NEGATIVE INDEX MATERIALS: A NEW FRONTIER IN OPTICS?

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Almost thirty years after the initial theoretical study of hypothetical materials with negative effective index of refraction by Veselago, Pendry et al. designed an artificial structure, a split ring resonator (SRR), which would exhibit negative permeability. It took another two years for the first experimental demonstration of negative refraction in a composite structure consisting of SRRs and metallic wires by Shelby et al. During the last four years, this new and exciting field is growing fast. In this paper, some recent important results are presented and commented upon.

1. Introduction

Thirty seven years ago, Veselago [1] studied theoretically the propagation of an electromagnetic wave of frequency ω in a hypothetical medium where both the permittivity, $\varepsilon(\omega)$, and permeability, $\mu(\omega)$, were negative. It follows immediately from Maxwell's equations

$$\vec{k} \times \vec{E} = \frac{\omega}{c}\mu\vec{H}, \tag{1}$$

$$\vec{k} \times \vec{H} = -\frac{\omega}{c}\varepsilon\vec{E} \tag{2}$$

that the set of three vectors $\vec{E}$, $\vec{H}$, $\vec{k}$ is left handed when $\varepsilon < 0$, $\mu < 0$ and right handed when $\varepsilon > 0$, $\mu > 0$. As a result, in a negative ε, μ medium, the wave vector $\vec{k}$ is opposite to the Poynting vector $\vec{S} = (c/4\pi)\left(\vec{E} \times \vec{H}\right)$. In other words, the energy flows in such a medium in a direction opposite to that of phase. Then, the refraction of an electromagnetic (e/m) wave at a plane interface separating a regular medium from one with negative ε, μ is in the "wrong" side as shown in Fig. 1.

This follows immediately from the equality of $k_{||}$ of the incoming, reflected, and refracted waves combined with the relation $\vec{S}$ opposite to $\vec{k}$ in the negative ε, μ medium. It is easy to see that Snell's law is still valid, if we choose the negative square root for the index of refraction, n:

$$n = -\sqrt{\varepsilon \cdot \mu}, \tag{3}$$

when $\varepsilon < 0$ and $\mu < 0$. For this reason, media with $\varepsilon < 0$ and $\mu < 0$ are also referred to as negative index materials (NIM). Such materials, if available, can act as flat lenses; they boost also the evanescent waves and, consequently, they can provide resolution (in the near field) much better than the refraction limit.

As we shall see below, negative $\varepsilon(\omega)$ and $\mu(\omega)$can be realized only in dispersive media and only for some limited range(s) of ω. For such isotropic dispersive media, we have the following generalizations of some well-known formulae of electromagnetism:

$$\langle u \rangle = \frac{1}{8\pi}\left[\frac{\partial(\varepsilon\omega)}{\partial\omega}\left\langle \vec{E}^2 \right\rangle + \frac{\partial(\mu\omega)}{\partial\omega}\left\langle \vec{H}^2 \right\rangle\right], \tag{4}$$

$$\langle \vec{p} \rangle = \frac{\varepsilon\mu}{c^2}\left\langle \vec{S} \right\rangle + \frac{\vec{k}}{8\pi}\left[\frac{\partial\varepsilon}{\partial\omega}\left\langle \vec{E}^2 \right\rangle + \frac{\partial\mu}{\partial\omega}\left\langle \vec{H}^2 \right\rangle\right] = \frac{\langle u \rangle}{\omega}\vec{k}, \tag{5}$$

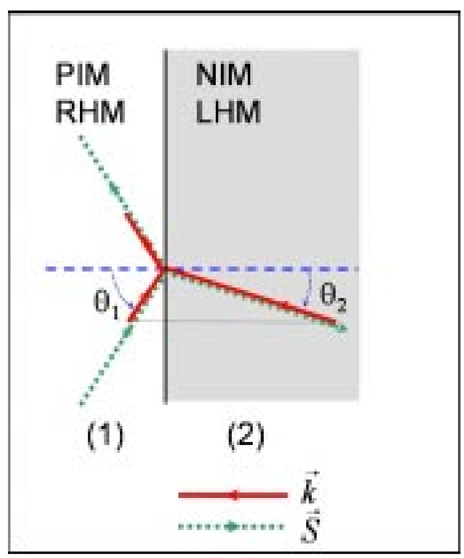

Fig. 1. Electromagnetic wave refraction at the flat interface separating a negative (NIM) from (regular) positive index material (PIM)

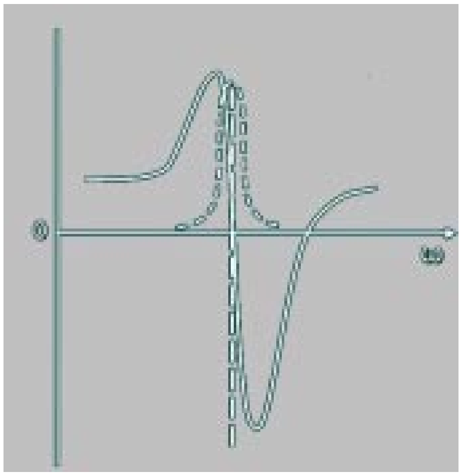

Fig. 2. Real (solid line) and imaginary (dashed line) parts of a response function near a sharp strong resonance

$$\left\langle \vec{S} \right\rangle = \langle u \rangle \vec{v}_g, \tag{6}$$

$$\vec{v}_g = \frac{\vec{v}_p}{\alpha}, \ \ \vec{v}_p = \frac{c}{|n|}\vec{k}_0, \tag{7}$$

$$\alpha \equiv 1 + \frac{d\ell n|n|}{d\ell n\omega} \qquad n\alpha > 0, \tag{8}$$

where $\langle u \rangle$ is the energy density of the e/ m field, $\langle p \rangle$ is its momentum density, v_g and v_p are its group and phase velocities, respectively, and n is the index of refraction; the symbol $\langle\rangle$ denotes time average.

2. Realization

There are no natural materials having both $\varepsilon(\omega)$ and $\mu(\omega)$negative over a common frequency range. Thus, the question is whether artificial structures can be fabricated exhibiting this feature. The key concept for achieving this is a resonance. As shown in Fig. 2, a strong sharp resonance can drive the response function to negative values.

Such resonances can be classified in two categories: in the first one, the resonance is associated with some material and/or structural features of the composite medium; as a result, the resonance wavelength in the material, $\lambda_m = 2\pi c/|n|\omega_m$, is not necessarily comparable to the periodicity length, α, of the composite medium; actually λ_m can be much larger than α, a feature which allows us to replace the actual system by an effective *uniform* medium. In the second category, the resonance is purely geometrical, i.e., it takes place when λ_m is comparable to both α and the scatterer's size. This occurs, e.g., in the so-called photonic crystals exhibiting a photonic gap when a Mie type resonance takes place combined with $\lambda_m \simeq 2\alpha$. The analysis becomes very complicated when both categories of resonances occur at similar frequencies making the optimization of the design problematic; nevertheless, one should not exclude the possibility that future structures may take advantage of the combination of the two types of resonances.

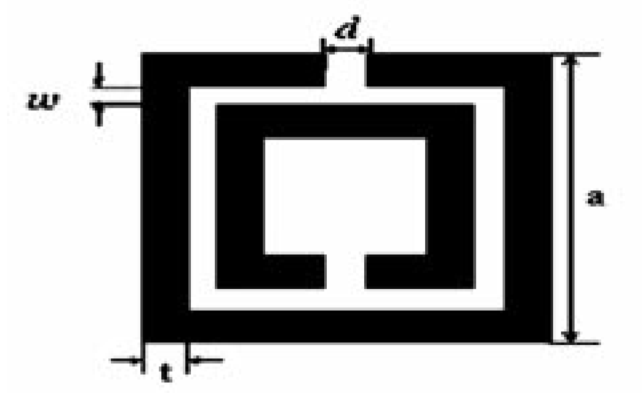

Fig. 3. Double split ring resonator. For a 3 GHz resonance, optimum values are $\alpha = 7.2$ mm, $d = w = 0.4$ mm, $t = 1$ mm

Pendry et al. [2] proposed a design based on the above ideas. The negative $\varepsilon(\omega)$ was to be produced by a set of periodically placed parallel metallic wires, for which

$$\varepsilon(\omega) = 1 - \frac{\omega_p^2}{\omega^2 + i\omega/\tau}; \tag{9}$$

the effective plasma frequency, ω_p, is greatly reduced from its bulk value for two reasons: (a) the effective average concentration of electrons is reduced by a factor $\pi r_0^2/\alpha^2$ (where r_0 is the radius of each wire and α^2 is the area of a 2d unit cell); (b) the effective mass of electrons is greatly enhanced, because the energy of the magnetic field due to the oscillating current is added to the kinetic energy of electrons.

If the metallic wires are of finite length, the electrons are somehow bound, and, as a result, an additional harmonic restoring force is generated. This is revealed in the permittivity given by (9) by replacing ω^2 by $\omega^2 - \omega_0^2$.

Pendry's proposal for the magnetic resonance was a system of two split metallic rings as in Fig. 3. This system possesses both inductance, L, and capacitance, C, and, as such, exhibits a resonance at $\omega_m = (LC)^{-1/2}$ which can be excited by a magnetic

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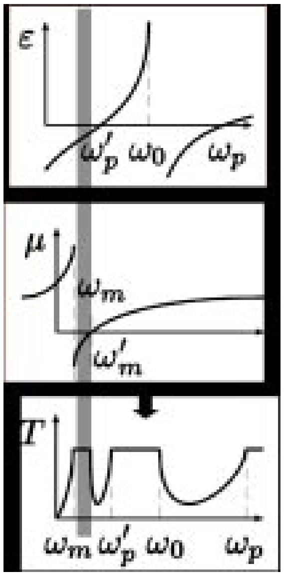

Fig. 4. In the frequency region where both $\varepsilon(\omega)$ and $\mu(\omega)$ are negative (shaded), a left-handed peak in the transmission coefficient is expected, as shown schematically in the bottom panel

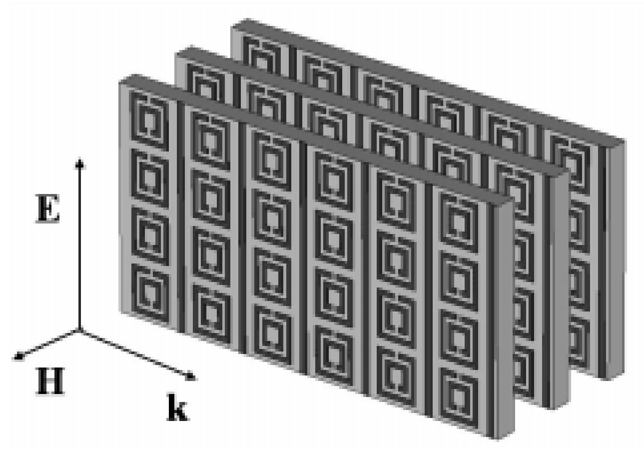

Fig. 5. Composite structure consisting of wires and SRRs on several boards; the orientation of the e/m field vectors is also shown

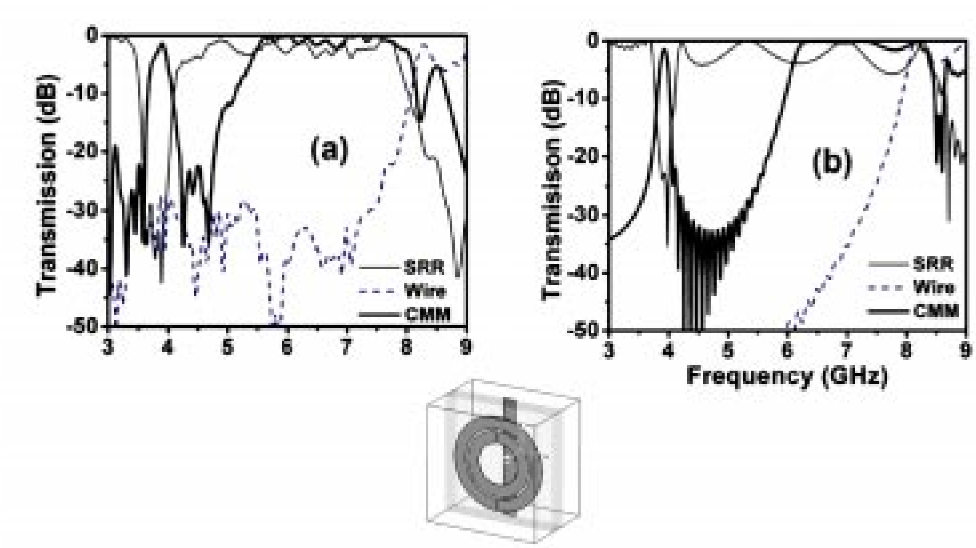

Fig. 6. Transmission coefficient vs frequency for a composite material (CMM, heavy solid line), SRRs only (light solid line), and wires only (dashed line). The left panel gives experimental results, and the right panel shows the theoretical calculations for the same optimized design

field normal to the plane of the rings. Hence, the permeability would be of the form

$$\mu(\omega) = 1 - \frac{\omega_{0m}^2}{\omega^2 - \omega_m^2 + i\omega/\tau_m}. \quad (10)$$

In Fig. 4, we plot schematically the expected behavior of $\varepsilon(\omega)$, $\mu(\omega)$, and $T(\omega)$ of a composite system consisting of wires and SRRs (see Fig. 5); $T(\omega)$ is the transmission coefficient.

Notice that the ω dependence of permittivity in Fig. 4 is the sum of the infinite wire response and the cut wire response (the latter coming from the SRRs which act as cut wires as well).

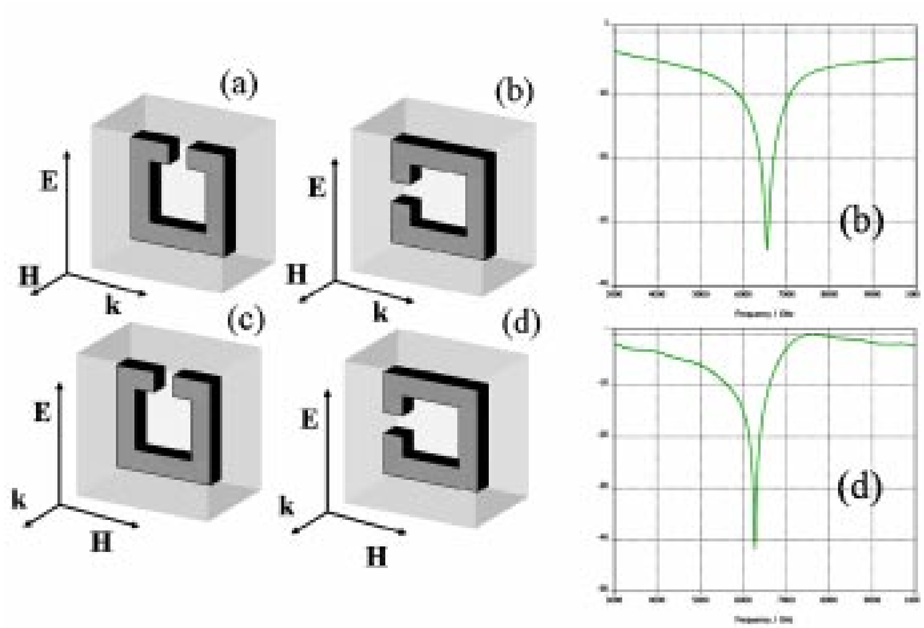

Fig. 7. Various orientations of the e/m field vectors relative to the SRRs. Magnetic resonance is excited by: the magnetic field only, case (a); the magnetic and electric field, case (b); the electric field only, case (d). In configuration (c), there is no excitation of the magnetic resonance. In the left panels, transmission vs frequency is plotted showing the magnetic resonance excitation for cases (b) and (d)

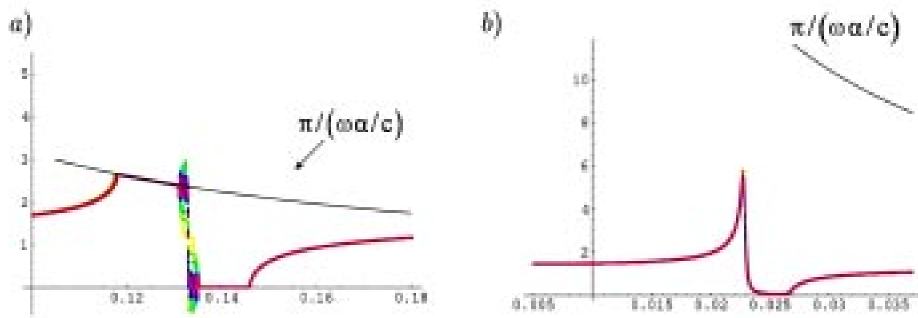

Fig. 8. Real part of the index n vs $\omega\alpha/c$ compared to $\pi/(\omega\alpha/c)$. The condition $n \ll \pi/(\omega\alpha/c)$ is violated in case (a), while it is obeyed in case (b)

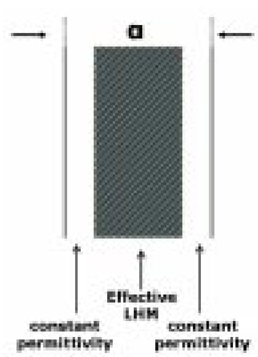

Fig. 9. Unit cell for a periodic effective medium

3. Results

Pendry's design was fabricated by the San Diego researchers [3] who demonstrated experimentally the negative refraction. Following this success, the field experienced an impressive growth in spite of initial objections (mostly based on misconceptions and inappropriate approximations). Our consortium [4], by optimizing the design, has obtained the highest transmission coefficient in a negative index band around 4 GHz. Indeed, we compare these experimental data for the transmission coefficient with theoretical results in Fig. 6. The agreement is very good. The theoretical

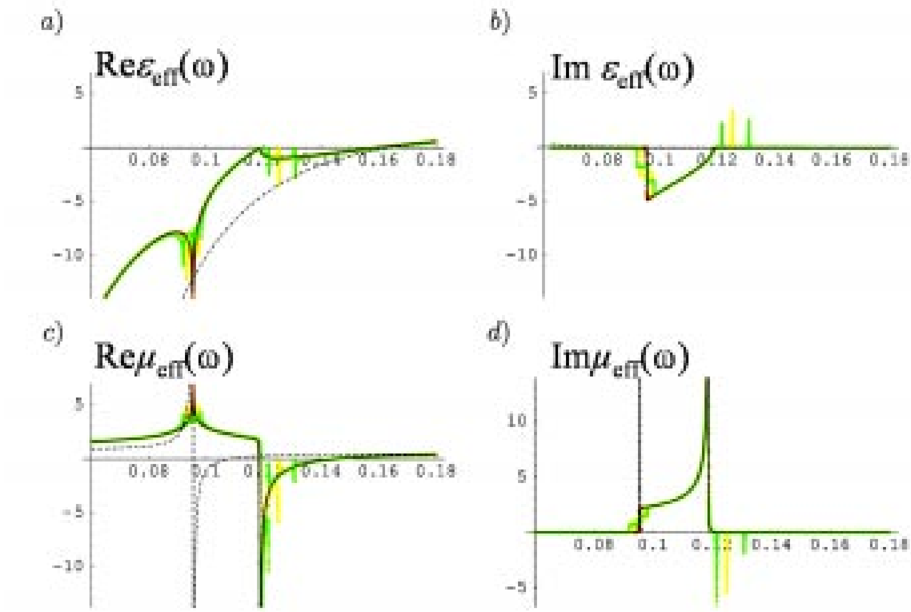

Fig. 10. When $\lambda_m \lesssim \alpha$, the effective $\varepsilon(\omega)$ and $\mu(\omega)$ for a uniform medium (solid lines) exhibit the anomalous ω-dependence (e.g., negative Imε, panel b) which disappears for a periodic effective medium (dashed lines)

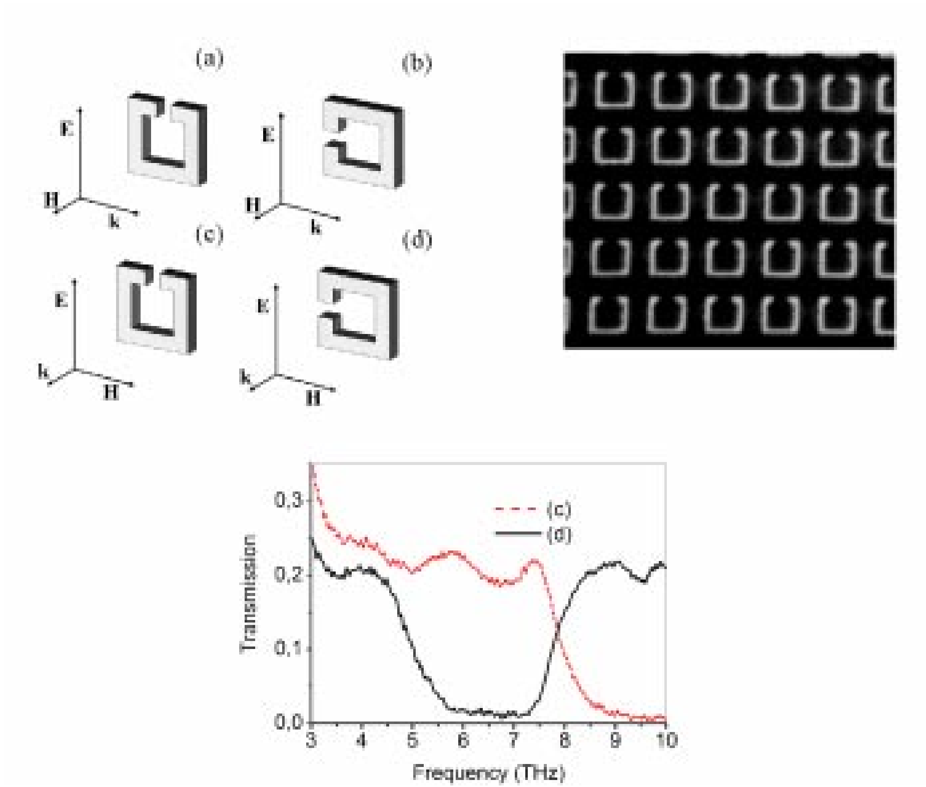

Fig. 11. Miniaturized single SRRs on boards (five layers) were fabricated. The transmission coefficient vs frequency for the almost normal incidence was measured for two polarizations demonstrating the electric field excitation of a magnetic resonance at 6 THz for polarization (d)

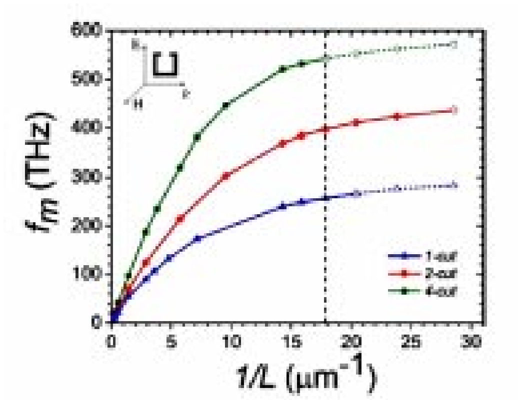

Fig. 12. Increase of the magnetic resonance frequency $f_m = \omega_m/2\pi$ vs the inverse size, $1/L$, of a single SRR with only one cut, two cuts (on opposite sides), and four cuts (on all sides)

results were obtained by employing several methods (TM = transfer matrix, MWS = microwave studio, FDTD = finite difference time domain).

It is worth to point out that the magnetic resonance of the SRR can be excited by an electric field as well, as shown in Fig. 7. This occurs when the electric field is parallel to the split sides of the SRRs; then, because of the asymmetry, a circulating current is set up by the electric field.

To describe the composite medium by an effective uniform $\varepsilon(\omega)$ and $\mu(\omega)$, the wavelength, λ, inside the medium, $\lambda = 2\pi c/\omega|n|$, must be much larger than twice the period, α, of the structure along the propagation direction; this implies that $n \ll \pi/(\omega\alpha/c)$. Because of the resonance value of n, this inequality is not always obeyed. In Fig. 8, we show a case where the real part of n, as it increases with ω, reaches a point where $n = \pi c/\omega\alpha$; beyond this point the effective uniform medium breaks down, and n is forced to follow the curve $\pi c/\omega\alpha$, until it comes again below this curve (Fig 8,a).

To face this breakdown, one can simulate the structure by a periodic (instead of uniform) effective medium, whose unit cell is shown in Fig. 9.

In Fig. 10, we plot the theoretical results (obtained by the TM method) for the effective $\varepsilon(\omega)$ and $\mu(\omega)$ as obtained by fitting the transmission and reflection either to a uniform or periodic effective medium. All the anomalous features present in the uniform effective medium case disappear from the periodic effective medium case, which exhibits the textbook behavior as in Eqs. (9) and (10). This shows that the anomalous features are simply consequences of the breakdown of the uniform effective medium approximation.

4. Towards Future Goals

To realize the full potential of the artificial negative index materials, the NIM band must be pushed to higher frequencies (possibly all the way to optical frequencies); furthermore, two- and three-dimensional designs must be developed and fabricated.

To push the magnetic resonance frequency to higher values one must reduce the size of SRRs. It is expected that both their effective capacitance and their effective inductance are proportional to the size, L, of the SRR (assuming that all other lengths scale linearly with L); thus, $\omega_m \sim 1/L$.

Our group at FORTH [5] fabricated, by microelectronic techniques, five layers of single SRRs on polyimide boards as shown in Fig. 11; the size of each single SRR is 5×5 μm^2 and the unit cell is $7\times7\times5$ μm^3. The transmission coefficient, $T(\omega)$, through the five layers (almost normal incidence) for both polarizations

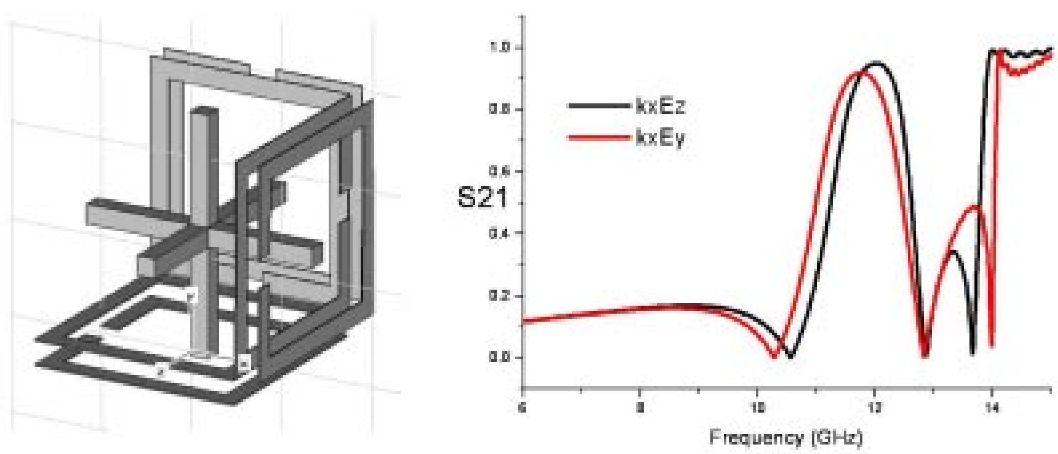

Fig. 13. Three-dimensional design (left) which shows the almost identical response for two polarizations

was measured. Polarization (d), which allows the excitation of a magnetic resonance by an electric field, shows a strong reduction in $T(\omega)$ for $\omega \simeq 6T$ Hz, while polarization (c) which is not coupled to the magnetic resonance does not exhibit any significant variation in this frequency range.

The Karlsruhe group in collaboration with the Ames group [6] succeeded in constructing a single layer of single SRRs with a unit cell $450 \times 450\,\mathrm{nm}^2$ which produced a magnetic resonance (excited by an electric field of the appropriate polarization) at 100 THz. More recent unpublished results by the same collaboration have pushed the resonance frequency to 200 THz, i.e. $\lambda = 1.5\,\mu$.

Theoretical calculations of the magnetic resonance frequency of single SRRs with one, two (on opposite sides), and four (on all sides) cuts vs the size of the SRRs show that the expected $1/L$ behavior is followed up to a point. But, as $1/L$ becomes larger, the curves tend to saturate, as shown in Fig. 12. No convincing explanation for this saturation is available.

For higher dimensional structures, we need symmetric elementary units to avoid the complicating phenomenon of the electric field excitation of the magnetic resonance. In Fig. 13, a design for a three-dimensional unit cell is shown; such a design has almost the same response for both polarizations, as it is demonstrated in Fig. 13.

1. *Veselago V.G.* // Sov. Phys. Usp. **10** (1968) 509.
2. *Pendry J.B., Holden A.J., Robbins D.J., Stewart W.J.* // IEEE Trans. Microwave Theory Tech. **47** (1999) 2075.
3. *Shelby R.A., Smith D.R., Schultz S.* // Science **292** (2001) 77.
4. *Aydin K., Guven K., Kafesaki M. et al.* // Opt. Lett. **29** (2004) 2623—2625.
5. *Katsarakis N., Konstantinidis G., Kostopoulos A. et al.* // Opt. Lett. (to be published).
6. *Linden S., Enkrich Ch., Wegener M. et al.* // Science **306** (2004) 1351—1353.

МАТЕРІАЛИ З НЕГАТИВНИМ ІНДЕКСОМ РЕФРАКЦІЇ: НОВІ РУБЕЖІ В ОПТИЦІ?

Є.Н. Економу

Р е з ю м е

Через 30 років після піонерських теоретичних робіт В.Г. Веселаго про гіпотетичні матеріали з негативним ефективним індексом рефракції Пандрі із співробітниками створив штучну структуру, масив розімкнутих кільцевих резонаторів (РКР), котра проявляє негативну сприйнятливість. Через два роки було вперше продемонстровано негативну рефракцію в складних структурах, що складаються з РКР з металічних дротів. В останні 4 роки ця нова і цікава область стрімко розвивається. У цій роботі представлено деякі важливі результати, отримані останніми роками, та проведено їхнє обговорення.

МАТЕРИАЛЫ С ОТРИЦАТЕЛЬНЫМ ИНДЕКСОМ РЕФРАКЦИИ: НОВЫЕ РУБЕЖИ В ОПТИКЕ?

Е.Н. Эконому

Р е з ю м е

Спустя 30 лет после пионерских теоретических работ В.Г. Веселаго по гипотетическим материалам с отрицательным эффективным индексом рефракции, Пандри с сотрудниками создал искусственную структуру, массив разомкнутых кольцевых резонаторов (РКР), которая проявляет отрицательную восприимчивость. Через два года была впервые продемонстрирована отрицательная рефракция в сложных структурах, состоящих из РКР и металлических проволок. В последние 4 года эта новая и интересная область стремительно развивается. В статье представлены некоторые важные результаты, полученные в последние годы, и дано их обсуждение.