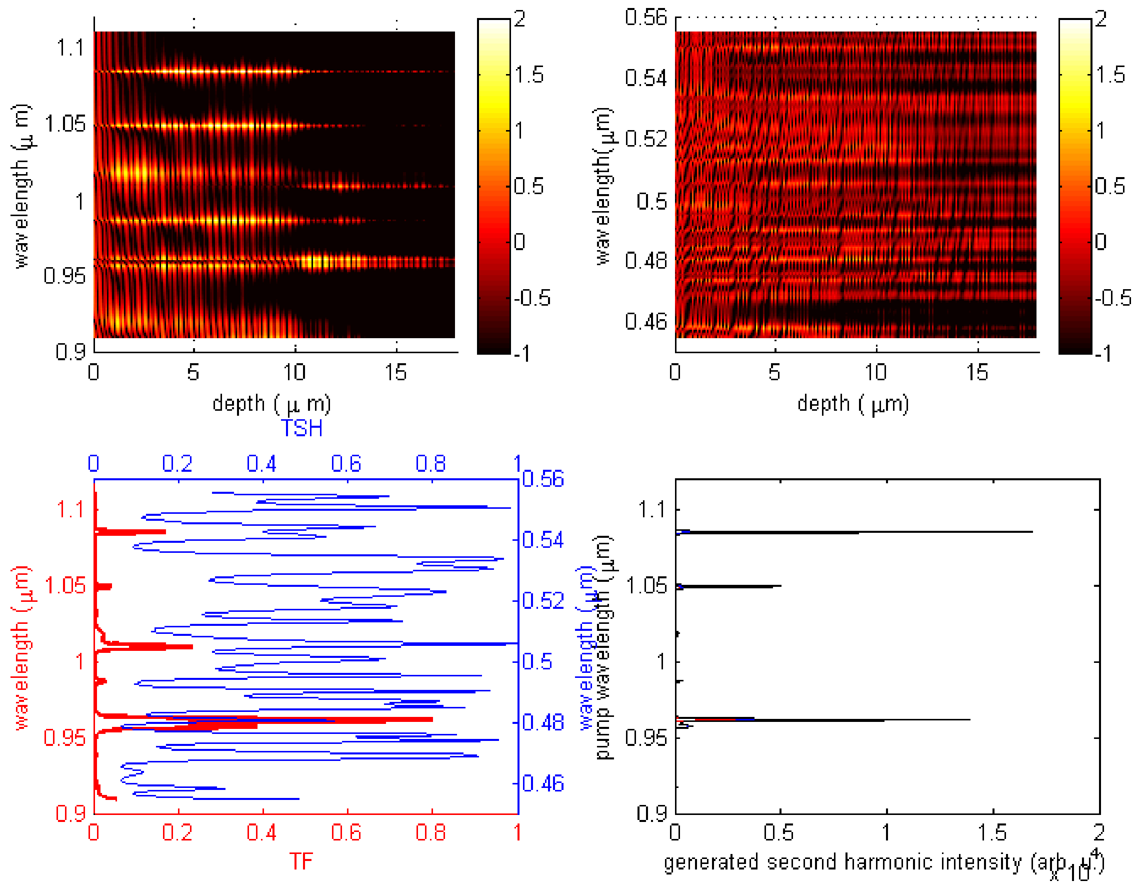
For periodic finite structures, tuning the pump at the band edge resonance, PM is always achieved if the second harmonic is tuned to the second resonance*

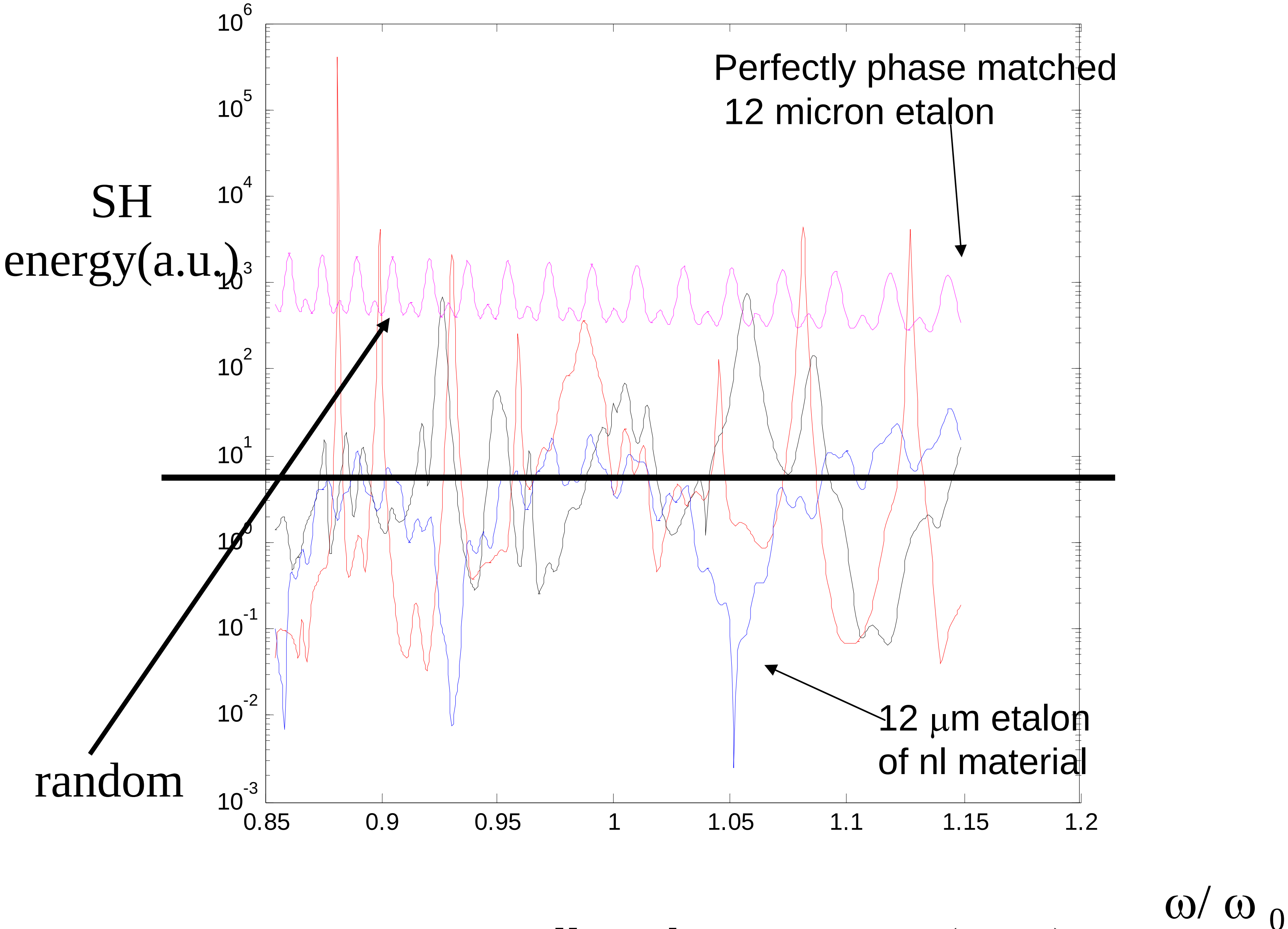
* Centini et al. Opt. Letters (2004)

* Centini et al. Phys. Rev. E **60**, 4891 (19

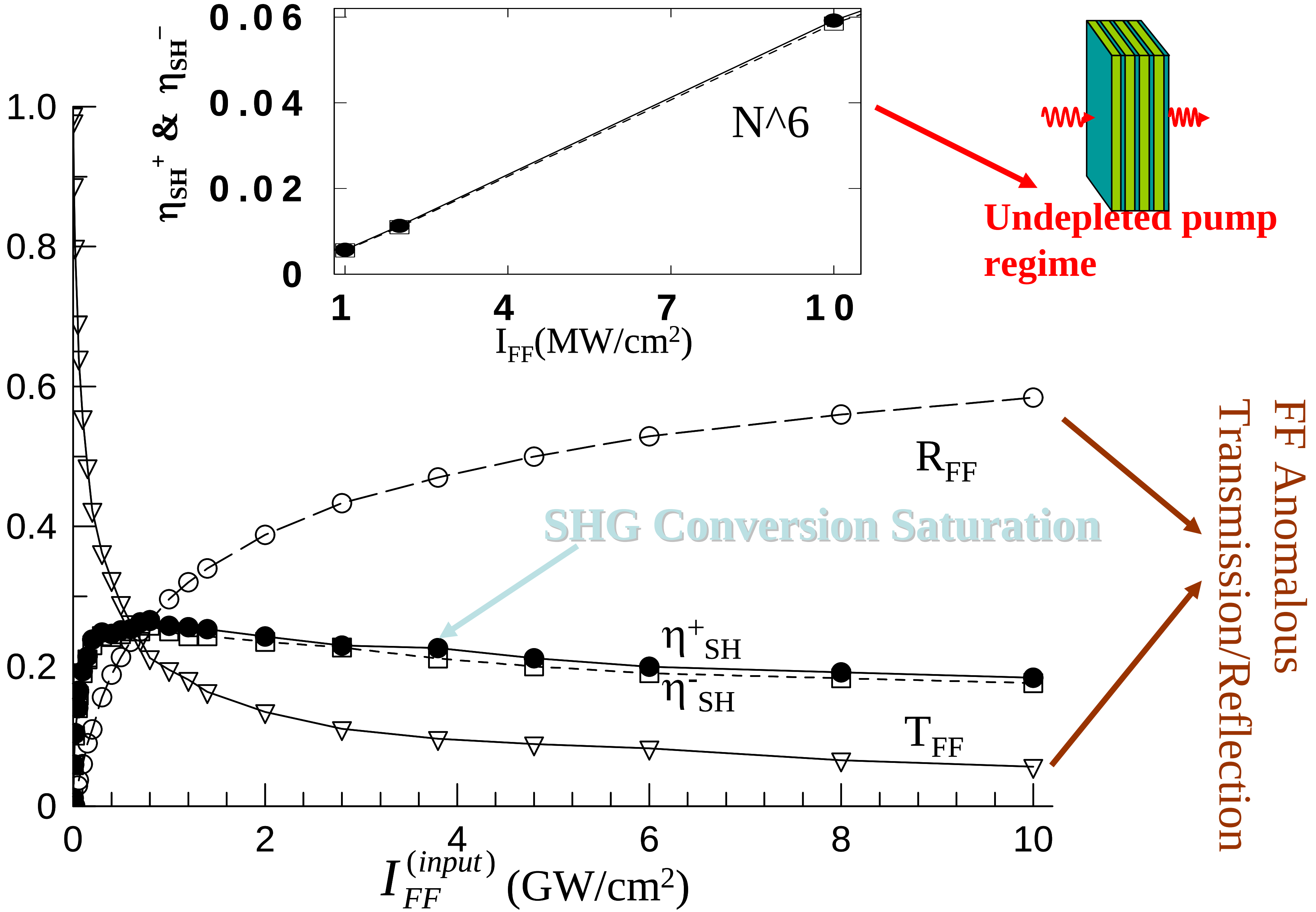
Huge conversion for random sequences

The random code is generated assuming that each layer has 50% of probability to be H or L.
nL=1.45 ;nH=sqrt(4.9048 + 0.11768/(λ^2-0.0475) – 0.027169 λ^2 (dielectric constant of LiNbO3 ordinary wave). Structure is obtained by alternating 250 “unit” layers





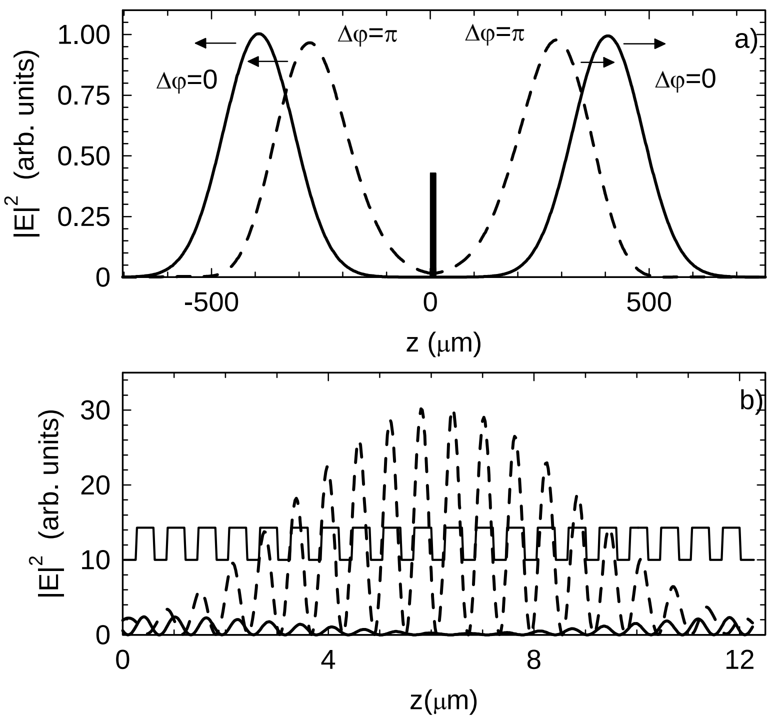
Coll. with D.Wiersma (Lens)

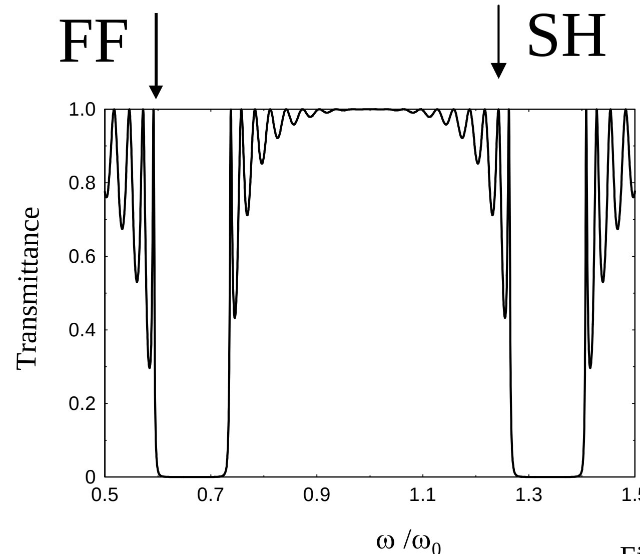


D'Aguanno *et al.* **"Energy Exchange Properties during SHG in finite,1-D ,PBG structures with deep g**

Second Harmonic Generation for counter-propagating pumps

Control of SHG process





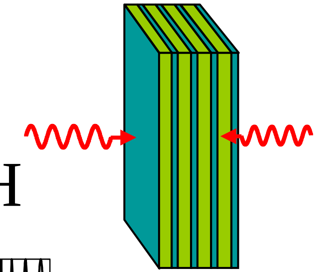
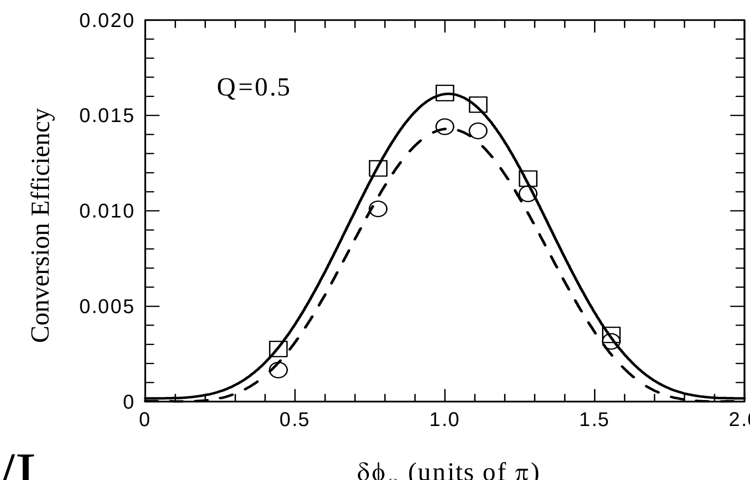


Fig.1(a)



$$Q=I_{pump2}/I_{pump1}$$

Fig.(6a)

Second-harmonic generation in local modes of a truncated periodic structure

J. Trull and R. Vilaseca

Departament de Física i Enginyeria Nuclear, Universitat Politècnica de Catalunya, 08222 Terrassa (Barcelona), Spain

Jordi Martorell and R. Corbalán

Departament de Física, Universitat Autònoma de Barcelona, 08193 Bellaterra (Barcelona), Spain

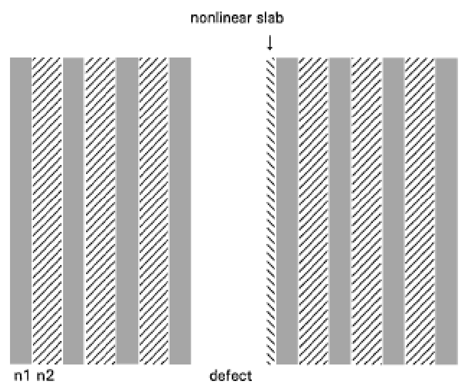
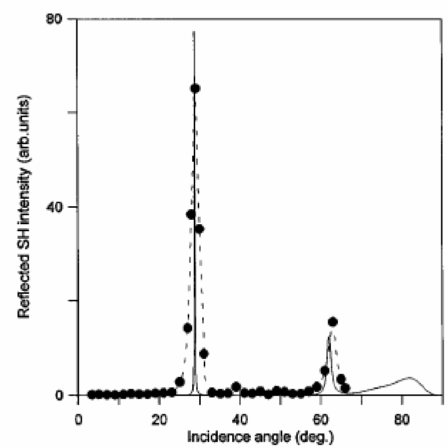


Fig. 1. Two one-dimensional stacks of alternating layers of indices n_1 and n_2 separated by an air gap or defect layer. The monolayer of nonlinear molecules is pictured on the front surface of the multilayer stack at the right. The widths of the dielectric layers and the defect layer are drawn to scale of the experimental parameters.



with 35-ps pulses from an active–passive mode-locked Nd:YAG laser. The average energy of the pulse was

Experiments on SHG

Y. Dumeige et al APL 2001, JOSAB 2002

$AlGaAs/Al_2O_3$

$$I_2^T \approx N^6$$

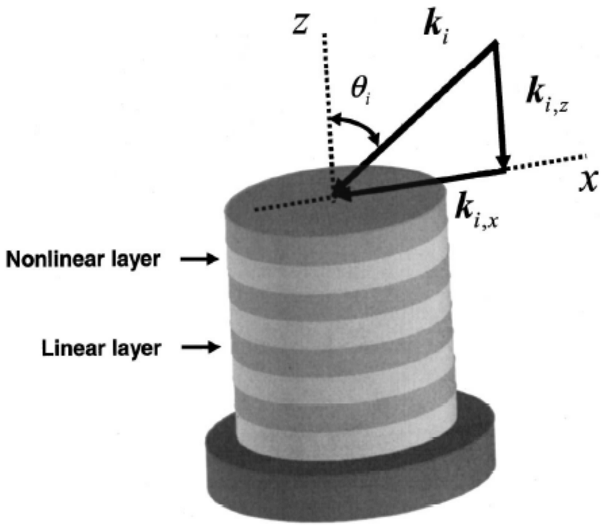


Fig. 1. Geometry of the PC studied. The incident wave vector and the incident angle are represented with respect to the two characteristic axes (x, z). The incident wave vector $\mathbf{k}_i$ is expanded into two orthogonal projections ($\mathbf{k}_{i,x}$, $\mathbf{k}_{i,z}$) related to the symmetry of the structure.

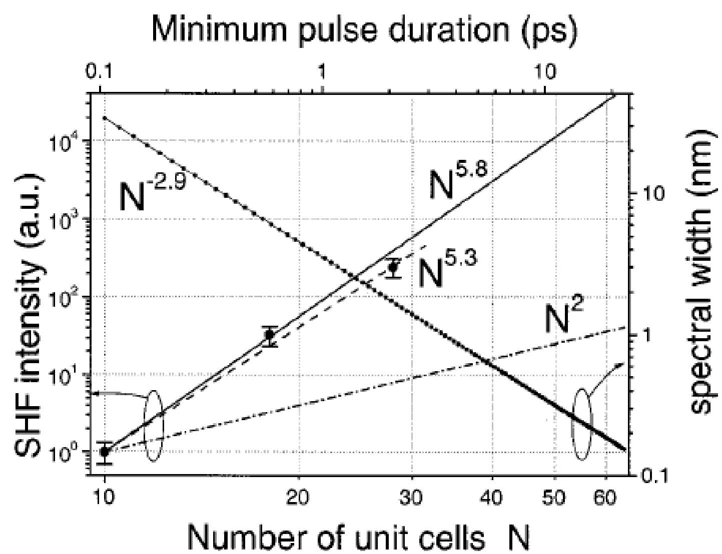


Fig. 9. SHF intensity, normalized to the value of the 10-period sample, plotted as a function of the number of PC unit cells N. The circles and the dashed curve correspond to the experimental results and the best fit, respectively. The solid curve is the theoretical prediction, whereas the usual quadratic law is represented by the dotted–dashed curve for comparison. The dotted curve is the calculated spectral width of the FF transmission resonance.

Simultaneous perfect phase matching for second and third harmonic generations in ZnS/YF3 photonic crystal for visible emissions

Weixin Lu, Ping Xie, Zhao-Qing Zhang, George K. L. Wong, and Kam S. Wong

Optics Express, Vol. 14, Issue 25, pp. 12353-12358

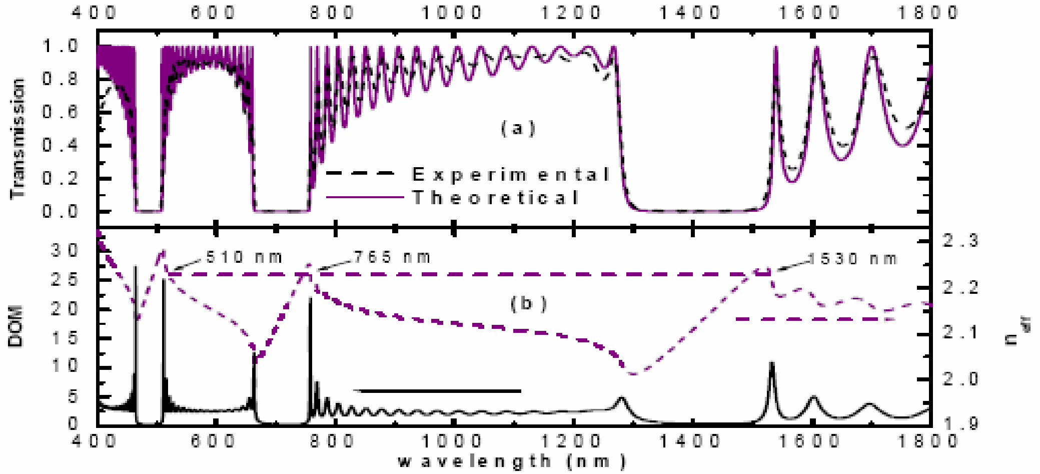


Fig. 1. (a) The experimentally measured (dotted line) and the corresponding theoretically calculated (solid line) transmission. (b) Effective refractive index (solid line) and density of modes (DOM) (dashed line) as a function of wavelengths of the same PC.

M. Centini, et al,
"Simultaneously phase-matched enhanced second and third harmonic generation," Phys. Rev. E 64, 046606 (2001)

Giant second-harmonic generation in a one-dimensional GaN photonic crystal

J. Torres, D. Coquillat,* R. Legros, J. P. Lascaray, F. Teppe, D. Scalbert, D. Peyrade, Y. Chen, O. Briot,
M. Le Vassor d'Yerville, E. Centeno, D. Cassagne, and J. P. Albert
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(Received 8 July 2003; revised manuscript received 8 October 2003; published 20 February 2004)

PHYSICAL REVIEW B **69**, 085105 (2004)

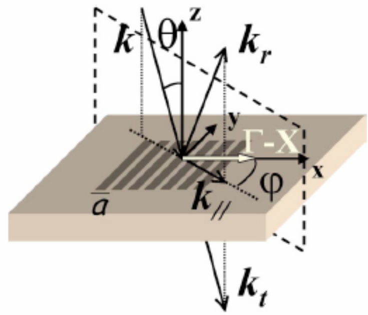
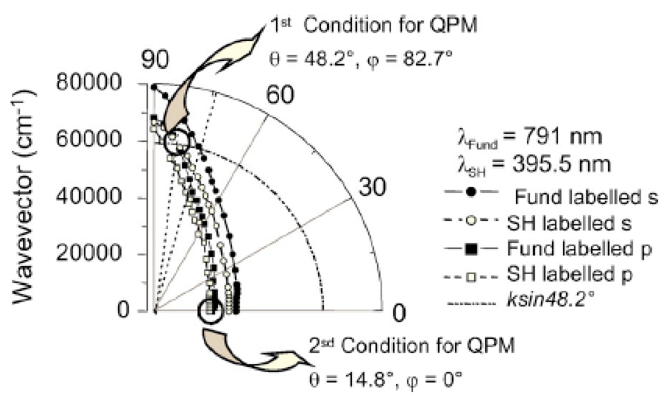


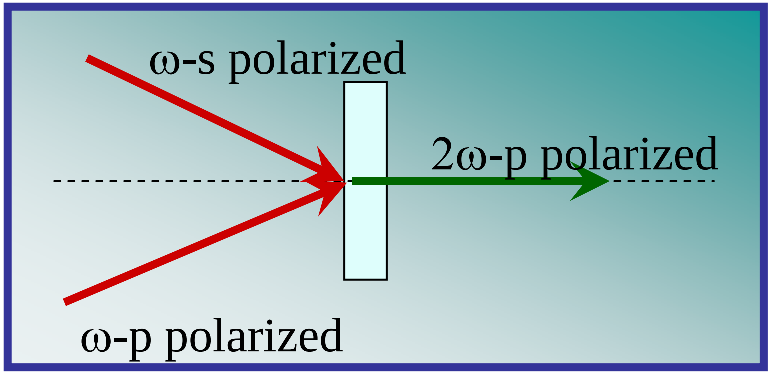
FIG. 1. Schematic diagram illustrating the 1D GaN PhC in planar waveguide geometry, and the coordinate system used in this study. Here a is the periodicity of the PhC.

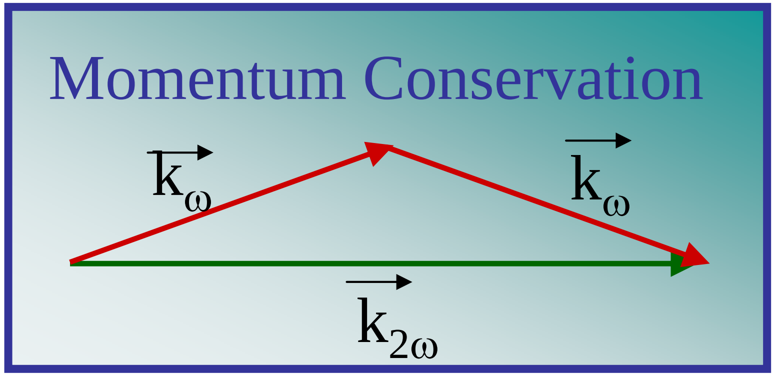


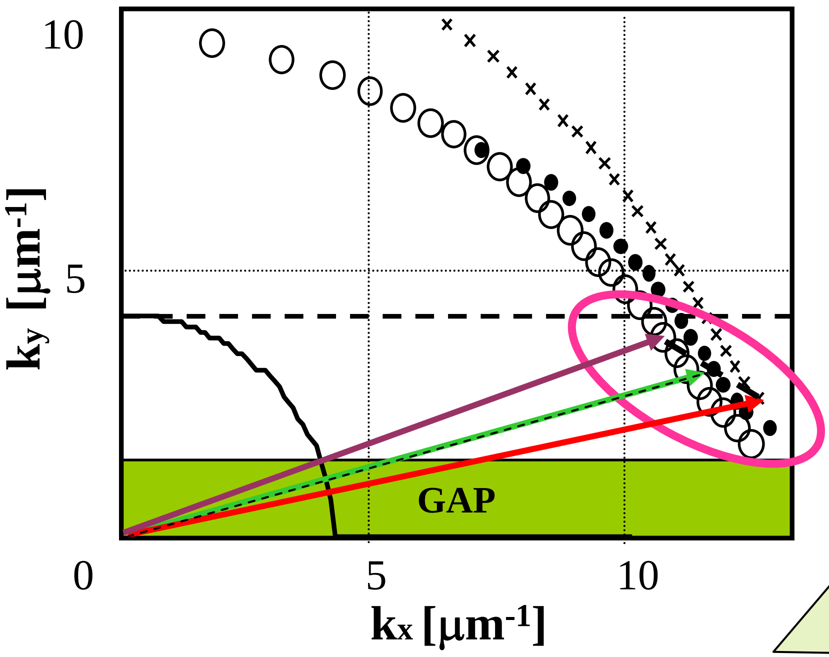
Experiment on Type II SHG :AlGaAs/Al_2O_3

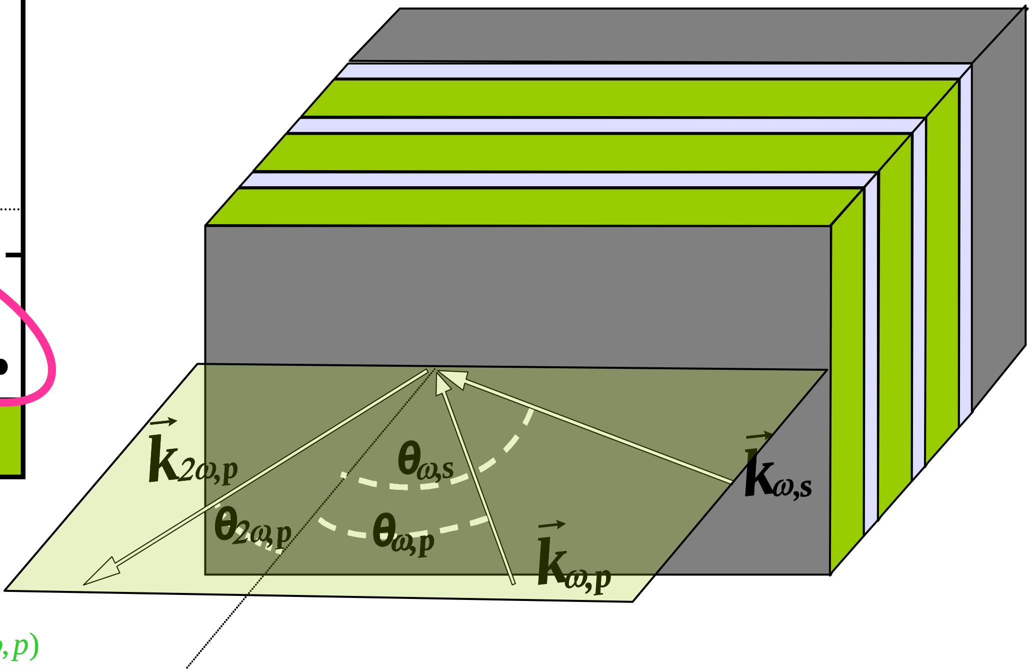
λp=1.55 microns

Non collinear Type II second harmonic generation



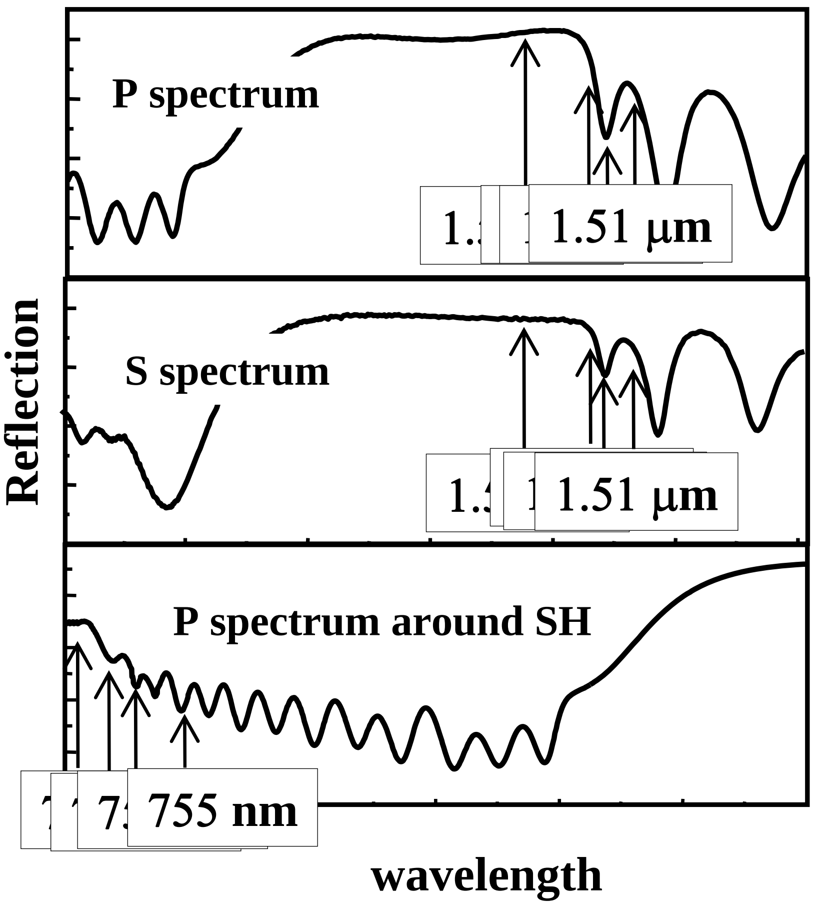


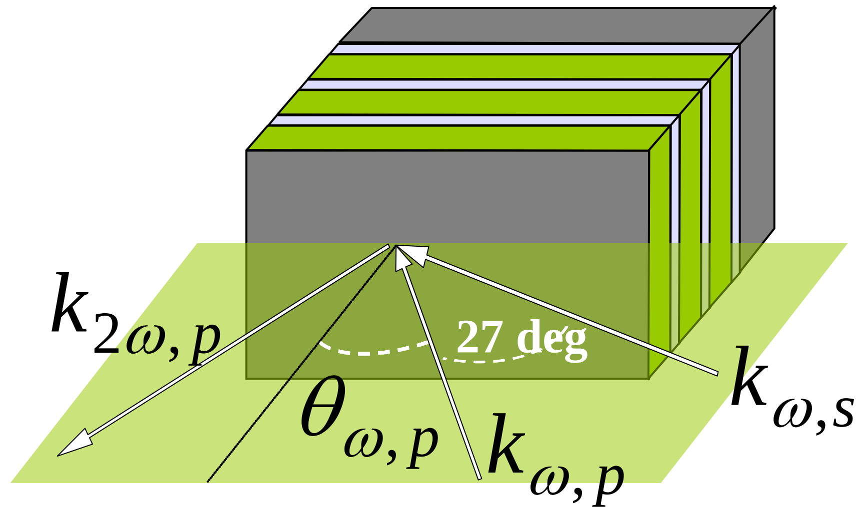


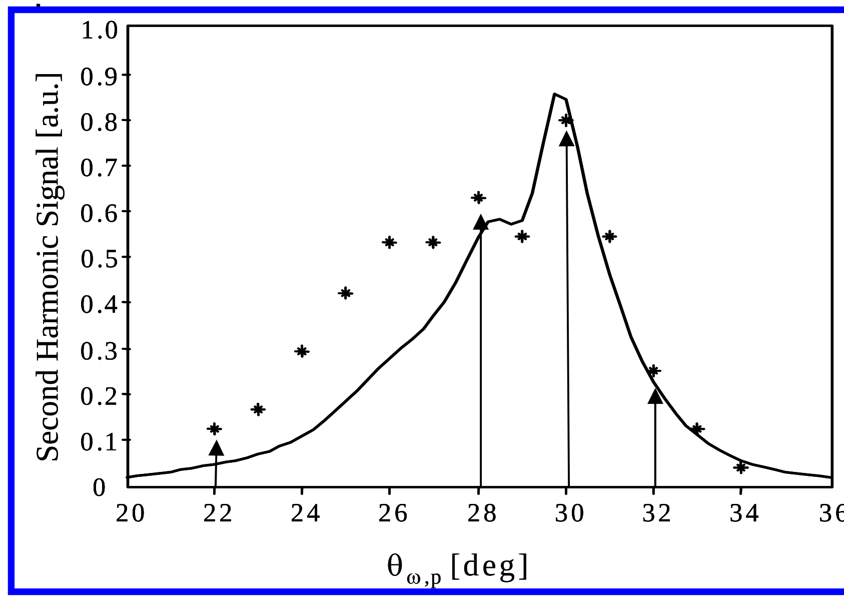


$$\frac{1}{2}\left[\vec{k}^{(\omega,s)} + \vec{k}^{(\omega,p)}\right] = \frac{1}{2}\vec{k}^{(2\omega,p)}$$

Angular measurements:







Almost no energy inside the photonic crystal

*Bosco et al APL 05

Nonlinearities in metals

- In metals bulk production of second harmonic radiation is usually inhibited due to symmetry reasons and only an electric quadrupole and a magnetic dipole terms may be active
- These reasons do not exhist at surface due to the lack of symmetry in a sheet region of the order of a few Å's thickness between vacuum and the metal. In this case the discontinuity of the electric field is important
- Therefore, although the production of radiation has a very low efficiency ($\sim 10^{-10}$) it is interesting to examine what happens

$$\ddot{\mathbf{P}} + \gamma\dot{\mathbf{P}} = \frac{\omega_p^2}{4\pi}\mathbf{E} + \frac{e}{mc}\dot{\mathbf{P}} \times \mathbf{H}.$$

the term E x B originating from the Lorentz force acts on the current produced at the first order by B (it is nonlinear in B). It is a bulk current in the metal and does not irradiate except if there is a boundary.

The other term is a surface electric quadrupole term. It is present in nonlinear reflection from a metal and originates from the discontinuity of the normal component of E at the interface which generates nonlinear currents. The surface currents are a tangential component and a bulk one directed along z.

The SH polarization from the perpendicular and parallel surface currents can be written as

$$\mathbf{P}_z^{surface}(2\omega) \propto a(\omega)\mathbf{E}_z(\omega)\mathbf{E}_z(\omega) \qquad \mathbf{P}_x^{surface}(2\omega) \propto b(\omega)\mathbf{E}_x(\omega)\mathbf{E}_z(\omega)$$

Dependence on fundamental beam polarization direction

- A typical apparatus is shown in the figure

- Since the surface nonlinear sources are in the simplest theory proportional to $(\nabla \cdot \mathbf{E})\mathbf{E}$, then incident fields polarized only in the plane of the surface (normal to the plane of incidence) do not contribute to the harmonic signal.

- However the bulk terms proportional to $\mathbf{E} \times \mathbf{H}$ lead to harmonic generation for all incidence polarizations

Dependence on fundamental beam polarization direction

- **Neglecting bulk terms, a [cos(ϕ)]^4 dependence on the polarization is predicted, where ϕ is the electric field angle relative to the plane of incidence.**

- **Deviations from this relation are indicative of contributions from the bulk term, and in particular the ratio**

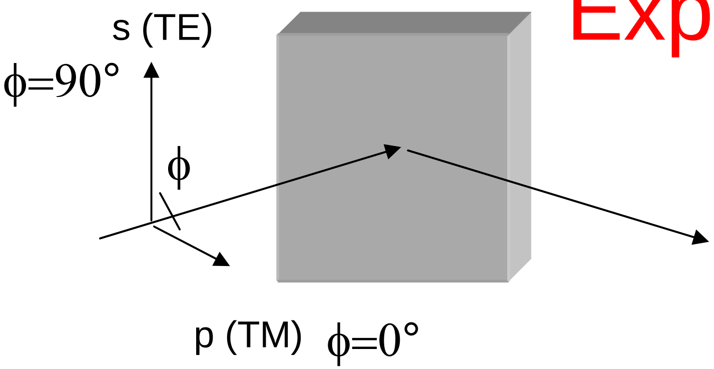
- $$M = I^{SH}(\phi=90°) / I^{SH}(\phi=0°)$$

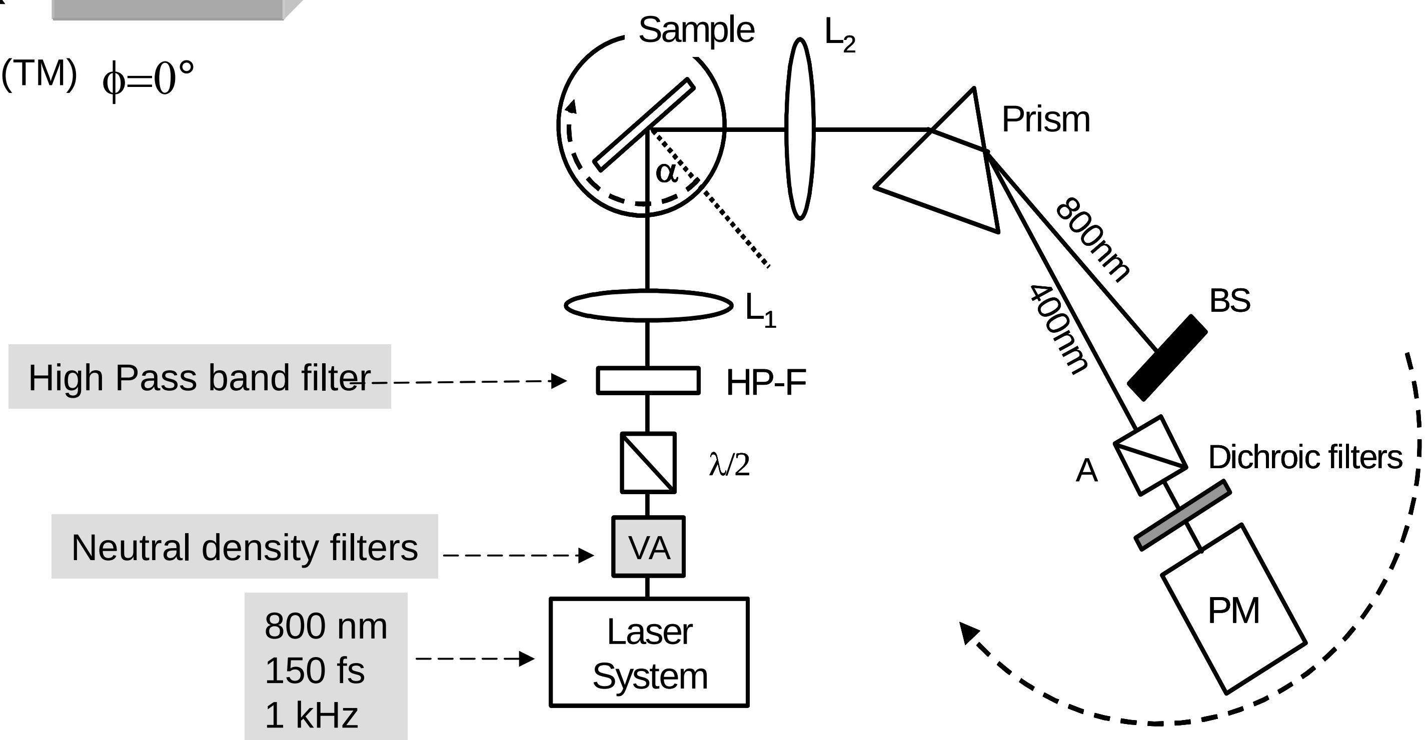
- **yields information about the relative strengths of the surface versus bulk terms**

- **F.Brown et al., PRL 14 (1965) 1029**
- **F.Brown and R.E.Parks, PRL 16 (1966) 507**
- **N. Bloembergen et al., PR 174 (1968) 813**

Nonlinearities in metals

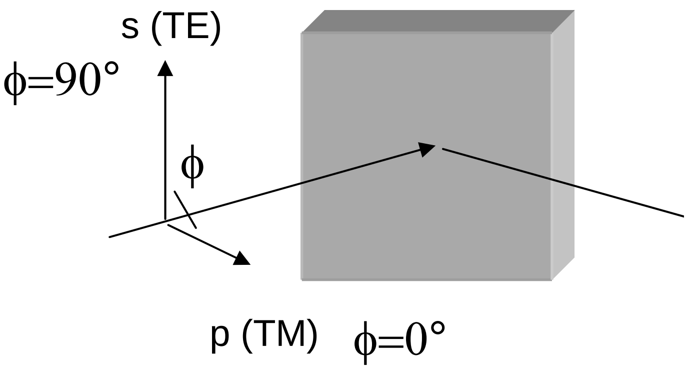
Experimental Setup

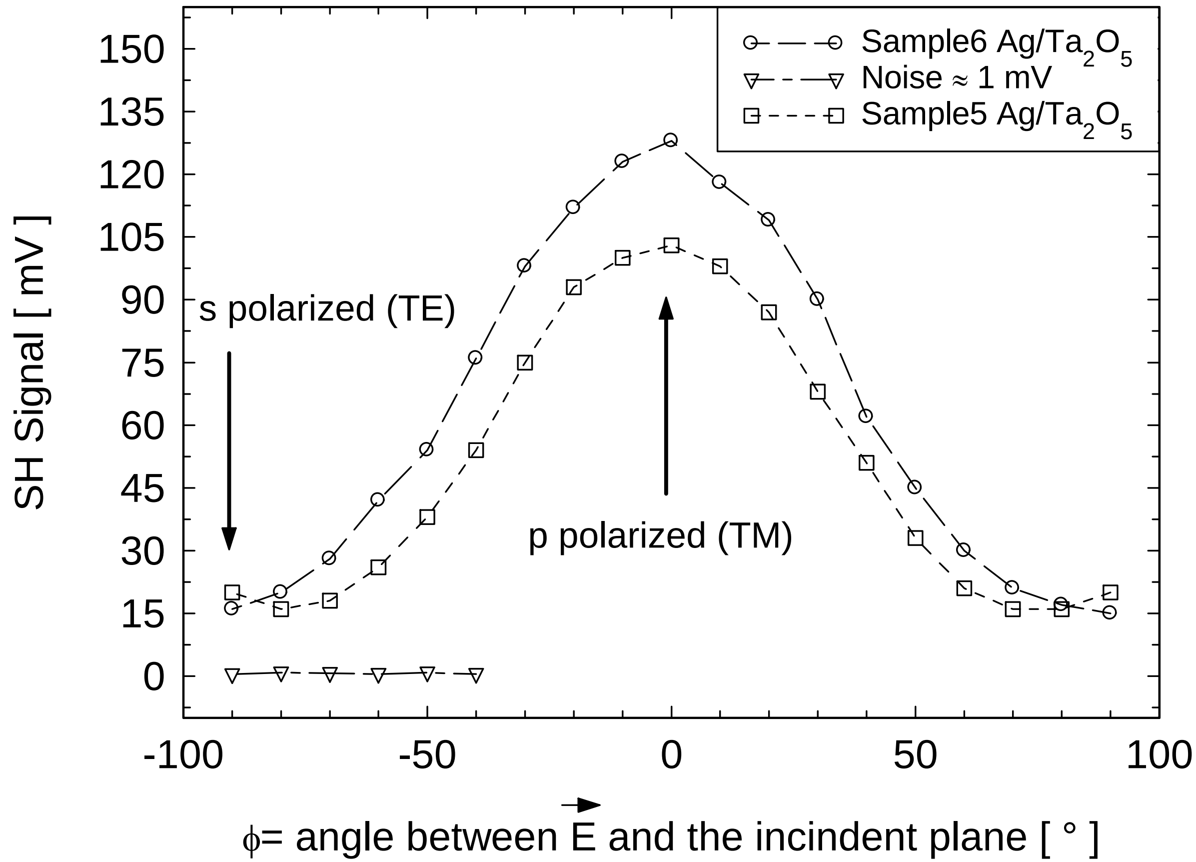




Metallo-dielectric multilayer SH production in reflection

When the fundamental wave is s polarized, only the bulk contribution is present

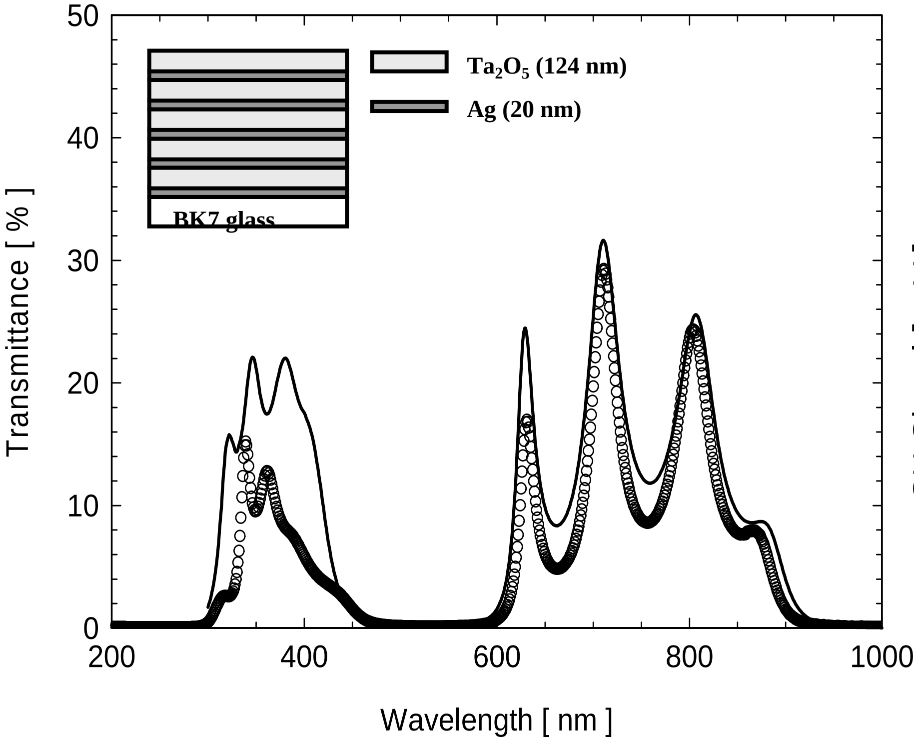


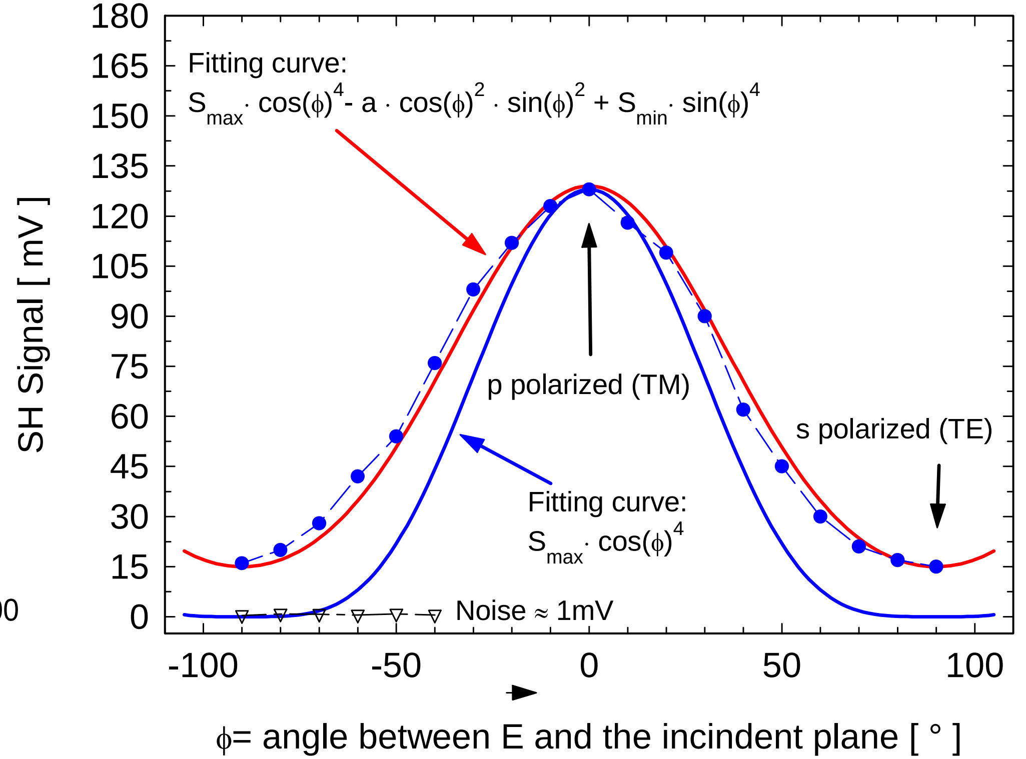


The surface contribution is TM when the fundamental is TM(P), if the fundamental is TE there is no discontinuity of the normal component of E and the SH vanishes.

The bulk is always present, but only at the interface.

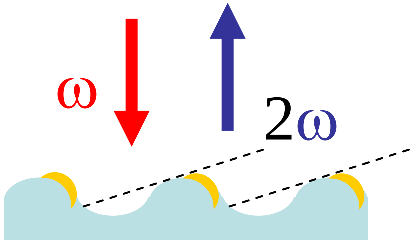
If fundamental is TM surface and bulk are present; if fundamental is TE only bulk is present.



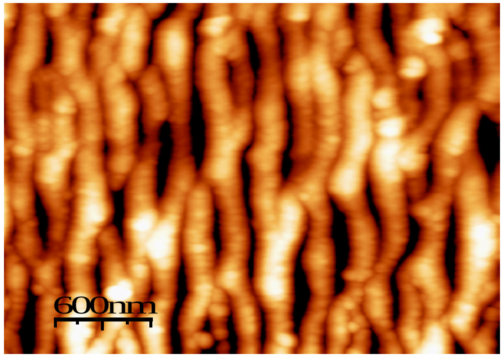


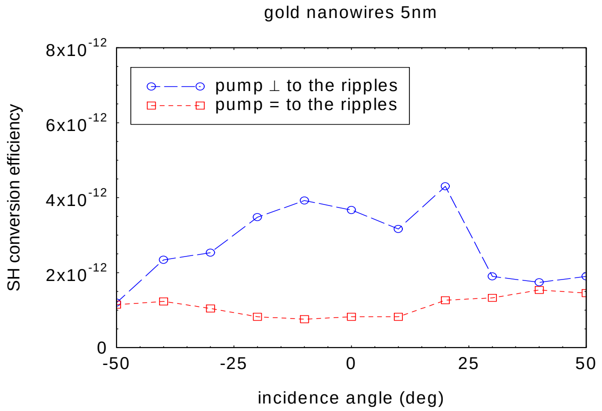
Second Harmonic Generation from nanostructured metals

S.H. signal

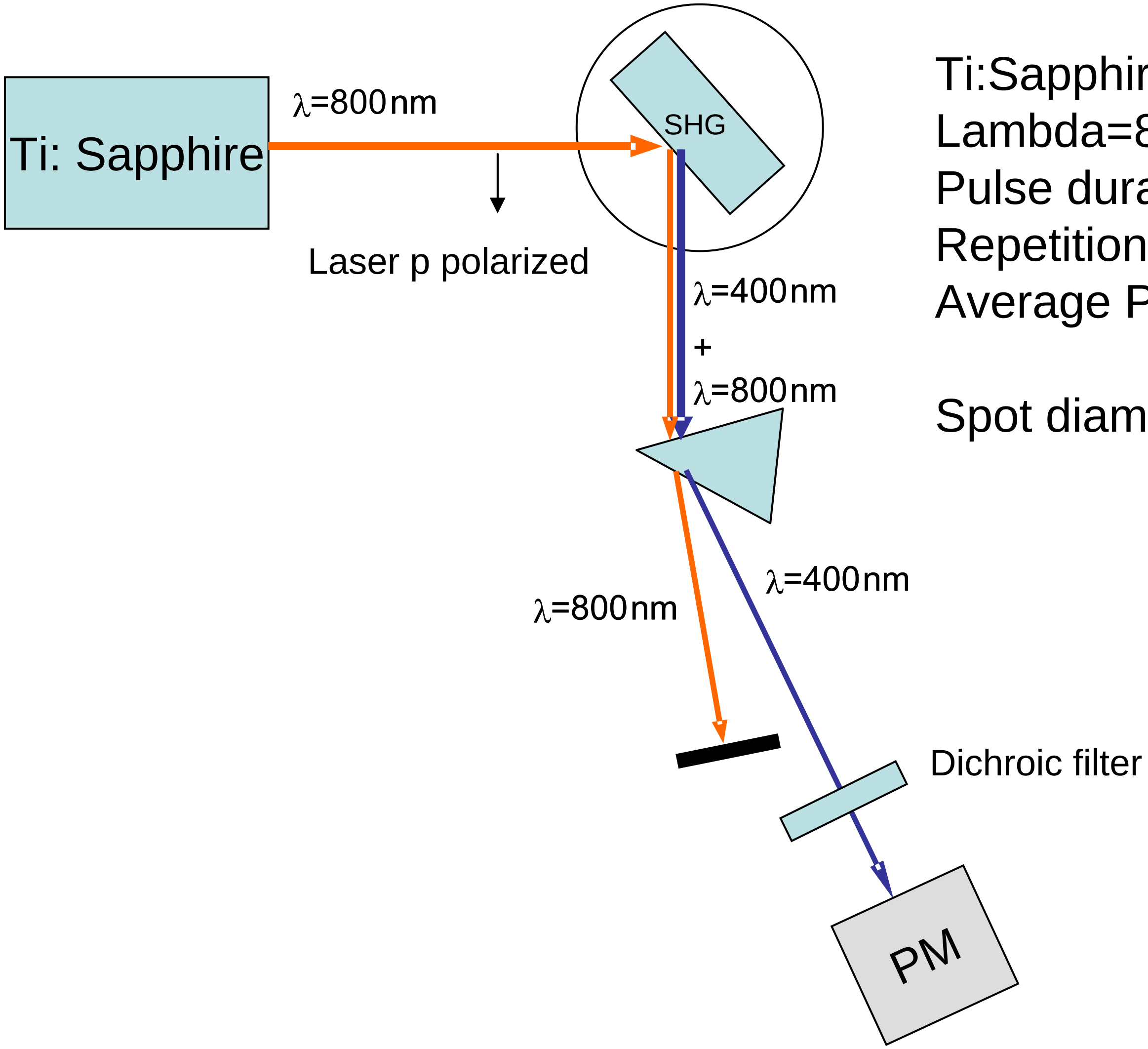


Sample A
Gold 5 nm/glass





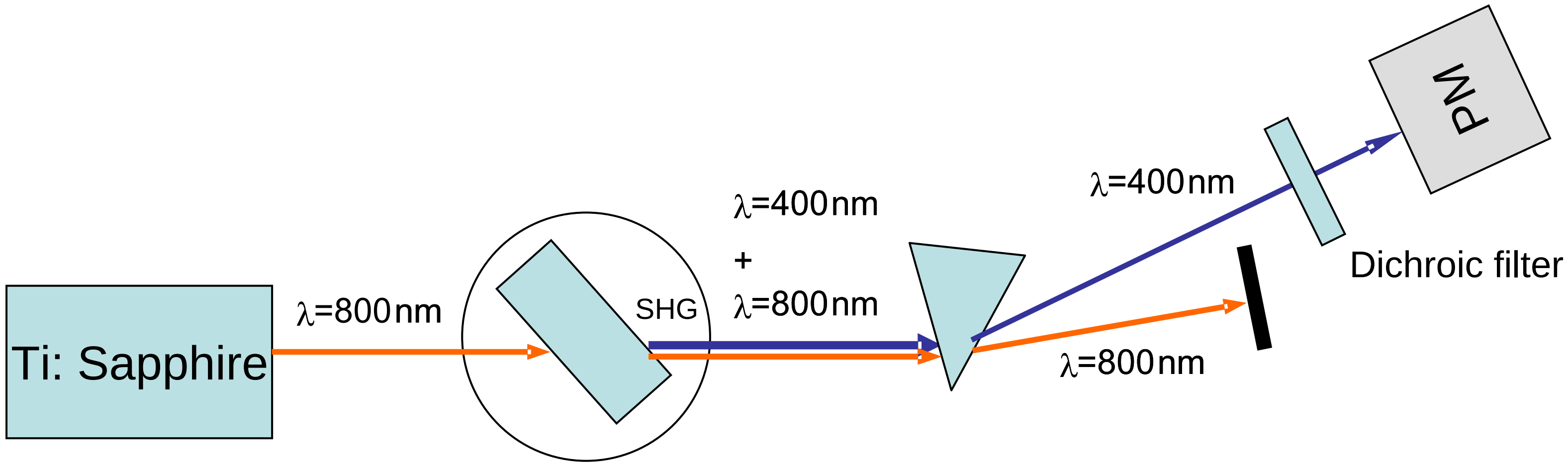
Second Harmonic Generation Set-up (Reflection)



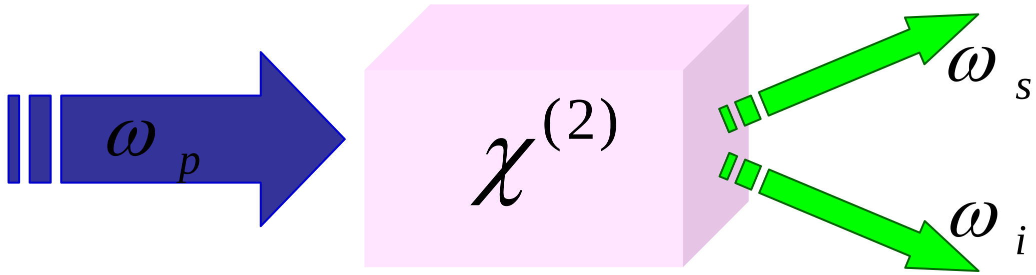
Ti:Sapphire
Lambda=800nm
Pulse duration =130fs
Repetition rate=1kHz
Average Power=78mW

Spot diameter on the sample=1.6mm

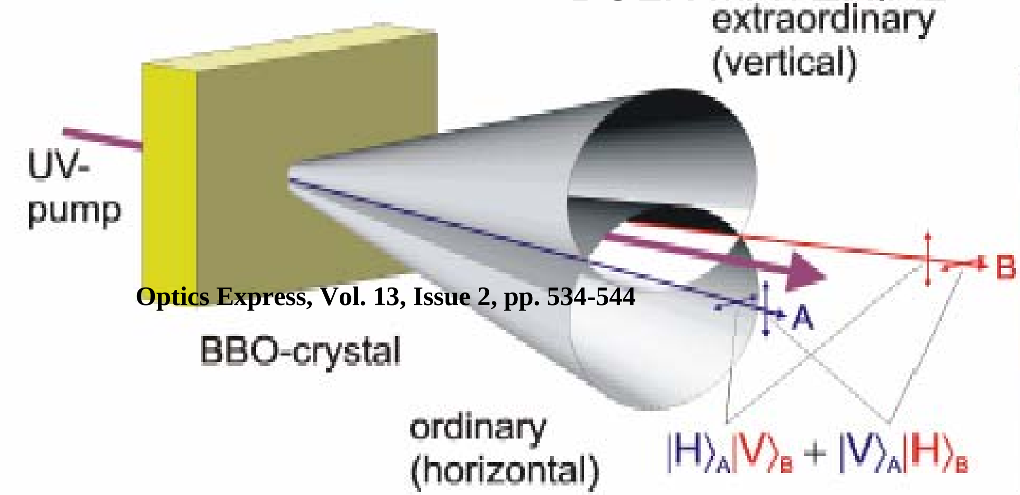
Second Harmonic Generation Set-up (Transmission)



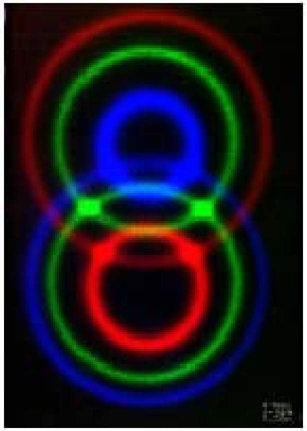
ENTANGLED PHOTON GENERATION



BULK MATERIAL



Optics Express, Vol. 13, Issue 2, pp. 534-544



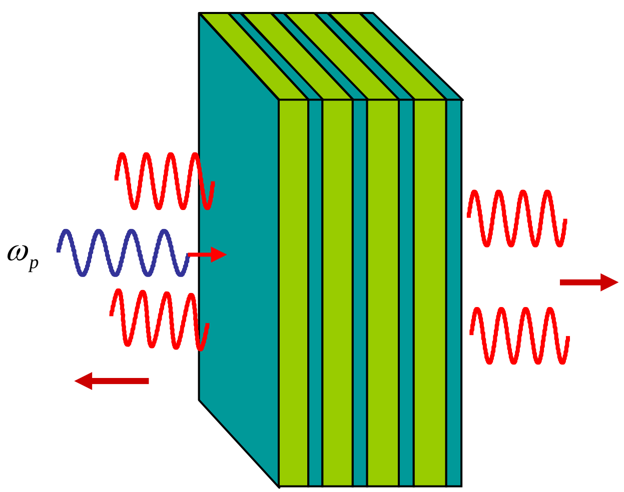
Kwiat et al. PRL 75 4337 (1995)

- ×Low efficiency
- ×Size
- ×Spatial and frequency filtering needed

STATE of the ART: 10^6 twin photons/s at low temperature in a Photonic Crystal Fiber

ENTANGLED PHOTON GENERATION In PhC

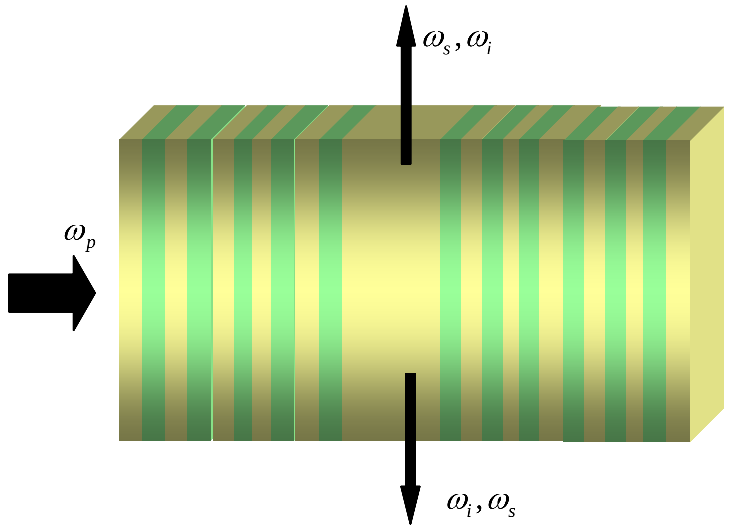
1D Ph Crystal



$$E_n = A_n^{(+)}(z)\Theta_n^{(+)}(z\ ) + A_n^{(-)}(z)\Theta_n^{(-)}(z\ )$$

- ✓Size
- ✓High brightness per mode
- ✓Narrow linewidth

1+1 D Ph Crystal



- ✓Size
- ✓High brightness per mode
- ✓Narrow linewidth
- ✓Guided entangled photon source

Conclusions

We showed that it is possible to enhance or suppress the nonlinear effect by properly choosing the parameters of the structure. Despite of what happens in bulk materials, the key feature for high efficiency of the nonlinear process is localization and overlap of interacting fields in nonlinear layers.

Structures, suitable for degenerate and nondegenerate generations of entangled photon pairs, were suggested using GaN/ALN layers and AlGaAs/Al2O3.

- INTERlink Project -Miur.Italy

-Phoremost NoE. EU

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