Basics of single negative and double negative metamaterials

Ekaterina Shamonina

UNIVERSITY OF OSNABRÜCK  UNIVERSITY OF ERLANGEN-NÜRNBERG
Basics of single negative and double negative metamaterials

Contents

• Definition of single negative and double negative media
• Properties and resulting applications
• Common $\varepsilon$-negative and $\mu$-negative media
• How they work

• Example: Near field imaging with silver superlens
Basics of single negative and double negative metamaterials

Contents

• Definition of single negative and double negative media
• Properties and resulting applications
  • Common $\varepsilon$-negative and $\mu$-negative media
  • How they work
• Example: Near field imaging with silver superlens
Basics of single negative and double negative metamaterials

Contents

• Definition of single negative and double negative media
• Properties and resulting applications

• Common $\varepsilon$-negative and $\mu$-negative media
• How they work

• Example: Near field imaging with silver superlens
Basics of single negative and double negative metamaterials

Contents

• Definition of single negative and double negative media
• Properties and resulting applications
• Common $\varepsilon$-negative and $\mu$-negative media
• How they work
• Example: Near field imaging with silver superlens
Basics of single negative and double negative metamaterials

Contents

• Definition of single negative and double negative media
• Properties and resulting applications

• Common $\varepsilon$-negative and $\mu$-negative media
• How they work

• Example: Near field imaging with silver superlens

• Near field imaging with magnetic metamaterials (Anna Radkovskaya’s lecture)
What are metamaterials?

μετά = beyond (Greek)
What are metamaterials?

μετά = beyond (Greek)

Metamaterials are not natural materials
What are metamaterials?

μετά = beyond (Greek)

**Metamaterials** are engineered composites

http://physicsweb.org/articles/world/16/5/3/1#pwpia2_05-03
What are metamaterials?

μετά = beyond (Greek)

Metamaterials are engineered composites that exhibit superior properties not found in nature and not observed in the constituent materials.
What is a metamaterial?

Metamaterial is an artificial material in which electromagnetic properties (\(\varepsilon, \mu\)) can be controlled.

It is made up of periodic arrays of metallic resonant elements. Both the size of the element and the unit cell are small relative to the wavelength.
**What is a metamaterial?**

*Metamaterial* is an artificial material in which electromagnetic properties ($\varepsilon, \mu$) can be controlled.

It is made up of periodic arrays of metallic resonant elements. Both the size of the element and the unit cell are small relative to the wavelength.

**Why is it important?**

Because it makes possible the manipulation of fields and waves at a subwavelength scale.
Near field: evanescent waves

An object of width $w$ can be represented in terms of Fourier components up to $k_x = \frac{2\pi}{w}$

Near field imaging requirement:
Transfer Function $T(k_x) = 1$ for all Fourier components!
Near field: evanescent waves

An object of width $w$ can be represented in terms of Fourier components up to $k_x = \frac{2\pi}{w}$

If $w < \lambda$  $k_x > k = \frac{2\pi}{\lambda}$ wave is evanescent:

NEAR FIELD IMAGING REQUIREMENT:
Transfer Function $T(k_x) = 1$ for all Fourier components!
Near field: evanescent waves

An object of width $w$ can be represented in terms of Fourier components up to $k_x = \frac{2\pi}{w}$

If $w < \lambda$ and $k_x > k = \frac{2\pi}{\lambda}$, the wave is evanescent:

Wave with $\vec{k} = (k_x, k_z)$.

Wave $\exp[-j\vec{k} \cdot \vec{r}] = \exp[-j(k_z z + k_x x)]$

$|\vec{k}| = \sqrt{k_z^2 + k_x^2}$ so that $k_z = \sqrt{k^2 - k_x^2} = \sqrt{\omega^2 \mu \varepsilon - k_x^2}$

If $k_x > k = \frac{2\pi}{\lambda} = \frac{\omega}{v} = \omega \sqrt{\mu \varepsilon}$, then $k_z$ is imaginary and the wave is evanescent.

NEAR FIELD IMAGING REQUIREMENT:
Transfer Function $T(k_x) = 1$ for all Fourier components!
Definition of single negative and double negative media

\[ k = \pm \frac{\omega}{c} \sqrt{\varepsilon \mu}, \quad n = \pm \sqrt{\varepsilon \mu} \]

\( \varepsilon > 0, \mu > 0 \)

“double positive“
(conventional materials)
real \( k \)
real \( n \)
Definition of single negative and double negative media

\[ k = \pm \frac{\omega}{c} \sqrt{\varepsilon \mu}, \quad n = \pm \sqrt{\varepsilon \mu} \]

\( \varepsilon < 0, \mu > 0 \)

„single negative“
(plasmas, rod MM)
complex \( k \)
complex \( n \)

\( \varepsilon > 0, \mu > 0 \)

„double positive“
(conventional materials)
real \( k \)
real \( n \)
Definition of single negative and double negative media

\[ k = \pm \frac{\omega}{c} \sqrt{\varepsilon \mu}, \quad n = \pm \sqrt{\varepsilon \mu} \]

\( \varepsilon < 0, \mu > 0 \)
- „single negative“
- (plasmas, rod MM)
- complex \( k \)
- complex \( n \)

\( \varepsilon > 0, \mu > 0 \)
- „double positive“
- (conventional materials)
- real \( k \)
- real \( n \)

\( \varepsilon > 0, \mu < 0 \)
- „single negative“
- (ferrites, ring MM)
- complex \( k \)
- complex \( n \)
Definition of single negative and double negative media

\[ k = \pm \frac{\omega}{c} \sqrt{\varepsilon \mu}, \quad n = \pm \sqrt{\varepsilon \mu} \]

- \( \varepsilon < 0, \ \mu > 0 \)  
  "single negative"  
  (plasmas, rod MM)  
  complex \( k \)  
  complex \( n \)

- \( \varepsilon < 0, \ \mu < 0 \)  
  "double negative"  
  (ring-rod MM)  
  real \( k \)  
  real \( n \)

- \( \varepsilon > 0, \ \mu > 0 \)  
  "double positive"  
  (conventional materials)  
  real \( k \)  
  real \( n \)

- \( \varepsilon > 0, \ \mu < 0 \)  
  "single negative"  
  (ferrites, ring MM)  
  complex \( k \)  
  complex \( n \)
\[ k = \pm \frac{\omega}{c} \sqrt{\varepsilon \mu}, \quad n = \pm \sqrt{\varepsilon \mu} \]

- **Conventional materials**
  - \( \varepsilon > 0, \mu > 0 \)
  - Real \( k \)
  - Real \( n > 0 \)

**Diagram:**
- Air
- Conventional materials
\[ k = \pm \frac{\omega}{c} \sqrt{\varepsilon \mu}, \quad n = \pm \sqrt{\varepsilon \mu} \]

- **Plasmas, Rod Metamaterials**: \( \varepsilon < 0, \mu > 0 \)  
  - Complex \( k \)  
  - Complex \( n \)

- **Conventional Materials**: \( \varepsilon > 0, \mu > 0 \)  
  - Real \( k \)  
  - Real \( n > 0 \)

**No propagation!**
\[ k = \pm \frac{\omega}{c} \sqrt{\varepsilon \mu}, \quad n = \pm \sqrt{\varepsilon \mu} \]

- **Plasmas, rod metamaterials**
  - \( \varepsilon < 0, \mu > 0 \)
  - Complex \( k \)
  - Complex \( n \)
  - **No propagation!**

- **Conventional materials**
  - \( \varepsilon > 0, \mu > 0 \)
  - Real \( k \)
  - Real \( n > 0 \)

- **Ferrites, ring metamaterials**
  - \( \varepsilon > 0, \mu < 0 \)
  - Complex \( k \)
  - Complex \( n \)
  - **No propagation!**
\[ k = \pm \frac{\omega}{c} \sqrt{\varepsilon \mu}, \quad n = \pm \varepsilon \mu \]

- Plasmas, rod metamaterials: \( \varepsilon < 0, \ \mu > 0 \)
  - Complex \( k \)
  - Complex \( n \)
  - No propagation!

- Conventional materials: \( \varepsilon > 0, \ \mu > 0 \)
  - Real \( k \)
  - Real \( n > 0 \)

- Ring-rod metamaterials: \( \varepsilon < 0, \ \mu < 0 \)
  - Real \( k \)
  - Real \( n < 0 \)

- Ferrites, ring metamaterials: \( \varepsilon > 0, \ \mu < 0 \)
  - Complex \( k \)
  - Complex \( n \)
  - No propagation!
\[ k = \pm \frac{\alpha}{c} \sqrt{\varepsilon \mu}, \quad n = \pm \sqrt{\varepsilon \mu} \]

conventional materials
\( \varepsilon > 0, \mu > 0 \)
real \( k \)
real \( n > 0 \)
\[ k = \pm \frac{\omega}{c} \sqrt{\varepsilon \mu}, \quad n = \pm \sqrt{\varepsilon \mu} \]

- **plasmas, rod metamaterials**
  - \( \varepsilon < 0, \ \mu > 0 \)
  - complex \( k \)
  - complex \( n \)
  - no propagation!

- **conventional materials**
  - \( \varepsilon > 0, \ \mu > 0 \)
  - real \( k \)
  - real \( n > 0 \)
  - no propagation!

- **ferrites, ring metamaterials**
  - \( \varepsilon > 0, \ \mu < 0 \)
  - complex \( k \)
  - complex \( n \)
  - no propagation!
\[ k = \pm \frac{\omega}{c} \sqrt{\varepsilon \mu}, \quad n = \pm \sqrt{\varepsilon \mu} \]

- **Conventional Materials**
  - \( \varepsilon > 0, \mu > 0 \)
  - Real \( k \)
  - Real \( n > 0 \)

- **Plasmas, Rod Metamaterials**
  - \( \varepsilon < 0, \mu > 0 \)
  - Complex \( k \)
  - Complex \( n \)

- **Negative Refraction**
  - **Ring-Rod Metamaterials**
    - \( \varepsilon < 0, \mu < 0 \)
    - Real \( k \)
    - Real \( n < 0 \)

- **No Propagation**
  - **Conventional Materials**
    - \( \varepsilon > 0, \mu > 0 \)
    - Real \( k \)
    - Real \( n > 0 \)

- **Ferrites, Ring Metamaterials**
  - \( \varepsilon > 0, \mu < 0 \)
  - Complex \( k \)
  - Complex \( n \)
Double negative media

Various names, the same physics

- Double Negative Medium  DNM
- Veselago Medium
- Negative Refractive Index Medium  NRIM
- Negative Index Medium  NIM
- Left-Handed Medium  LHM
- Backward Wave Medium  BWM
Double negative media: why “left-handed”? 

\[ \begin{align*} 
n &< 0 \quad \text{“left handed”} \\
n &> 0 \quad \text{“right handed”} 
\end{align*} \]
Double negative media: why “left-handed”? 

- $n > 0$: “right handed” 
- $n < 0$: “left handed”
Double negative media: why “left-handed”? 

- If $n > 0$, the medium is “right handed”. 
- If $n < 0$, the medium is “left handed”.

The diagrams illustrate the concept with hands pointing in different directions to indicate the handedness.
Double negative media: why “backward wave media”?

“forward wave”

“backward wave”
Double negative media: why “backward wave media”?

\[ S = \frac{1}{2} \text{Re}[E \times H^*] \]

“forward wave”
Double negative media: why “backward wave media”? 

\[ S \uparrow \uparrow k \] 

“forward wave” 

\[ H \] 

\[ k \] 

\[ E \] 

\[ S = \frac{1}{2} \text{Re}[E \times H^*] \] 

\[ v_g \uparrow \uparrow v_{ph} \] phase and group velocities in the same direction
Double negative media: why “backward wave media”?

\[ S_{\uparrow\uparrow}k \] “backward wave”

\[ S_{\downarrow\downarrow}k \] “forward wave”

\[ S = \frac{1}{2} \text{Re}[E \times H^*] \]

phase and group velocities in the same direction
Double negative media: why “backward wave media”?

“forward wave”

\[ S = \frac{1}{2} \text{Re}[E \times H^*] \]

phase and group velocities in the same direction

“backward wave”

\[ S = \frac{1}{2} \text{Re}[E \times H^*] \]

opposite phase and group velocities
More on terminology for single/double negative media

\[\begin{array}{c|c|c|c}
\varepsilon < 0, \mu > 0 & \varepsilon > 0, \mu > 0 \\
	ext{„single negative“} & \text{„double positive“} \\
	ext{„ENG“} & \text{„DPS“} \\
\hline
\varepsilon < 0, \mu < 0 & \varepsilon > 0, \mu < 0 \\
	ext{„double negative“} & \text{„single negative“} \\
	ext{„DNG“} & \text{„MNG“} \\
\end{array}\]
Negative refraction

Maxwell’s equations and appropriate boundary conditions!
Negative refraction

Maxwell's equations and appropriate boundary conditions!

\[ n = -\sqrt{\varepsilon \mu} = -1 \]
Positive refraction vs negative refraction

\[ \mathbf{S} \uparrow \uparrow \mathbf{k} \] “forward wave”

wedge \( \varepsilon = 2.2, \mu = 1 \)

\[ \mathbf{S} \uparrow \downarrow \mathbf{k} \] “backward wave”

wedge \( \varepsilon = -1, \mu = -1 \)
AN INTRODUCTION
TO THE
THEORY OF OPTICS

BY

ARTHUR SCHÜSTER,
Ph.D. (Heidelberg), Sc.D. (Cantab.), F.R.S.
PROFESSOR OF PHYSICS AT THE UNIVERSITY OF MANCHESTER.

LONDON
EDWARD ARNOLD
41 & 43, MADDOX STREET, BOND STREET, W.

1904

[All Rights reserved]
direction to the wave velocity. If there is a convection of energy forward, the waves must therefore move backwards. In all optical media where the direction of the dispersion is reversed, there is a very powerful absorption, so that only thicknesses of the absorbing medium can be used which are smaller than a wave-length of light. Under these circumstances it is doubtful how far the above results have any application. But Professor Lamb† has devised mechanical ar-

arrangements in which without absorption there is a negative wave velocity. One curious result follows: the deviation of the wave on entering such a medium is greater than the angle of incidence, so that the wave normal is bent over to the other side of the normal as indicated in Fig. 179. This is seen at once by considering that the traces on the
Historic remark…

<table>
<thead>
<tr>
<th>Year</th>
<th>Author</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1904</td>
<td>Lamb, Schuster</td>
<td></td>
</tr>
<tr>
<td>1944</td>
<td>Mandelstam</td>
<td></td>
</tr>
<tr>
<td>1968</td>
<td>Veselago: $\varepsilon &lt; 0$ and $\mu &lt; 0 \rightarrow n &lt; 0$</td>
<td></td>
</tr>
</tbody>
</table>
### Historic remark...

<table>
<thead>
<tr>
<th>(\varepsilon &lt; 0)</th>
<th>Silver ((\omega &lt; \omega_p))</th>
</tr>
</thead>
<tbody>
<tr>
<td>1961 Rotman: rods</td>
<td></td>
</tr>
<tr>
<td>1996 Pendry: rods</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>(\mu &lt; 0)</th>
<th>Backward waves, negative refraction</th>
</tr>
</thead>
<tbody>
<tr>
<td>1904 Lamb, Schuster</td>
<td></td>
</tr>
<tr>
<td>1944 Mandelstam</td>
<td></td>
</tr>
<tr>
<td>1968 Veselago: (\varepsilon &lt; 0) and (\mu &lt; 0 \rightarrow n &lt; 0)</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>(\mu &lt; 0)</th>
<th>1956 Thompson: ferrites</th>
</tr>
</thead>
<tbody>
<tr>
<td>1981 Hardy: split rings</td>
<td></td>
</tr>
<tr>
<td>1999 Pendry: split rings</td>
<td></td>
</tr>
</tbody>
</table>
Historic remark…

\[ \varepsilon < 0 \]
\[ \mu < 0 \]

silver \((\omega < \omega_p)\)

1961 Rotman: rods
1996 Pendry: rods

Backward waves, negative refraction

1904 Lamb, Schuster
1944 Mandelstam
1968 Veselago: \(\varepsilon < 0 \text{ and } \mu < 0 \rightarrow n < 0\)

2000 Smith et al.: \(\varepsilon < 0 \text{ and } \mu < 0 \rightarrow n < 0\)

2001 Shelby et al.: first experiment

1956 Thompson: ferrites
1981 Hardy: split rings
1999 Pendry: split rings
Artificial medium with $\varepsilon<0$: realisation?

Drude response

bulk metal

$$\varepsilon = 1 - \frac{\omega_p^2}{\omega^2}$$

plasma frequency

$$\omega_p = \frac{Ne^2}{\varepsilon_0 m}$$
Artificial medium with $\varepsilon<0$: realisation?

Drude response

bulk metal

$\varepsilon = 1 - \frac{\omega_p^2}{\omega^2}$

plasma frequency

$\omega_p = \frac{Ne^2}{\varepsilon_0 m}$

Drude-like response

"wire medium"

$\varepsilon = 1 - \frac{\omega_p^2}{\omega^2}$

$\omega < \omega_p$

$\varepsilon < 0$

Brown 1950’s, Rotman 1960’s, Pendry 1996
Artificial medium with $\varepsilon < 0$: realisation?

**Drude response**

bulk metal

$$\varepsilon = 1 - \frac{\omega_p^2}{\omega^2}$$

plasma frequency

$$\omega_p = \frac{Ne^2}{\varepsilon_0 m}$$

**Drude-like response**

"wire medium"

$$\varepsilon = 1 - \frac{\omega_p^2}{\omega^2}$$

plasma frequency

$$\omega_p = \frac{2\pi c^2}{\varepsilon_0 \ln(a/r)}$$

tunable, depends on the geometry!

Brown 1950's, Rotman 1960's, Pendry 1996
Artificial medium with $\mu<0$: realisation?

single split ring: LC circuit

$\omega_0$
Artificial medium with $\mu<0$: realisation?

**Single split ring: LC circuit**

**Impedance**
\[ Z = j(\omega L - \frac{1}{\omega C}) = j\omega L \left(1 - \frac{\omega_0^2}{\omega^2}\right) \]

**Resonant frequency**
\[ \omega_0 = \frac{1}{\sqrt{LC}} \]

**Voltage**
\[ V = -\frac{\partial \Phi}{\partial t} = j\omega_0 H \pi r^2 \]

**Current**
\[ I = \frac{V}{Z} = \frac{V}{j(\omega L - 1/\omega C)} = \frac{j\omega_0 H \pi r^2}{j\omega L} \frac{\omega^2}{\omega^2 - \omega_0^2} \]

Pendry 1999
Artificial medium with $\mu<0$: realisation?

\[ Z = j(\omega L - \frac{1}{\omega C}) = j\omega L \left( 1 - \frac{\omega_0^2}{\omega^2} \right) \]

\[ I = \frac{V}{Z} = \frac{-V}{j(\omega L - 1/\omega C)} = \frac{j\omega \mu_0 H\pi r^2}{j\omega L} \frac{\omega^2}{\omega^2 - \omega_0^2} \]

\[ \omega_0 = \frac{1}{\sqrt{LC}} \]

\[ V = -\frac{\partial \Phi}{\partial t} = j\omega \mu_0 H\pi r^2 \]

Pendry 1999
Artificial medium with $\mu < 0$: realisation?

**Single split ring: LC circuit**

$$I = \frac{V}{Z} = \frac{V}{j(\omega L - 1/\omega C)} = \frac{j\omega H \pi r^2}{j\omega L} \frac{\omega^2}{\omega^2 - \omega_0^2}$$

Resonant frequency $\omega_0 = \frac{1}{\sqrt{LC}}$

**“Split ring medium”**

$$\mu = 1 - F \frac{\omega^2}{\omega^2 - \omega_0^2}$$

Frequencies $\omega_0$, $\omega_\infty$ depend on the geometry!

Pendry et al. 1999
Exploring magnetic „atoms“: Split Ring Resonators

Kafesaki et al. 2005

Marques et al. 2003

Aydin et al. 2005
Exploring magnetic „atoms“: the family grows…

O’Brien & Pendry 2005
Guo et al. 2005
Hsu et al. 2004
Bulu et al. 2005

Radkovskaya et al. 2007
Kafesaki et al. 2005
Kafesaki et al. 2005
Isotropic magnetic „atoms“

in 2D

Gay-Balmaz, Martin 2002
Chen et al. 2006

in 3D

Baena 2005
Gay-Balmaz, Martin 2002
Padilla 2005
Double negative media: realisations

Combination of SRRs (\(\mu<0\)) & wires (\(\varepsilon<0\))

\[ E \]

\[ H \]

Smith et al. 2000
Double negative media: realisations

SRR+rod

Omega-particles

Smith et al. 2000

Saadoun, Engheta 1992, 1994

Shelby et al. 2001

Tretyakov 1996

chiral particles
From SRR to wire pairs ...
From microwave to optical frequencies …

source: Wegener, Linden, von Freymann, lecture course 2007, Karlsruhe
Double negative metamaterials: realisation

\[ \mu < 0 + \varepsilon < 0 = n < 0 \]
Why Metamaterials?

Physics
Extension of electromagnetism
“inverse” electromagnetic phenomena
- inverse Snell’s law (negative refraction of rays)
- inverse Doppler shift, Cherenkov radiation, Goos-Haenchen shift
- “growing” evanescent waves

Applications
- “Perfect lens”, subwavelength imaging in photonics
- Invisibility and cloaking
- “Nano-circuits”, miniaturised waveguide components
- Medical imaging
Superlens

2000 Pendry:

metamaterial plate with $n<0$
acts as a perfect lens
2000 Pendry:

metamaterial plate with $n<0$
acts as a perfect lens

also for a sub-$\lambda$-object?
2000 Pendry:

metamaterial plate with $n<0$
acts as a perfect lens

also for a sub-$\lambda$-object?

Explanation: Near field
$n>0$ exponential decay
$n<0$ exponential growth

Mechanism: surface resonant modes
couple to the evanescent part of the
object spectrum and lead to the
reconstruction of the object in the image plane
Surface modes in metamaterials?
Surface modes in metamaterials?

Domains of existence of surface modes

TM: "transverse magnetic"
TE: "transverse electric"
Silver slab as a superlens: surface plasmon-polaritons

- Coupled modes of surface plasmon-polariton resonances
- Two spatial resonances; flat transfer function in-between

$\lambda=360\text{nm}$

Shamonina et al., Electr. Lett. (2001)
Silver slab as a superlens: surface plasmon-polaritons

\[ \omega < \omega_p \]

\[ \varepsilon < 0 \]

Surface electromagnetic eigenmode of the medium
Silver slab as a superlens: surface plasmon-polaritons

\[ \omega \leq \omega_p, \quad \varepsilon < 0 \]

Surface electromagnetic eigenmode of the medium

slow waves of short wavelength!

cannot interact with propagating waves!
Silver slab as a superlens: surface plasmon-polaritons

Surface electromagnetic eigenmode of the medium

Slow waves of short wavelength!

Cannot interact with propagating waves!

Do interact with the evanescent, near field components of the Fourier spectrum of an object!

Surface plasmon-polariton

ϕ < ϕₚ

ε < 0
SLAB: TWO SURFACES

\[ \zeta_e = -\tanh \frac{\kappa_2 d}{2} \]

\[ \zeta_e = -\coth \frac{\kappa_2 d}{2}. \]

\[ k_{z1} = -j\kappa_1, \quad k_{z2} = -j\kappa_2. \]

\[ k_{z1} = \sqrt{\varepsilon_r 1 \mu_r 1 k_0^2 - k_x^2}, \quad k_{z2} = \sqrt{\varepsilon_r 2 \mu_r 2 k_0^2 - k_x^2}. \]
ARBITRARY VALUES OF $\mu, \varepsilon$

\begin{align*}
\varepsilon_{r2} &= 1 - \frac{\omega_p^2}{\omega^2} \\
\mu_{r2} &= 1 - \frac{F\omega^2}{\omega^2 - \omega_0^2}.
\end{align*}
SPP dispersion for single interface. $F = 0.56$, $\omega_0 = 0.8\omega_p$. (a) $\omega - k_x$ diagram. (b) $\mu - \varepsilon$ diagram.

\[
\varepsilon_{r2} = 1 - \frac{\omega_p^2}{\omega^2}, \quad \mu_{r2} = 1 - \frac{F\omega^2}{\omega^2 - \omega_0^2}.
\]
ARBITRARY VALUES OF $\mu, \varepsilon$

SPP dispersion for single interface. $F = 0.56$, $\omega_0 = 0.6\omega_p$.

(a) $\omega - k_x$ diagram. (b) $\mu - \varepsilon$ diagram.

$$
\varepsilon_{r2} = 1 - \frac{\omega_p^2}{\omega^2} \quad \mu_{r2} = 1 - \frac{F\omega^2}{\omega^2 - \omega_0^2}.
$$
ARBITRARY VALUES OF $\mu, \varepsilon$

SPP dispersion for single interface. $F = 0.56, \omega_0 = 0.8\omega_p$. (a) $\omega - k_x$ diagram. (b) $\mu - \varepsilon$ diagram.

$$\varepsilon_{r2} = 1 - \frac{\omega_p^2}{\omega^2} \quad \mu_{r2} = 1 - \frac{F\omega^2}{\omega^2 - \omega_0^2}.$$
SLAB, ARBITRARY VALUES OF $\mu, \varepsilon$
In order to achieve a cut-off at 10 $k_0$ (for resolution $\lambda/10$)
a slab of $d=0.1 \lambda$ tolerates a loss of 0.002,
a slab of $d=0.67 \lambda$ tolerates a loss of not more than $10^{-19}$!

Podolskiy and Narimanov (2005)
Smith et al. (2003), French et al. (2006).
Silver lens: Near-perfect?

(a)\[ \text{intensity } |E|^2 \] vs. \( x \) [nm] for different materials:
- object
- ES
- full
- air

(b)\[ \text{intensity } |E|^2 \] vs. \( x \) [nm] for different materials:
- object
- ES
- full
- air

- d=40 nm \textit{thick}!
- d=20 nm
Silver lens: perfect without losses? No!
Experiment: SiC Superlens for IR

- Resolution $\lambda/20$ ($\lambda=10.85\mu m$)
- Optical Signal Processing

Near-Field Microscopy Through a SiC Superlens

T. Taubner, D. Korobkin, Y. Urzhumov, G. Shvets, R. Hillenbrand

Fig. 1. Near-field microscopy through a 880-nm-thick superlens structure: (A) Experimental setup. (B) Scanning electron micrograph (mirrored) of the object plane, showing holes in a 60-nm-thick Au film. (C) Infrared amplitude in the image plane at $\lambda = 10.85 \mu m$ where superlensing is expected. (D) Infrared phase contrast ($\lambda = 11.03 \mu m$). (E) Control image showing infrared amplitude at $\lambda = 9.25 \mu m$ (no superlensing).
Fig. 3: The image of a 2D arbitrary object “NANO”. (a) FIB (Focused Ion Beam) image of the object “NANO” after fabrication on Cr film. (b) Superlensing image (scale bar 2um) and (c) its cross-section profile. (d) Control imaging result of the same object (scale bar 2um) and (e) its average line cross-section.

N Fang et al., Science 308, 2005
Silver slab: Poynting vector optics

\[ \varepsilon = -1 - j\alpha, \quad \alpha = 10^{-4} \]

NEGATIVE REFRACTION?!
(only $\varepsilon$ is negative!)

Shamonina et al., PIERS 2002
NEGATIVE REFRACTION?! (only $\varepsilon$ is negative!)

$$\omega = \omega_p k_x/c$$

- Light line
- Surface plasmon-polariton
- Group velocity near zero!

Poynting vector
Multilayered superlens

60nm slab too thick for $\lambda=360$nm!

Microscopic picture: "Poynting vector optics"

$\lambda=360$nm, $\varepsilon=-1-j\alpha$, $\alpha=0.1$


Shamonina et al., PIERS 2002
Multilayered superlens

Microscopic picture: "Poynting vector optics"

Numerical simulation (CST Microwave Studio)

E. Tatartschuk (Erlangen)
Magnifying multilayered superlens

Microscopic picture: “Poynting vector optics“

flat “near-sighted“ lens

The image is not magnified

![Diagram of a flat near-sighted lens.]

- ! -

![Diagram of a cylindrical far-sighted lens.]

cylindrical „far-sighted“ lens

The image is magnified and can be captured with an optical microscope

Magnifying multilayered superlens: simulation

Microscopic picture: „Poynting vector optics”

Numerical simulation (CST Microwave Studio): E. Tatartschuk (Erlangen)
Cylindrical multilayered superlens: experiment

Magnifying Superlens in the Visible Frequency Range

Igor I. Smolyaninov, Yu-Ju Hung, Christopher C. Davis

Cylindrical multilayered superlens: experiment

Far-Field Optical Hyperlens Magnifying Sub-Diffraction-Limited Objects

Zhaowei Liu,* Hyesog Lee,* Yi Xiong, Cheng Sun, Xiang Zhang†

Metamaterials

Physics

Extension of electromagnetism

“inverse” electromagnetic phenomena
  - inverse Snell’s law (negative refraction of rays)
  - inverse Doppler shift, Cherenkov radiation, Goos-Haenchen shift
  - “growing” evanescent waves

Applications

- “Perfect lens”, subwavelength imaging in photonics
- Invisibility and cloaking
- “Nano-circuits”, miniaturised waveguide components
- Medical imaging
Basics of single negative and double negative metamaterials

Summary

• Definition of single negative and double negative media
• Properties and resulting applications

• Common ε-negative and μ-negative media
• How they work

• Example: Near field imaging with silver superlens
Basics of single negative and double negative metamaterials

Summary

• Definition of single negative and double negative media
• Properties and resulting applications

• Common \( \varepsilon \)-negative and \( \mu \)-negative media
• How they work

• Example: Near field imaging with silver superlens
• Near field imaging with magnetic metamaterials (Anna Radkovskaya’s lecture)
Fig. 9.24 2D microwave cloaking structure with a plot of the material parameters implemented. $\mu_r$ (red line) is multiplied by a factor of 10 for clarity. $\mu_\theta$ (green line) = 1, $\varepsilon_z = 3.423$. The SRRs of cylinder 1 (inner) and cylinder 10 (outer) are shown in expanded schematic form. From Schurig et al. (2006).
Positive and negative Goos-Haenchen Shift

\( n_1 > 0 \)

0\( \leq n_2 \leq n_1 \) positive lateral shift

(a)

\( n_1 > 0 \)

\( n_2 < 0 < n_1 \) negative lateral shift

(b)

\( \varepsilon_r = 9, \mu_r = 1 \)

\( \varepsilon_r = 3, \mu_r = 1 \)

\( \varepsilon_r = 9, \mu_r = 1 \)

\( \varepsilon_r = -3, \mu_r = -1 \)

Ziolkowski 2003