Excitation of surface waves in a photonic crystal with negative refraction: The role of surface termination

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Results of the excitation of surface waves in a photonic crystal (PC) with negative refraction slab are presented. The role of the surface termination of the finite PC in exciting surface waves and enhancing the transmission is examined. It is demonstrated that only for a specific surface termination, the surface waves in the PC are excited in the same fashion as in a homogeneous medium. The existence of peaks well above the propagating modes in the calculated transfer function confirms the excitation of the surface modes and explains the high intensity of the field at the image plane as well as in both interfaces. The dependence of the excitation of the surface waves on the system length is analyzed. Finally, the unexpected high sensitivity of the focus position to the surface termination is evaluated.

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Recent experimental and theoretical results[1–3] have confirmed the existence of negative refraction in specific type of materials known as the left-handed materials (LHM) or metamaterials. These materials exhibit an unusual phenomenon: they refract the electromagnetic (EM) waves in the negative direction (i.e., towards the same side of the surface normal). Such phenomenon was predicted years ago by Veselago who showed that in a LHM **k**, **E**, and **H** form a left-handed set of vectors.[4] Pendry suggested years later that a slab of material with a negative permittivity ϵ and negative permeability μ can be used as a perfect lens.[5] Moreover, he argued that the negative medium amplifies the evanescent waves making the restoration of both propagating and evanescent waves in the image possible. Many recent papers[6–9] have studied negative refraction and have focused on the superlensing phenomenon or on the diffraction limit problem. Knowing that the key feature of these phenomena is the excitation of the surface waves, many groups have studied the excitation of surface waves and their role in the imaging for homogeneous medium[10–12] as well as in PCs.[13–17] Surface waves are localized waves parallel to the surface and decay exponentially away from the surface in either perpendicular direction. By virtue of the excited surface waves, the lens reconstitute the near field as well as the far field emanated from a point source to form the focus on the opposite side of the slab. Such a slab would amplify waves that carry information about features smaller than the wavelength. Surface waves are also found to enhance the total field intensity improving the transmission of optical devices. Furthermore, they are used to guide the EM emission in metallic and PCs in the forward direction which is called the beaming phenomenon by coupling the excited surface waves to the propagating ones. In the transfer function, defined by the amplitude ratio of a plain wave component at the focus and the source, the excitation of the surface waves is manifested as poles for parallel momentum above the propagating modes. Due to some losses inside the structure, these poles are damped and merely appear as finite peaks.[18,19] For a perfect lens the transfer function is unity for all parallel momenta. However, for the near perfect lens the transfer functions show an order of unity $[o(1)]$ behavior at small parallel momenta $k_{\parallel}$ which turns into exponential decay for large $k_{\parallel}$. The crossover between $o(1)$ behavior and exponential decay for a given lens defines the maximum parallel momentum $k_{\parallel,\max}$. Qualitatively, $k_{\parallel,\max}$ indicates the highest evanescent wave still restored by the lens, and thus defines the maximum attainable subwavelength resolution. Despite the discrete nature of the PC, the band structure indicates that the PC should behave as a homogeneous medium with a refractive index $n<0$ in a specific range of frequency. Since the band structure is obtained for an infinite periodic structure while the imaging phenomenon implies a limited thickness of the structure which involves moreover a not well-defined interfaces, the excitation of the surface waves on PC can be challenging. In a homogeneous medium, appropriate conditions lead to the excitation of surface waves easily. In general, surface waves do not necessarily exist in the finite PC slab; specific surface termination is needed to support surface waves. By surface termination we mean that the unit cells of the periodic PC located at the surfaces of the slab are chosen different from the bulk unit cells. In this paper, surface termination are done by cutting away part of the bars that are in the first and/or the last row of the PC. Homogeneous and PCs media do share the basic mechanism related to the superlensing phenomenon. Indeed, if negative refraction index $n \approx -1$ is expected from the band structure, focus formation would result in the PC as well as in the homogeneous medium. Due to the negative refraction at the interfaces we expect in both media an internal focus inside the structure. The existence of such internal focus excludes the possibility of the channeling behavior or what is called the funneling effect[20,21] suggested by some authors. However, in contrast with the homogeneous medium in which the position of the focus is at the expected position dictated by Snell's law, the position of the focus in the PC case may depend on the surface termination of the PC. So far, two major studies have been reported concerning the excitation of surface waves in PC. In the first one,[13–15] authors were concentrated on the enhancement of the transmission by coupling the surface waves to the propagating modes. In the second studies,[16,17] authors explored the physical mechanism of the excitation of surface waves. However, to our knowledge, the few papers studying the excitation mechanism of the surface waves in PC do not have real excitation of surface waves. In Ref. 16, the authors claimed enhancing the transmission by exciting the surface waves. However, in the transfer function presented, none of the cut had a peak in the transfer function beyond the maximum wave vector of the propagating modes k_0 implying an absence of surface waves excitation. In other studies,[17] the authors claimed "unrestricted subwavelength lensing" in a PC with a refractive index equal to −1 implying recovering all the parallel momentum up to infinity at the image plane. Such a situation cannot be true unless the lens is perfect with a transfer function equal to one for all the parallel momenta. In this paper, we demonstrate that it is possible to excite the surface waves in a PC for a specific surface termination. For comparison, excitation of surface waves in homogeneous medium is also presented. The existence of poles (peaks) in the transfer function as well as the field distribution that demonstrates the excitation and high intensity of surface waves are shown. This excitation is shown to depend on the specific cut as well as on the length of the system. Surprisingly, this study shows that the position of the focus is very sensitive to the surface termination of the PC.

The PC examined in this paper consists of a two-dimensional triangular array of dielectric bars with a dielectric constant $\varepsilon=9.61$ in air. The dimensions of each bar in the x and y directions are $r_x=0.40a$ and $r_y=0.80a$, respectively, where a is the lattice constant. This structure was studied earlier for negative refraction and superlensing behavior.[22] For this structure, the photonic band structure as well as the equifrequency surfaces (EFS) predict an isotropic negative refractive index of $n\approx-1$ for a frequency of $0.345\frac{c}{a}$ where c is the vacuum speed of light. This frequency was used in all the PC simulations. A point source is placed at a distance A from the first interface of the PC slab of thickness D and a focus is expected on the opposite side of the slab at a distance B from the second interface. Finite-difference-time domain (FDTD) method was used to observe time and space evolution of the emitted EM waves.[23] In all the FDTD simulations, perfect matched layer (PML) (Ref. 24) boundary conditions were used. The source emitted an almost monochromatic TM polarization (E along the rods) of desired dimensionless frequency. In addition, the transfer-matrix method (TMM) (Ref. 25) that utilizes periodic boundary condition was used to compute the stationary transfer function for a monochromatic plane wave incident normally to the PC slab surface. In the following discussion dimensionless units are used where all the lengths are measured in units of the linear size a of the unit cell and all the frequencies are in units of the vacuum speed of light divided by a. In this unit, the vacuum speed of light becomes $c=1$ and the wavelength $\lambda=2\pi/\omega$.

In order to examine different surface termination, a point source was placed at a distance of 0.125λ in front of our finite PC which has a lateral width 15.87λ and a thickness 2.7λ. By checking different cuts of the surface rods [see the inset of Fig. 1(a)] we tried to find optimum transmission while exciting surface waves at both interfaces of the PC with $k_\parallel > k_0$. Figure 1(a) shows the focus peak intensity versus the different cuts and Fig. 1(b) shows the corresponding

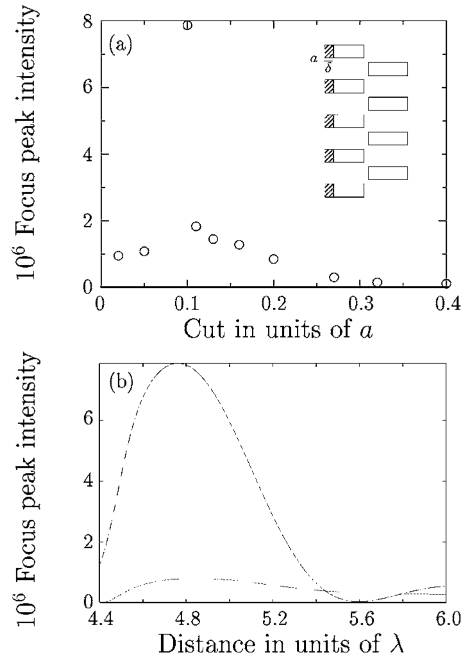

FIG. 1. (a) Focus peak intensity versus the cut in units of lattice constant. The inset shows the definition of the surface cut. (b) The corresponding field intensity at the focus plane versus the distance from the second interface for the two cuts: the solid and dotted lines correspond to the $\delta=0.10a$ and $\delta=0.20a$ cuts, respectively.

field intensity of the focus for two chosen cuts with $\delta=0.1a$ and $\delta=0.20a$, respectively. Among the several cuts shown in Fig. 1(a), the cut of $\delta=0.10a$ has the highest focus peak intensity which is more than 4 times higher than the focus peak intensity of the other surface terminations and almost 8 times higher than the one with $\delta=0.20a$ [Fig. 1(b)]. In order to determine the source of the high intensity of the focus peak, transfer functions for both cuts were examined. Figure 2 shows the dependence of the transfer function on the parallel momentum $k_\parallel$. The transfer function shows the expected crossover behavior between $o(1)$ for the propagating modes and the exponential decay afterwards. For the propagating modes, the $\delta=0.10a$ cut case (solid line) has the transfer function close to one, while the $\delta=0.20a$ case (dotted line) shows some deviation from the $o(1)$ behavior with a minimum at around $k_\parallel/\omega=0.7$. With a refractive index

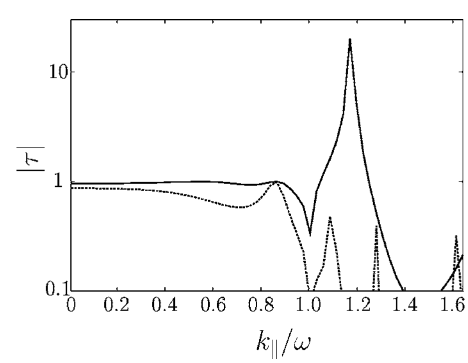

FIG. 2. Transfer function versus the parallel momentum for $\delta=0.10a$ cut (solid line) and $\delta=0.20a$ cut (dotted line).

$n=-1$ and matched interfaces (i.e., the impedance equal to one), a homogeneous and isotropic structure should not show any reflection at the interfaces. However, for a discrete PC which has in addition a limited thickness as well as nonunity effective impedance, reflection takes place at the interfaces.[26] Thus, Fabry-Perot oscillations can be seen in the transfer function. The small dip in Fig. 2 is due to the Fabry-Perot oscillation. After the exponential decay, both cuts present peaks in the transfer function that are associated with surface waves. The $\delta=0.10a$ cut has a significant peak at $k_{\parallel,\max}=1.2k_0$ with an intensity 20 times higher than the propagating modes. $k_{\parallel,\max}=1.2k_0$ corresponds to the highest evanescent wave still restored by the PC lens, and consequently, a maximum attainable subwavelength resolution of $\Delta x_{\min}\sim 2\pi/1.2k_0$. The $\delta=0.20a$ cut has multiple peaks with much lower amplitude as compared to the $\delta=0.10a$ case and weaker contribution to the transmission is expected. The observed higher transmission can result from the propagating mode if they are closer to one without Fabry-Perot structure or it can result from the peak of the surface waves at $k_\parallel > k_0$ if the intensity of this peak is considerable. If the intensity of this surface waves peak is several times higher than the propagating modes which is the case in Fig. 2, most likely the high intensity of the transmission results from the contribution of surface waves. In that case the contribution of the propagating modes will not be significant. Further insight about the field distribution when the surface waves are excited can be gained from Fig. 3(a) which plots the time average field distribution across the structure. For all the field plots, the three horizontal lines represent the source line, the first, and the second interfaces of the PC, respectively. 365 time steps per period T $\left(T=\frac{2\pi}{\omega}\right)$ and a fine discretization of 40 and 160 mesh per unit cell were used. As expected, high intensity of the field distribution is observed at both interfaces as well as in the image plane for the $\delta=0.10a$ cut case. The profile of surface waves as standing waves in the parallel direction and of decaying waves in the normal direction to the interface are clearly shown at both interfaces for this specific cut compared to the continuum case of Fig. 3(b). For this case, the surface modes propagate in the parallel direction with an expected parallel momentum higher than k_0. For comparison purposes and in order to see if the surface waves in PCs are excited as the surface plasmon polaritons (SPP) (Refs. 10 and 12) in homogeneous media, the time average field intensity of a point source located at 0.125λ from the first interface of a homogeneous structure with a negative refractive index $n=-1$ and the same structure parameters as the PC are plotted in Fig. 3(b). The field patterns of the surface waves on both interfaces looks similar in the two cases although with different internal field patterns. Despite the discrete nature of the PC, this similarity implies that the surface modes in periodic crystals can be excited in the same way as in the continuum. It is important to notice that in both homogeneous and PC media, the local enhancement of the internal focus is observed. However, this internal focus is not as clear for the PC case due to the spatial modulation of the field distribution caused by the discrete nature of the PC. It can be seen more clearly in Fig. 4.

We further investigate the effect of the thickness of PC on the excitation of the surface modes. The same type of cut (i.e., $\delta=0.10a$) was simulated for different thicknesses of the PC. The time average field distribution for three structures of thicknesses 1.2λ, 1.5λ, and 2.1λ are plotted in Fig. 4 [in addition to the thickness 2.7λ in Fig. 3(a)]. From Fig. 3 and Fig. 4, we can observe that for this specific cut there are only particular thicknesses for which the surface waves are excited. For instance, surface waves were supported by the $\delta=0.10a$ cut structure with thickness of 1.2λ and 2.7λ as shown in Fig. 4(a) and Fig. 3(a), respectively. However, the two other thicknesses did not support surface waves. For the cases in which the surface waves were excited, the time average field shows exponential decay of the surface waves perpendicular to the surface and standing waves parallel to the interfaces; for the cases that did not support any surface waves, no localized standing waves along the interfaces were observed. Note that by changing the thickness of the system the appearance of the field distribution was changed. Thus, exciting surface waves in PCs do not depend only on a specific cut but also on the thickness of the system. It can be concluded that both the surface termination as well as the thickness of the structures are important parameters which must be chosen appropriately to excite surface modes on the PC. Another important feature found in all the different structures studied and plotted in Figs. 3 and 4 is the internal focus inside the structure. This supports the assertion that the left-handed behavior in our case is not due to the channeling

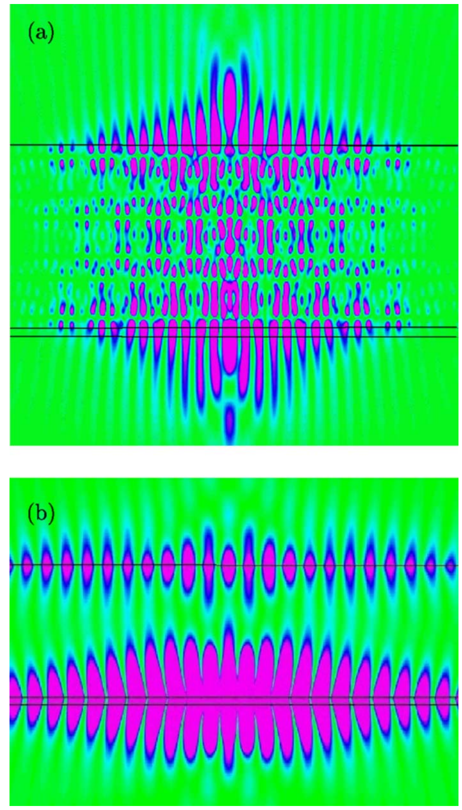

FIG. 3. (Color online) (a) Time average field intensity of a point source localized at 0.125λ from the first interface propagating inside a PC with the following parameters: cut $\delta=0.10a$, lateral width$=15.87\lambda$ and thickness$=2.7\lambda$. (b) Time average field intensity of a point source propagating through a homogeneous medium with the same parameter as the PC.

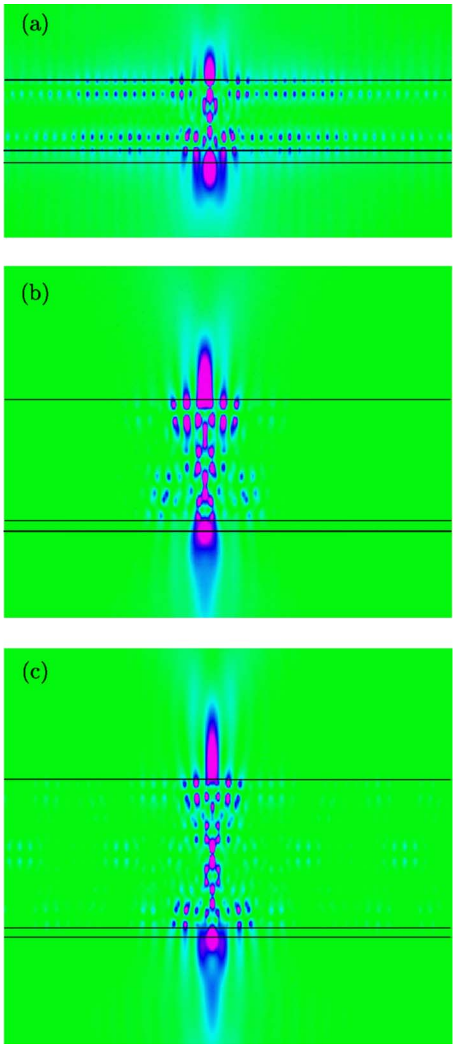

FIG. 4. (Color online) Time average field intensity of a point source localized at 0.125λ from the first interface propagating inside a PC with lateral width of 15.87λ and a cut of $\delta=0.10a$ for three different thickness 1.2λ (a), 1.5λ (b), and 2.1λ (c).

effect[20,21] or what is known in other PC structure as a funneling effect but is really due to the negative refraction happening at the interfaces of the PC.

According to the band structure, we expect the PC to behave like an homogeneous medium. Thus, the structure fulfills the lensing equation $A+B=D$ and shows a focus. The position of the focus changes accordingly to the Snell's law by changing the thickness of the slab. Many factors influence the position of the focus: the value of the refractive index inside the structure, the impedance which might give reflection at the interface, and the surface termination of the structure. In this particular PC, we did notice that the position of the focus is surprisingly dependent on the surface termination of the structure. To investigate the effect of the surface termination on the position of the focus, the complete structure as well as two others with different cuts were simulated. Figure 5 presents the time average field intensity of a point source propagating in a PC with a thickness of 2.7λ. Figure 5(a) shows the complete structure (without any cut) and Fig. 5(b) and Fig. 5(c) show the results for the $\delta=0.20a$ and $\delta=0.40a$ cuts, respectively. For the complete structure [Fig. 5(a)], after the internal focus occurring inside the structure, the field spreads out towards the second interface where

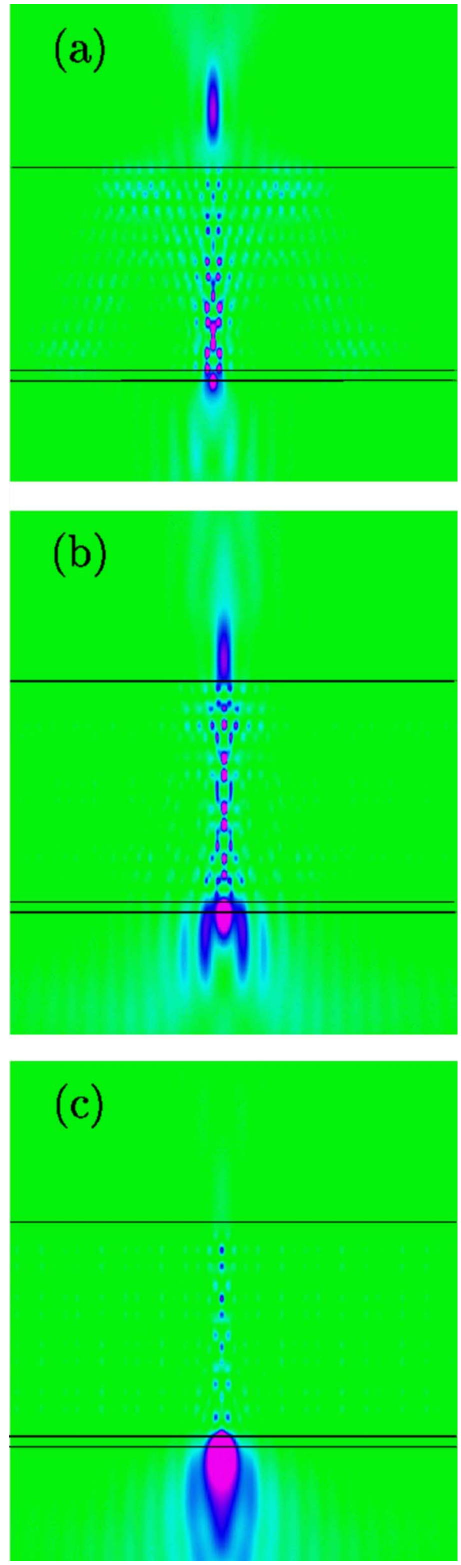

FIG. 5. (Color online) Time average field intensity of a point source localized at 0.125λ from the first interface propagating inside a PC with lateral width of 15.87λ and a thickness of 2.7λ for the complete structure (a), $\delta=0.2a$ cut (b), and $\delta=0.40a$ cut (c).

some reflection from the second interface occurs. Some reflections back and forth from the two interfaces take place and a clear focus is formed outside the structure located at 0.69λ. When a cut is introduced, the field distribution inside the PC is affected and the back and forth reflection seen in the complete structure disappears for the other two cuts of $\delta=0.20a$ and $\delta=0.40a$. The main difference among the three cases is the position of the focus. For the $\delta=0.20a$ cut [Fig. 5(b)], the focus moves further towards the second interface at approximately 0.28λ and for the $\delta=0.40a$ cut [Fig. 5(c)] the focus disappears completely. It is important to note that the change of the position of the focus is large compared to what would be expected from the shortened structure. The simulations were repeated with only one interface cut. The field distributions for such structures were effected and the position of the focus changed.

In conclusion, we succeeded in exciting the surface waves in the PC. Despite the discrete nature of the PC, the excited surface waves exhibited typical profile of standing waves parallel to the interfaces and of decaying waves in the normal direction, similar to what was earlier observed in the homogeneous medium. The surface waves were excited only for specific cuts. The transfer function results confirmed the excitation of surface waves for the specific surface termination and explained the high intensity at the image plane as well as in both interfaces. From the dependence of the surface wave excitation on the thickness of the PC it can be inferred that a cut which supports surface waves for a given thickness of the structure would not necessarily support them for another thickness. Thus, we can conclude that there is no universal cut to support surface waves. Additionally, the surface wave excitation is dependent on the system length. It was demonstrated, for this particular PC, that introducing a cut effects the reflection at the interface, the field distribution and surprisingly the position of the focus.

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[1] R. A. Shelby, D. R. Smith, and S. Schultz, Science **292**, 779 (2001).
[2] S. Foteinopoulou, E. N. Economou, and C. M. Soukoulis, Phys. Rev. Lett. **90**, 107402 (2003).
[3] D. R. Smith, W. J. Padilla, D. C. Vier, S. C. Nemat-Nasser, and S. Schultz, Phys. Rev. Lett. **84**, 4184 (2000).
[4] V. G. Veselago, Usp. Fiz. Nauk **92**, 517 (1964) [Sov. Phys. Usp. **10**, 509 (1968)].
[5] J. B. Pendry, Phys. Rev. Lett. **85**, 3966 (2000).
[6] C. Luo, S. G. Johnson, J. D. Joannopoulos, and J. B. Pendry, Phys. Rev. B **65**, 201104(R) (2002).
[7] E. Cubukcu, K. Aydin, E. Ozbay, S. Foteinopoulou, and C. M. Soukoulis, Phys. Rev. Lett. **91**, 207401 (2003).
[8] E. Cubukcu, K. Aydin, E. Ozbay, S. Foteinopoulou, and C. M. Soukoulis, Nature (London) **423**, 604 (2003).
[9] P. V. Parimi, W. T. Lu, P. Vodo, and S. Sridhar, Nature (London) **426**, 404 (2003).
[10] X. S. Rao and C. K. Ong, Phys. Rev. B **68**, 113103 (2003).
[11] X. S. Rao and C. K. Ong, Phys. Rev. E **68**, 067601 (2003).
[12] R. Ruppin, J. Phys.: Condens. Matter **13**, 1811 (2001).
[13] P. Kramper, M. Agio, C. M. Soukoulis, A. Birner, F. Muller, R. B. Wehrspohn, U. Gosele, and V. Sandoghdar, Phys. Rev. Lett. **92**, 113903 (2004).
[14] E. Moreno, F. J. Garcia-Vidal, and L. Martin-Moreno, Phys. Rev. B **69**, 121402(R) (2004).
[15] J. Bravo-Abad, F. J. Garcia-Vidal, and L. Martin-Moreno, Phys. Rev. Lett. **93**, 227401 (2004).
[16] S. Xiao, M. Qiu, Z. Ruan, and S. He, Appl. Phys. Lett. **85**, 4269 (2004).
[17] X. Wang and K. Kempa, Phys. Rev. B **71**, 085101 (2005).
[18] G. Gómez-Santos, Phys. Rev. Lett. **90**, 077401 (2003).
[19] D. R. Smith, D. Schurig, M. Rosenbluth, S. Schultz, S. A. Ramakrishna, and J. B. Pendry, Appl. Phys. Lett. **82**, 1506 (2003).
[20] Z.-Y. Li and L.-L. Lin, Phys. Rev. B **68**, 245110 (2003).
[21] L.-S. Chen, C.-H. Kuo, and Z. Ye, Phys. Rev. E **69**, 066612 (2004).
[22] R. Moussa, S. Foteinopoulou, Lei Zhang, G. Tuttle, K. Guven, E. Ozbay, and C. M. Soukoulis, Phys. Rev. B **71**, 085106 (2005).
[23] A. Taflove, *Computational Electrodynamics-The Finite Difference Time-Domain Method* (ArtechHouse, Boston, 1995); K. S. Yee, IEEE Trans. Antennas Propag. **14**, 302 (1966).
[24] J.-P. Berenger, J. Comput. Phys. **114**, 185 (1994); IEEE Trans. Antennas Propag. **44**, 110 (1996).
[25] P. Markoš and C. M. Soukoulis, Phys. Rev. E **65**, 036622 (2002).
[26] Z. Ruan, M. Qiu, S. Xiao, S. He, and L. Thylen, Phys. Rev. B **71**, 045111 (2005).