In- and out-of-plane propagation of electromagnetic waves in low index contrast two dimensional photonic crystals

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Propagation of electromagnetic waves through a two-dimensional triangular lattice has been studied for different values of refractive index contrast between the constituent dielectrics, and for angles of incidence both in and out of the plane of periodicity. Transmission results have been obtained both experimentally and with the transfer matrix technique, and good agreement has been found between the two. Comparison with band structure calculations has also been made. © 2001 American Institute of Physics. [DOI: 10.1063/1.1330250]

I. INTRODUCTION

Photonic band gap materials are artificial periodic dielectric structures that can control the properties of light in a manner analogous to the way semiconductor crystals control the properties of electrons. Such dielectric structures, sometimes referred to as “photonic crystals,” can forbid the propagation of electromagnetic (EM) waves in certain frequency ranges called “photonic band gaps,” which are said to be “complete” when the prohibition applies to all propagation directions and polarizations. The specific properties of the band gaps (for example, their positions and widths) can be engineered by altering the geometry or lattice spacing of the dielectric structure, by adjusting the index contrast between the constituent dielectrics, or by introducing impurities or defects. The resulting versatility and unique properties of photonic crystals makes them promising for applications such as waveguides, mirrors, cavities, antenna substrates, etc.

In previous work, it was demonstrated that complete band gaps can exist in both two- and three-dimensional photonic crystals in specific cases. The two-dimensional (2D) photonic crystals, in particular, have attracted experimental interest because of their advantage of simpler fabrication, especially in the case of the small lattice spacings required to produce band gaps at optical frequencies. A standard fabrication method for air-dielectric 2D photonic crystals is the etching of holes in a slab of material, either by patterning a dielectric film on a substrate or by fabricating a channel-array glass. It is well-known that photonic crystals with relatively low refractive index contrast do not usually possess complete photonic band gaps. However, an advantage of these materials is that the directional stop bands occur at higher frequencies and are narrower than those of corresponding photonic crystals with higher index contrast. Additionally, the optical properties of low-index-contrast photonic crystals can be useful for certain technological applications, such as optical filters and optical power limiters.

In a majority of past studies, the optical properties of 2D photonic crystals were investigated mainly for EM waves propagating within the plane of periodicity. Recently, however, a lot of interest has focused on the fabrication of optical fibers that rely entirely on the existence of photonic band gaps rather than on total internal reflection. In these optical fibers, light is guided perpendicularly to the 2D periodic plane through a defect in the photonic crystal lattice.

This novel technological application of a photonic crystal material suggests the need to investigate the existence and properties of photonic band gaps for light propagating out of the periodic plane in 2D photonic crystals.

II. CALCULATION METHOD AND EXPERIMENT

In the present work, experimental measurements of the transmission of EM waves through a 2D triangular photonic crystal are compared with corresponding theoretical computations. The method used for the calculations is the transfer matrix technique developed by J. B. Pendry. In this method, a grid is used to divide the space occupied by the photonic crystal into small cells. Maxwell’s equations are discretized and solved on this grid lattice. One is able to calculate the transmission and reflection coefficient of a layer consisting of only a few grid points by integrating the fields on that layer along the direction of propagation. After repeating this process for adjacent layers throughout the entire sample, the total transmission coefficient is obtained by combining the coefficients of all the layers.

The experimental data were obtained by measuring the transmission of channel-glass photonic crystals similar to those described previously. Measurements were performed along various propagation directions, both in and out of the array plane. The real-space structure of these photonic...
crystals, along with the corresponding first Brillouin zone and its respective high-symmetry points, is depicted in Fig. 1.

III. RESULTS AND DISCUSSION

A series of experimental transmission spectra for several in-plane angles of incidence is shown in Fig. 2. These spectra were measured through a photonic crystal consisting of air cylinders in a glass matrix \( (n \approx 1.45) \) and having a lattice constant of 0.73 \( \mu \)m. The cylinder diameter is approximately \( 2/3 \) of the lattice constant and the cylinder length is approximately 300 \( \mu \)m. The samples measured were approximately 300 \( \mu \)m thick in the plane of periodicity. The spectra in Figs. 2~a and 2~b were obtained with a photonic crystal whose faces were oriented as shown in Fig. 1~a, for normal incidence and for incidence 19° away from the normal, respectively. The results in (c) and (d) correspond to a sample oriented as shown in Fig. 1~b, for incidence at 14° away from the normal and for normal incidence, respectively. The solid curves are for TM polarization and the dotted curves are for TE polarization.

The transmission calculated via the transfer matrix technique for the cases corresponding to Figs. 2(a)–2(d) is shown in Figs. 3(a)–3(d), respectively. The parameters used for the transfer matrix calculations were identical to those in the experiments except for cylinder height and the sample thickness. In the calculations, infinitely long cylinders were considered, and the photonic crystal consisted of only 25 unit cells in the direction normal to the surface. Thus, for a lattice spacing of 0.73 \( \mu \)m, the calculations effectively corresponded to a thickness of \( \sim 31 \mu \)m when the photonic crystal was oriented as shown in Fig. 1(a), and to \( \sim 18 \mu \)m when oriented as shown in Fig. 1(b). However, it should be noted that the sample thickness does not affect the position of the photonic band gap but only the strength of the attenuation. A related issue is that of sample termination, i.e., the positions of the sample faces relative to the centers of the nearest rows of cylinders. Although the sample termination could not be controlled experimentally due to the sub-micron spacing between cylinders, in the calculations each sample face was assumed to occur near the center of a row of cylinders, for convenience. Finally, it should also be noted that an "al-
A small discrepancy between the results of the transmission and band structure calculations arises from the limitations of the transfer matrix method. For this calculational method, a finite number of square cells (of the grid lattice used to discretize Maxwell’s equations) is required to fit within a unit cell of the 2D photonic crystal. Thus, the triangular lattice, which is essentially a centered rectangular lattice with a lattice ratio of $\sqrt{3}$, has to be approximated by one with a lattice ratio of 1.7. This would be responsible for a discrepancy up to 2%. Also, the finite (5 nm) step taken for the wavelength when calculating the transmission imposes an error on the value of the band gap edges obtained from the transmission results.

For TE polarization, the transmission calculated by the transfer matrix method determines the lowest-energy band gap to be from $\sim 1440$ nm to $\sim 1682$ nm for normal incidence along $\Gamma M$, as shown in Fig. 3(a), and from $\sim 1325$ nm to $\sim 1480$ nm for normal incidence along $\Gamma K$, as shown in Fig. 3(d). These results agree within 5% with those of the band structure calculation. However, the case of TM polarization is more complex. Although the transmission calculation determines a band gap from $\sim 1530$ nm to $\sim 1700$ nm for normal incidence along $\Gamma M$, as shown in Fig. 3(a), which is also within 5% of the result of the band structure calculation, the transmission calculation also predicts a band gap from $\sim 1235$ nm to $\sim 1456$ nm for incidence along $\Gamma K$, as shown in Fig. 3(d), where the band structure predicts no gap.

Experimentally, for TE polarization, the measured band gaps appear from $\sim 1335$ nm to $\sim 1559$ nm for normal incidence along $\Gamma M$ in Fig. 2(a), and from $\sim 1154$ nm to $\sim 1352$ nm for normal incidence along $\Gamma K$ in Fig. 2(d). For TM polarization, the measured band gaps appear from $\sim 1422$ nm to $\sim 1584$ nm for normal incidence along $\Gamma M$ in Fig. 2(a), and from $\sim 1127$ nm to $\sim 1358$ nm for normal incidence along $\Gamma K$ in Fig. 2(d). This represents a discrepancy between the measured and calculated transmission, described above, ranging from 7% to 13%. Similar agreement is also found for the cases of off-normal incidence, i.e., comparing Figs. 2(b) and 2(c) to Figs. 3(b) and 3(c), respectively. Known sources of experimental errors include uncertainties in the angle of incidence ($\pm 1$ degree), in the lattice spacing ($\pm 5\%$), and in the glass index of refraction ($\pm 5\%$), which is modified by the etching process. Since the position of the band gap is proportional to the lattice spacing, errors in the lattice spacing are linearly proportional to errors in the band gap position. Also, from a transfer matrix calculation, it was found that a 3% change in the glass index can cause a corresponding change of up to 3% in the positions of the band edges. These experimental uncertainties, in combination with the limitations of the transfer matrix method discussed above, can explain the discrepancies between the transmission spectra measured experimentally and those calculated with the transfer matrix method.

Nevertheless, the transmission (obtained both experimentally and through transfer matrix calculations) remains inconsistent with the band structure results by showing a gap at the $K$ symmetry point where the band structure predicts none. This type of apparent contradiction between the band structure results and the transmission (measured and calculated) of a photonic crystal has been observed in previous experiments.\(^{19,20}\) In Refs. 19 and 20, it is shown that even when a certain photonic crystal can host a particular mode, that mode will not be excited by the incident field if the field patterns which correspond to that mode have symmetry inconsistent with that of the incident field. This causes a pseudogap to appear in the transmission. Also, Sakoda\(^{21,22}\) has performed calculations for a triangular lattice of air cylinders in a glass matrix ($n \sim 1.65$). He showed that the second band for the TM polarization (which is degenerate at the $K$-point along the $\Gamma K$ direction) will not couple at all to an incoming field incident along the $\Gamma K$ direction when that direction is at normal incidence for the photonic crystal. Taking this into consideration, one expects the appearance of a pseudogap in the photonic crystals used in Fig. 2, between $\sim 1263$ nm and
1496 nm for TM polarization and normal incidence along the $\Gamma K$ direction. This is in very good agreement with the transmission results obtained both experimentally and through transfer matrix calculations, and also explains why for this incident direction the TM polarization gap appears to be wider than the TE polarization gap.

In the calculated TM polarization transmission spectrum of Fig. 3(c) there is another striking feature which deserves further discussion. Besides a gap from $\sim 1475$ nm to $\sim 1620$ nm there is a “dip” in the transmission for the wavelength region between $\sim 1130$ nm and $\sim 1295$ nm. This feature correlates well with a similar feature observed experimentally in Fig. 2(c). The transmission in the wavelength region of the dip differs significantly from zero and also shows interference fringes. By contrast, the transmission in the gap region for a sufficiently thick sample (as is the case for the sample used for the calculations) is always nearly zero. Additional transfer matrix calculations have shown that the attenuation associated with the “dip” is independent of sample thickness, while the attenuation associated with either a true gap or of a “pseudogap” [like the one in Fig. 3(d)] does depend on sample thickness. Also, further calculations have shown that the fringes within the “dip” as well as the magnitude of the attenuation in this spectral region are both sensitive to the sample termination. In addition, the band structure calculations show that propagating modes do exist in the wavelength region of the “dip.” These observations lead to the conclusion that the observed transmission “dip” is likely to result from the “partial” (i.e., incomplete) coupling between the incident wave and the propagating modes. (Note that, if the coupling were complete, one would observe ordinary transmission fringes oscillating up to the maximum transmission value of 1.)

The experimental transmission spectra for incidence directions outside the array plane for similar 2D triangular photonic crystals with several index contrasts are shown in Figs. 4, 5, and 6. The measured in-plane transmission corresponding to each index contrast is also included for reference. The photonic crystals used for these measurements were also based on channel glass materials, as above, except oriented so that the incident propagation direction of interest was along the surface normal for each sample. The particular direction chosen for the data in Figs. 4(a), 5(a) and 6(a) is $45^\circ$ away from the in-plane $\Gamma M$ direction. The lattice spacing of these photonic crystal samples was 0.56 $\mu$m, and the thickness of each sample was $\sim 300$ $\mu$m. Results for the highest index contrast achievable, namely for a photonic

FIG. 4. Measured transmission results for a 2D triangular structure of air cylinders in a glass ($n \sim 1.45$) matrix. The propagation direction is at $45^\circ$ out of the array plane with respect to $\Gamma M$ in (a), and within the array plane along $\Gamma M$ in (b). Two samples were used, each having the propagation direction at normal incidence. The lattice spacing is 0.56 $\mu$m and the sample thickness is 300 $\mu$m in each case. The solid curves are for TM polarization and the dotted curves are for TE polarization.

FIG. 5. Measured transmission results similar to Fig. 4, but for a structure consisting of water-filled ($n \sim 1.33$) cylinders in a glass ($n \sim 1.45$) matrix. The propagation direction is at $45^\circ$ out of the array plane with respect to $\Gamma M$ in (a), and within the array plane along $\Gamma M$ in (b). The additional absorption appearing near 1450 nm in both (a) and (b) is due to water. The solid curves are for TM polarization and the dotted curves are for TE polarization.
crystal consisting of air cylinders in a glass \((n \sim 1.45)\) matrix, are shown in Fig. 4. An intermediate index contrast was obtained in Fig. 5 by immersing the photonic crystal from Fig. 4 in water \((n \sim 1.33)\). (Additional absorptions due to the water, which overlap slightly with the attenuations associated with the photonic band gaps, appear in Fig. 5). The lowest index contrast (Fig. 6) was obtained in a photonic crystal consisting of glass \((n \sim 1.67)\) cylinders in a glass \((n \sim 1.65)\) matrix. (Note that all glass refractive indices are wavelength-dependent; the indices quoted, which were used in the calculations, are accurate at \(\sim 800\) nm). Also note that a more detailed study of the out-of-plane transmission for the lowest-index-contrast case has been published in Ref. 18.

The calculated transmission spectra corresponding to Figs. 4 and 6, obtained via the transfer matrix technique for incidence in the array plane and at a 45° angle outside the array plane with respect to the in-plane \(\Gamma M\) direction, are shown in Figs. 7 and 8. Although some fine structure is associated with the attenuations appearing in the transmission calculations for the lowest index contrast photonic crystal, at \(\sim 810\) nm for in-plane incidence and at \(\sim 725\) nm for out-of-plane incidence, only one (broader) attenuation is measured experimentally. This is at least partially a result of the slight disorder inevitably present in the actual photonic crystals, which results in inhomogeneous broadening of these narrow attenuation regions. Therefore, for comparison with the experimental results, only the center values of the calculated attenuation features were considered. Very good agreement is found between the measured transmission and the transfer matrix results. The discrepancies range from \(\sim 1\%\) to \(6\%\) for in-plane incidence and from \(\sim 6\%\) to \(9\%\) for out-of-plane incidence. The larger disagreement in the out-of-plane case is expected: The cylinders in the experimental sample had finite length and, for the measurements presented in Figs. 4(a), 5(a), and 6(a), they were cut in such a way that incidence is normal to the sample surface in each case, as mentioned. By contrast, the numerical method considered infinitely long cylinders that are parallel to the sample surface, which is oriented at 45° with respect to the incident wave vector. Since the boundary conditions imposed on the sample surface require that the parallel component of the incident wave vector be conserved in the crystal,23 the resulting transmission can be influenced by the orientation of the sample surface relative to the incident wave vector. In spite of these differences, however, good agreement was achieved between the transfer matrix calculations and the experimental results.

An important observation is that the photonic band gaps (or the corresponding attenuations) for all index contrast values persist for incidence out of the array plane but shift toward shorter wavelengths. However, the depths of all of the attenuations appear to decrease in the measured spectra. Such a decrease was not found in the transfer matrix results, except for the case of the lowest index contrast (corresponding to glass cylinders in a glass matrix), and then only for the lowest-frequency attenuation in the TM polarization. (The...
angular dependence of the strength of this attenuation was discussed in detail in Ref. 18. Another important point is that calculations for the highest-index-contrast case (corresponding to air cylinders in a glass matrix), performed for various lattice filling ratios, show that the widest gaps occur when the filling ratio is between 40% and 50% (defined as the sum of the cross-sectional areas of the cylinders over total area). The experiments presented here were done for a cylinder diameter approximately 2/3 of the lattice spacing, which corresponds to a 40% filling ratio. Thus, for the present case of the triangular lattice of air cylinders in a glass matrix, the optimal band gap is obtained.

IV. CONCLUSIONS

The transmission of 2D triangular lattices, with index contrast corresponding to air cylinders in a glass matrix and below, was studied both experimentally and through transfer matrix calculations. The experimental results show generally good agreement with the calculations, with discrepancies ranging from 1% to 13%. A narrow band gap region independent of incidence direction was found only for the photonic crystals consisting of air cylinders in a glass matrix, and only for in-plane propagation and TE polarization. None of the dielectric structures considered here possesses a complete 2D photonic band gap (independent of polarization, for in-plane propagation). For TM polarization, the appearance of a "pseudogap" was observed both in the measured spectra and in the transfer matrix calculations. The positions of the band gaps corresponding to the lowest index contrast structures, consisting of glass cylinders in a glass matrix, depend on the incident direction and are relatively narrow, making them useful for applications such as optical filters. The photonic crystals studied in this paper retain their band gaps even for out-of-plane propagation, but the gaps are shifted toward shorter wavelengths. The absence of overlap between the out-of-plane and the in-plane band gaps, even in the case of the photonic crystals consisting of air cylinders in a glass matrix, suggests that this particular structure is inappropriate for use as a photonic band gap optical fiber. However, previous transfer matrix calculations for a triangular photonic crystal with a dielectric contrast of 16 have shown the existence of a complete band gap for incidence up to 85° out of the array plane. Furthermore, understanding the propagation of EM waves at arbitrary directions through a photonic crystal should be a useful step toward incorporating finite-size photonic-crystal elements into the next generation of opto-electronic circuits.

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