

High transmittance left-handed materials involving symmetric split-ring resonators

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Abstract

We present theoretical and experimental results for a new design of highly symmetric, multigap split-ring resonators (SRRs), as well as for left-handed materials of a broad and high transmittance left-handed band, achieved by combining those symmetric SRRs with continuous wires. Studying in detail, both theoretically and experimentally, our proposed symmetric SRRs, we proved that they avoid the electric field excitation of the magnetic SRR resonance; thus they are appropriate for the creation of two-dimensional and three-dimensional left-handed materials. Finally, we propose critical design rules for the development of low-loss and broad-band left-handed materials.

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Artificial structures involving metallic and dielectric components that exhibit both negative effective permittivity, ϵ , and effective permeability, μ , over a common frequency range have received a lot of attention recently [1]. Such structures, usually referred to as left-handed materials (LHMs), show several amazing electromagnetic properties, such as negative index of refraction, opposite phase and group velocities, negative refraction, absence of reflection, flat lens focusing, opposite radiation pressure, etc. [2].

Natural materials with both ϵ and μ negative do not exist; hence the need for artificial structures. In constructing such structures the key concept is resonance. Indeed, near a resonance the response of a system to an electric field (as measured by the permittivity, ϵ) or to a magnetic field (as measured by the permeability, μ) exhibits the characteristic shape shown in Fig. 1. If the resonance is strong enough and sharp enough, there will be a frequency region (above the resonance frequency, ω_R) where $\text{Re}(\epsilon)$ or $\text{Re}(\mu)$ would be negative and $\text{Im}(\epsilon)$ or $\text{Im}(\mu)$ would be very small. It is worth to point out that metals for $\omega < \omega_p$ (where ω_p is the plasma frequency, typically in the ultraviolet) exhibit $\text{Re}(\epsilon) < 0$; actually their permittivity can be approximated by a Drude type formula, $\epsilon(\omega) = 1 - \omega_p^2/(\omega^2 + i\omega/\tau)$, where τ is a relaxation time.

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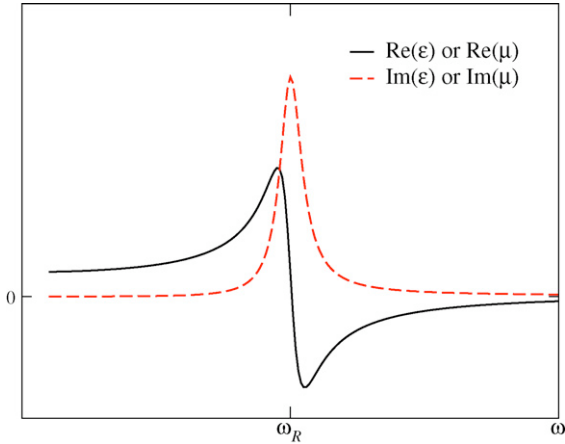


Fig. 1. Response of a system as a function of frequency, ω , for frequencies in the vicinity of a resonance frequency ω_R .

Hence, since one can easily have $\text{Re}(\epsilon) < 0$, employing metals, the most difficult task is to produce a structure exhibiting a strong magnetic resonance. A solution of this problem was advanced by Pendry [3]; Pendry proposed a metallic double split-ring structure, known as split-ring resonator (SRR), which behaves like an equivalent inductor-capacitor (LC) circuit with a magnetic resonance at $\omega_m = 1/\sqrt{LC}$ (ω_m depends on the size of the rings; for mm scale structures it falls in the microwaves).

Combining SRRs with appropriate metallic elements, one in principle can achieve left-handed behavior. To obtain though high transmission through the resulting LHM, the effective surface impedance, $z = \sqrt{\mu/\epsilon}$, should not be much different from one, a requirement which implies that ω_p should be close to ω_m . This requires a large reduction of ω_p , compared to that of a bulk metal, which can be achieved by parallel metallic wires of properly sized cross-section [4].

In implementing this split-ring resonator (SRR) and the parallel wires design, one has to take into account two complications: First, the SRRs exhibit an electric short-wire-like resonance (resonance in the electrical permittivity, ϵ), at a frequency ω_0 , in addition to their magnetic resonance at ω_m [5]; the frequency ω_0 , for symmetric relative to the applied electric field SRRs, roughly depends on the length of the SRRs in the direction of the applied electric field, being higher for shorter lengths. This resonant permittivity is added to the Drude-like permittivity of the parallel wires, producing an effective ω'_p which is lower than the ω_p of the parallel wires [5,6] and possibly lower than ω_m , thus wiping out the frequency range at which both ϵ and μ are expected to be negative. Second, for asymmetric structures with respect to the direction of the applied

electric field, $\mathbf{E}$, the induced circular currents giving rise to the magnetic resonance at ω_m can be excited by the electric field, giving rise to a resonant polarization at ω_m , parallel to the applied $\mathbf{E}$; as a result ϵ exhibits a resonance behavior at ω_m as well. This electric excitation of the magnetic resonance (EEMR) in some cases is very advantageous, but it is detrimental for the design of three-dimensional (3D) LHMs, as it imposes strong positive ϵ regimes where negative ϵ is required [7,8].

There is, therefore, a serious need for the realization of symmetric LH structures in order to target the fabrication of efficient three-dimensional LH metamaterials. In earlier attempts, P. Gay-Balmaz and O.J.F. Martin studied alternative magnetic resonant building blocks with a higher degree of isotropy [9], not fully isotropic unit cell though. Recently, Th. Koschny et al. proposed a three-dimensional isotropic LHM design that allows left-handed behavior for any direction of propagation and any polarization of the electromagnetic waves [10]. Finally, Ozbay et al. introduced the so-called “labyrinth” structure, which consists of four rings, and concluded through numerical and experimental studies that their structure does not show the EEMR effect [11].

In this work we designed and fabricated a highly symmetric double-ring SRR structure with two gaps in each of the rings. Moreover, we combined those SRRs with parallel metallic wires, and we achieved a wide and of high transmittance left-handed band, as was demonstrated by transmission measurements and corresponding simulations. Our SRR structure was analyzed both theoretically and experimentally, for two different orientations of the applied electric field (Fig. 2(a)) and the absence of the EEMR effect was demonstrated. The analysis of both the SRR structure and of the combined metamaterial (CMM) of SRRs and wires gave also guidelines for the achievement of a wide left-handed regime associated with high transmission intensities, issues that we discuss also in the present manuscript.

All the SRRs and SRRs and wires materials studied here have been fabricated using a conventional printed circuit board process with 30 μm -thick copper patterns on 1.6 mm thick FR-4 dielectric substrates. The FR-4 boards have a dielectric constant ϵ_b of ~ 3.3 and a dissipation factor $\tan \delta \sim 0.017$ at 1.5 GHz. Both the SRR-only structures and the CMMs consisting of SRRs and continuous wires (wires printed on the backside of the boards where the SRRs are printed) have been fabricated. The geometry and the dimensions for the SRRs are described in Fig. 2. After their fabrication, the

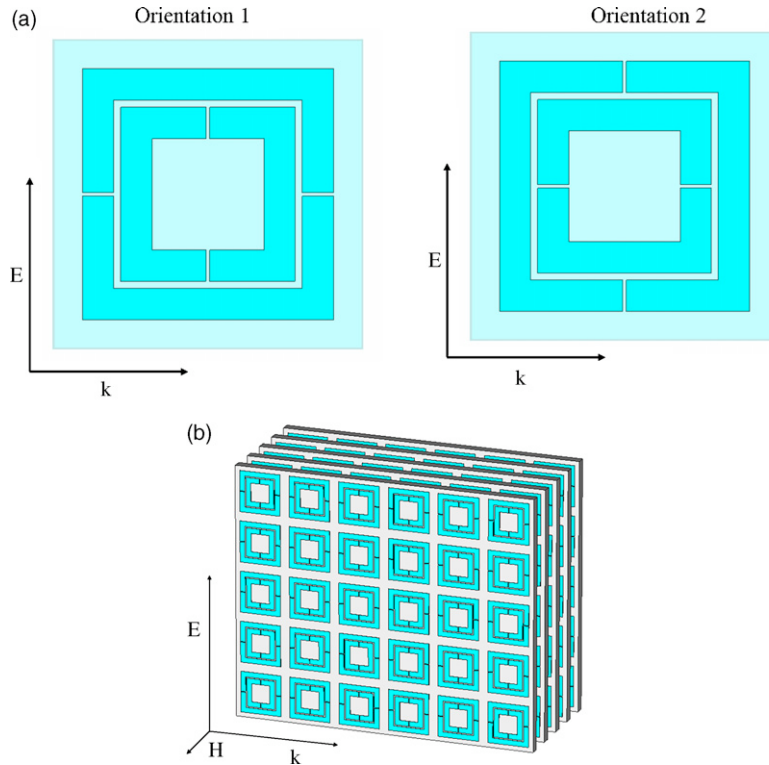


Fig. 2. (a) The unit cell of the proposed double ring design with two cuts per ring, for two orientations with respect to the incoming electromagnetic field, characterized by the wave vector, $\mathbf{k}$, the electric field, $\mathbf{E}$, and the magnetic field, $\mathbf{H}$ (perpendicular to the plane of $(\mathbf{E}, \mathbf{k})$ —not shown here). The system parameters are as follows: unit cell size along $\mathbf{E}$ and $\mathbf{k}$ axes is 8.8 mm, SRR linear size = 7.2 mm, rings' width = 0.9 mm, gaps width = 0.1 mm, rings distance in the double ring = 0.2 mm, metal thickness = 30 μm . The board where the SRRs are printed is FR-4, with thickness 1.6 mm. (b) The periodic arrangement of our SRRs.

FR-4 boards with the metallic structures have been stacked together in a one-dimensional periodic arrangement perpendicular to the $\mathbf{H}$ -axis (see Fig. 2(b)) in order to compose the SRRs and the SRRs and wires metamaterials. The transmission measurements of those metamaterials have been performed in free space using a Hewlett-Packard 8722 ES network analyzer and microwave standard-gain horn antennas.

In Fig. 3 we show theoretical transmission (S_{21}) results for the design and the two orientations shown in Fig. 2(a), for one unit cell along propagation direction (considering infinite systems along the other directions). The transmission has been calculated using the Finite Integration Technique, employed through the commercial software MicroWave Studio. The sharp characteristic dips at about 5.5 GHz, shown in Fig. 3, indicate a negative μ regime, which, combined with a positive ϵ , results to an imaginary propagation constant and, hence, to a dramatic reduction of the transmission. The negative μ origin of the ~ 5.5 GHz dips shown in Fig. 3 is confirmed by plots of the currents at the SRR rings, indicating strong circular currents in this regime; it is also confirmed by simulations

of the effective ϵ and μ parameters for the structure, presented later on in this manuscript.

From Fig. 3 one can see also that the resonance frequency ω_m is higher in orientation 1 than in orientation

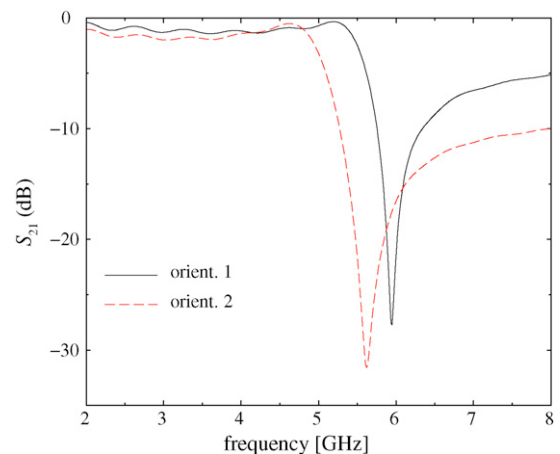


Fig. 3. Simulation results for the transmission, S_{21} (in dBs), of the two configurations shown in Fig. 2(a).

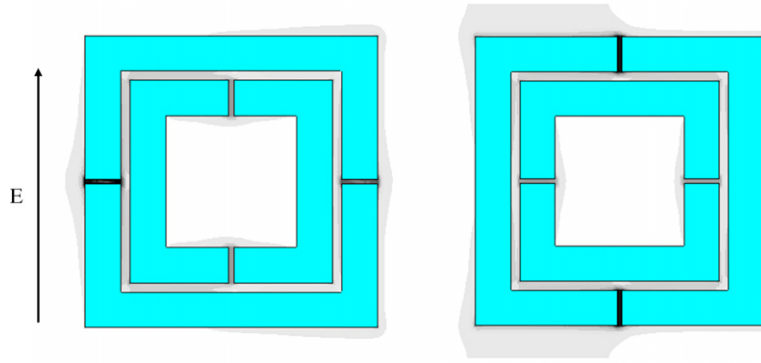


Fig. 4. The size of the electric field ($|E|$) at the magnetic resonance frequency shown in Fig. 3. Left panel shows the field for orientation 1 and right panel for orientation 2. Dark (light) color indicates high (low) field values. For orientation 2 the field between the inner and the outer ring is stronger than in orientation 1, as well as the field among neighboring unit cells along the E direction, indicating a larger effective capacitance and, hence, a lower resonance frequency (assuming that the effective inductance is about the same).

2. We attribute this difference mainly to the higher effective capacitance in the configuration 2: The total effective capacitance is due not only to the gaps and the inter-rings regime; there is also a capacitance between SRRs belonging to neighboring unit cells along the E direction. This capacitance is larger for orientation 2, since, due to the gaps, there is accumulation of charges at the neighboring sides of the successive SRRs along E direction. This argument is supported by corresponding electric field, E , plots, shown in Fig. 4 for the two orientations. It is also supported from associated simulations of the same SRRs but with larger unit cell size along the E direction; there, the magnetic resonance frequencies of the two orientations coincide.

In Fig. 5 we show experimental and theoretical transmission results for our SRRs in a periodic arrangement, for both orientations 1 and 2. The

experimental results have been obtained for a system of $15 \times 15 \times 20$ unit cells (15×15 u.c. in the SRRs plane; this allows examination and direct comparison of both orientations using the same sample); the theoretical results concern a system of six SRRs along propagation direction, employing periodic boundary conditions along the other directions (note that the smaller number of unit cells along propagation direction in the theory, compared to the experiment, is dictated only by computational memory limitations; six unit cells though are enough for the stabilization of the negative μ regime). The geometry of the unit cell and the SRR parameters are mentioned in Fig. 2. The distance, a_H , between successive SRR boards along the perpendicular to the SRRs plane direction is 5.5 mm (=lattice constant along H direction).

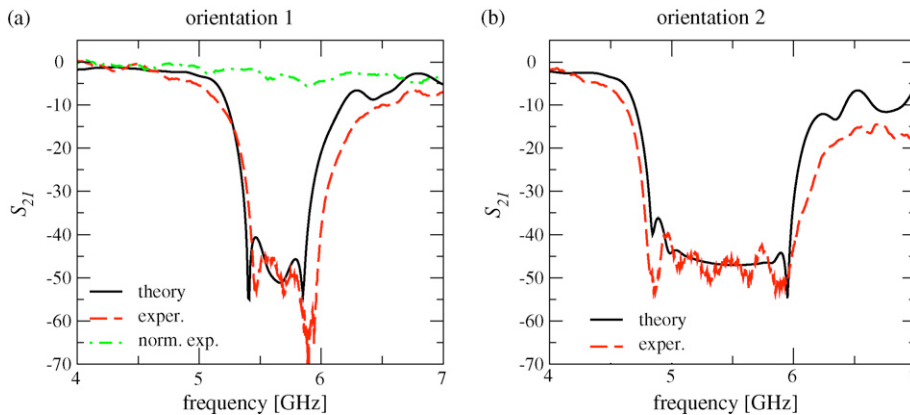


Fig. 5. Simulation transmission (S_{21}) results for a row of six SRRs along propagation direction (solid lines) and transmission measurements (in dB) through a system of $15 \times 15 \times 20$ SRRs (dashed lines) for each of the two orientations studied. The dotted-dashed line in left-panel shows the measured S_{21} for a third orientation, where the propagation vector $\mathbf{k}$ is perpendicular to the plane of the SRRs (normal incidence) and the external E as in orientation 1.

An important point to notice in Fig. 5 is the much broader negative μ regime for the orientation 2, compared to that of orientation 1. We attribute this broader and hence more desirable resonance of orientation 2 to the fact that, there, the neighboring sides of the neighboring along propagation direction SRRs are uninterrupted, permitting thus a stronger coupling of the corresponding currents, which is translated to a larger positive mutual inductance between neighboring along propagation direction SRRs.

Another point to notice in Fig. 5 is that the experimental data are in excellent agreement with the theoretical results, reproducing both the slight difference in the resonance frequency between the two orientations and the broader resonance of orientation 2.

To validate that our SRR structures are free from the EEMR effect, i.e. no electric resonance close to the magnetic resonance regime is present, and thus the structures are offered for the creation of 2D left-handed materials, we examined how our structures behave under normal incidence: In the left-panel of Fig. 5 we show also (see dotted–dashed line) the experimental transmission results for a third orientation, where the propagation vector $\mathbf{k}$ is perpendicular to the plane of the SRRs and the external $\mathbf{E}$ is as in orientation 1. In such an orientation the magnetic field is not expected to excite the magnetic resonance, since the magnetic flux through the SRR is zero; neither the electric field, $\mathbf{E}$, can excite this resonance, since there is mirror symmetry of the SRR with respect to the direction of $\mathbf{E}$ [7,8,12]. Indeed, the experimental measurements verified these expectations and showed that the transmission exhibits no dip for $\mathbf{k}$ perpendicular to the SRR plane (dotted–dashed curve), suggesting that the electric field does not excite the magnetic resonance.

To unambiguously confirm that the transmission dips around 5.5 GHz observed in our structures (see Fig. 5) are due to the effective negative magnetic permeability of the structures, and thus to proceed to the exploitation of those structures for the creation of left-handed materials, we calculated the effective permeability versus frequency from the theoretical transmission and reflection data, through the standard retrieval procedure [13]. The results for the real part of the effective μ for both orientations 1 and 2 are shown in Fig. 6, where the resonance structure of $\text{Re}(\mu)$ with the negative μ regime for each one of the two orientations is proved, and it also found to exhibit all the features deduced from Figs. 3 and 5.

The negative μ demonstrated in Fig. 6 shows the capability of our structures for the creation of left-handed behavior. This capability is demonstrated in

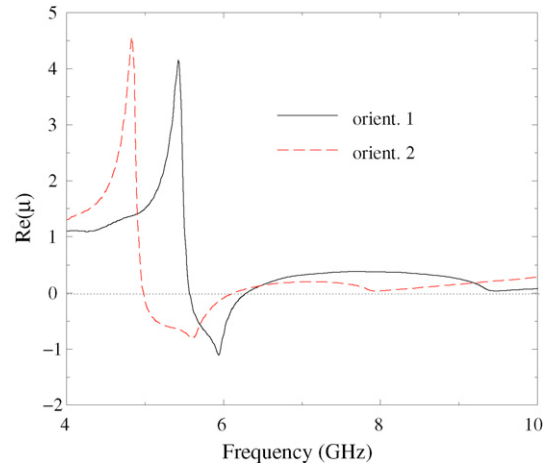


Fig. 6. Simulation results for the real part of the effective permeability, μ , of our SRR system, as obtained from the theoretical transmission and reflection data, through the standard retrieval procedure.

Fig. 7, where we present transmission measurements and simulations for metamaterials combining our multigap SRRs with continuous wires. The measurements concern two $15 \times 5 \times 20$ u.c. systems (5 u.c. along propagation direction; 20 u.c. along the $\mathbf{H}$ direction); each unit cell (see Fig. 7(c)) contains our multigap double-ring SRR, of the orientation 1 in respect to the incoming electromagnetic field, together with an uninterrupted wire of 2 mm width, to produce the required negative permittivity in a frequency range overlapping with the frequency range over which the permeability is negative (due to the magnetic resonance of the SRRs). In the first system (see Fig. 7(a)) the lattice constant along the perpendicular to the SRRs direction is 5.5 mm, while in the second (see Fig. 7(b)) it is 3.2 mm.

Examining the transmission for the system of Fig. 7(a), one can see that the left-handed regime is very wide – extending from 5.5 to 6.1 GHz – and of high transmittance. It is worthwhile to point out that the reflectance at the left-handed regime is less than 1 percent, indicating a very good impedance match between the metamaterial and the free space. The observed –5 dB reduction of LH transmission is almost exclusively due to losses in the dielectric substrate (and less due to losses in the metals), indicating that a lower loss substrate could lead to an almost fully transparent left-handed system. The origin of this good impedance ($z = \sqrt{\mu/\epsilon}$) match is the fact that the parameters of the system have been chosen in such a way as the negative μ regime of the SRRs to be close to the effective plasma frequency, ω'_p , of the combined SRRs and wires system [5,6], and thus the effective ϵ values to be small,

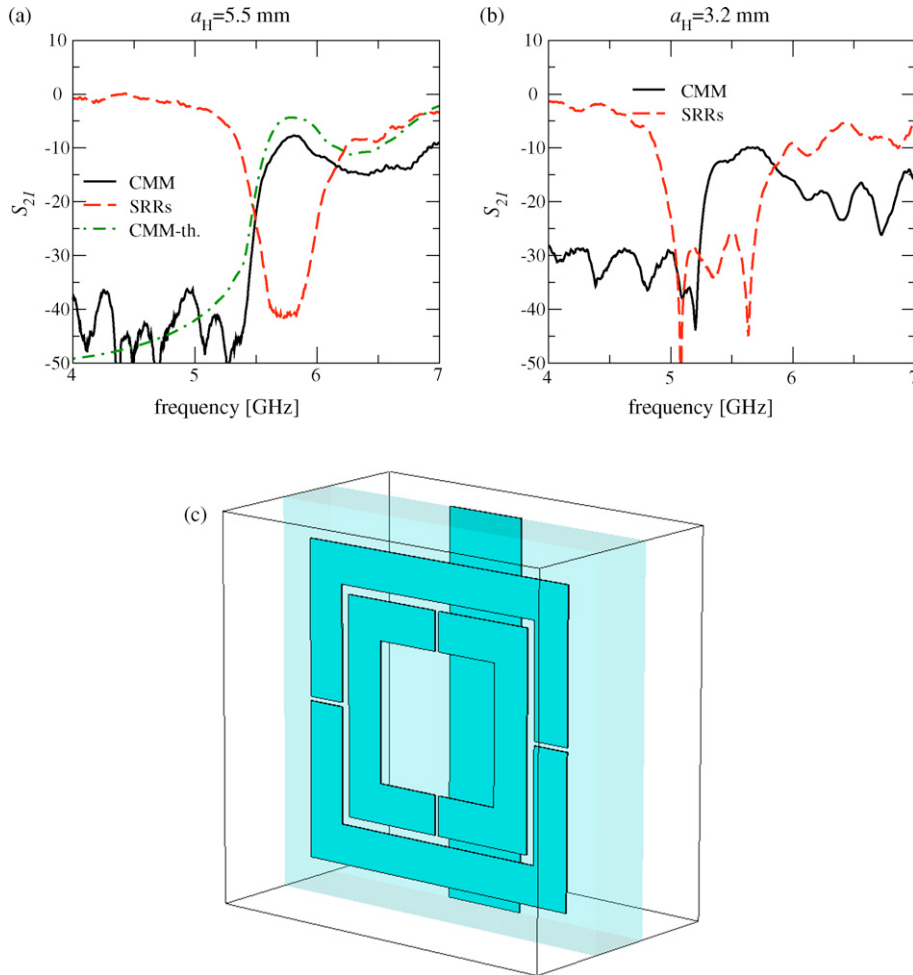


Fig. 7. The experimental transmission (S_{21}) data for our combined metamaterial (CMM) of SRRs and wires (solid lines) and for the only SRRs material (dashed lines). The unit cell of the SRRs and wires material (shown in panel (c)) contains our multigap double-ring SRR and a continuous wire of width 2 mm and thickness 30 μ m. The unit cell length along the perpendicular to the SRRs direction, a_H , is 5.5 mm for panel (a) and 3.2 mm for panel (b). The system consists of 5 unit cells along propagation direction. Dotted dashed line in panel (a) shows the corresponding theoretical result for the SRRs and wires material.

comparable to those of the negative μ (giving impedance close to unity). For given SRR parameters this can be achieved by adjusting properly the width and the distance of the continuous wires.

This good impedance match between the left-handed material and the free space is not maintained in the case of panel (b), where the lattice constant a_H has been reduced to 3.2 mm. This is because the reduction of a_H had as a result the increase of the effective plasma frequency, ω'_p (due to increase of the wires' plasma frequency, ω_p [4]) and thus lower negative ϵ values at the magnetic resonance, compared to the case of panel (a).

This less optimum impedance match between air and our metamaterial leads to lower transmission intensities at the left-handed regime, as can be seen comparing

panels (a) and (b) of Fig. 7. On the other hand, bringing the SRRs more close together, it resulted to a broadening of the negative μ regime (due to the larger coupling between the neighboring SRRs along the $\mathbf{H}$ -direction) and to a slight lowering of the magnetic resonance frequency (due to the increase of the “effective” inductance of each SRR, resulted from the increase of the flux, $\Phi = LI$, passing through the SRR due to the presence of the neighboring along $\mathbf{H}$ -direction rings; I is the current in the SRR). This decrease of the magnetic resonance frequency is responsible for the observed slight frequency mismatch between the $\mu < 0$ transmission dip of the SRRs system and the left-handed peak of the combined metamaterial (CMM) of SRRs and wires (in the CMM the continuous

wires prevent in some degree the inductive coupling between the neighboring SRR planes along $\mathbf{H}$ direction).

Detailed studies of our systems showed that the most critical parameter to achieve high transmittance in microwave left-handed materials (assuming low-loss component materials) is to adjust properly the impedance of the material. This can be done by adjusting the relative position of the magnetic resonance frequency, ω_m , of the SRRs and the effective plasma frequency, ω'_p , of the system. ω_m should not be that far below ω_p . The relative optimum distance of the two frequencies, which determines also how well resolved the LH regime could be in a transmission experiment, depends on the strength of the magnetic SRR resonance, i.e. on the negative μ values after the resonance (given a Drude like behavior for the low-frequency permittivity). Critical for this proper adjustment of the $\omega_m - \omega'_p$ distance is the correct determination of the effective plasma frequency of the system, by taking into account the contribution of the SRRs in this frequency [5]. Concerning only SRR systems, detailed study showed that in order to achieve wide negative μ regime it is important to have as large as possible inductive coupling among neighboring SRRs along the propagation direction. This can be achieved by permitting the strongest possible currents at the neighboring sides of the neighboring SRRs, e.g. by having those sides as long as possible. It is worth noticing though that a wide negative μ SRRs regime is not always translated to a broad left-handed regime if the SRRs are combined with wires, since SRRs-wires interaction can weaken the SRRs coupling.

Finally, we have to mention here that corresponding to those of Fig. 7 studies for SRRs and wires with the SRRs as in orientation 2 showed left-handed peaks of slightly smaller widths than those observed in Fig. 7, despite the wider and thus most promising negative μ regime observed in orientation 2. Those narrower left-handed peaks for the SRRs of the orientation 2 were due to the fact that, there, the effective plasma frequency of the system, ω'_p (for 2 mm width continuous wires), falls within the negative μ regime of the SRRs, being lower than that of the orientation 1. This lower ω'_p comes from the lower electric, short-wire-like, resonance of the orientation 2 SRRs (due to the larger length of the SRRs sides along the applied $\mathbf{E}$ -direction), which causes a larger downwards shift of the wires' plasma frequency [5] than the shift caused by the orientation 1 SRRs.

In conclusion, we demonstrated a broad left-handed regime associated with high transmission intensities in a metamaterial composed of symmetric SRRs combined

with continuous wires. We studied in detail the SRRs of those metamaterial for various polarizations of the incoming electromagnetic wave, proving that they are free from bianisotropy effects and therefore appropriate for the formation of two-dimensional and three-dimensional left-handed materials. The study of those SRRs and of the SRRs and wires metamaterial revealed the critical role of the coupling between SRRs in the achievement of a broad negative μ regime in a SRRs material; it also revealed the critical role of the metamaterial-air impedance matching for the achievement of a high transmittance left-handed regime, and identified parameters that can improve this impedance matching.

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