

Electric coupling to the magnetic resonance of split ring resonators

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We study both theoretically and experimentally the transmission properties of a lattice of split ring resonators (SRRs) for different electromagnetic (EM) field polarizations and propagation directions. We find unexpectedly that the incident electric field \mathbf{E} couples to the magnetic resonance of the SRR when the EM waves propagate perpendicular to the SRR plane and the incident \mathbf{E} is parallel to the gap-bearing sides of the SRR. This is manifested by a dip in the transmission spectrum. A simple analytic model is introduced to explain this interesting behavior. © 2004 American Institute of Physics. [DOI: 10.1063/1.1695439]

Metamaterials with a negative index of refraction have attracted recently great attention due to their fascinating electromagnetic (EM) properties. It was Veselago that introduced the term “left-handed substances” in his seminal work published in 1968.¹ He suggested that in a medium for which the permittivity ϵ and permeability μ are simultaneously negative, the phase of the EM waves would propagate in a direction opposite to that of the EM energy flow. In this case, the vectors \mathbf{k} , \mathbf{E} , and \mathbf{H} form a left-handed set and therefore Veselago referred to such materials as “left-handed.” The interest in Veselago’s work was renewed since Pendry *et al.* proposed an artificial material consisting of the so-called split-ring resonators (SRRs) which exhibit a band of negative μ values in spite of being made of nonmagnetic materials, and wires which provide the negative ϵ behavior.² Based on Pendry’s suggestion and targeting the original idea of Veselago, Smith *et al.* demonstrated in 2000 the realization of the first left-handed material (LHM) which consisted of an array of SRRs and wires, in alternating layers.³ Since the original microwave experiment by Smith *et al.* several composite metamaterials (CMMs) were fabricated^{4,5} that exhibited a pass band in which it was assumed that ϵ and μ are both negative. This assumption was based on transmission measurements of the wires alone, the SRRs alone, and the CMMs. The occurrence of a CMM transmission peak within the stop bands of the SRR and wire structures was taken as evidence for the appearance of LH behavior. Further support to this interpretation was provided by the demonstration that such CMMs exhibit negative refraction of EM waves.^{6,7} Moreover, there is a significant amount of numerical work^{8–11} in which the transmission and reflection data are calculated for a finite length of metamaterial. A retrieval procedure can then be applied to obtain the effective metamaterial parameters ϵ and μ , under the assumption that it can be

treated as homogeneous. This procedure was applied in Ref. 12 and confirmed that a medium composed of SRRs and wires could indeed be characterized by effective ϵ and μ whose real parts were both negative over a finite frequency band, as was the real part of the refractive index n . However, it was recently shown^{13,14} that the SRRs exhibit resonant electric response in addition to their resonant magnetic response. As a result of this electric response and its interaction with the electric response of the wires, the effective plasma frequency, ω_p' , is much lower than the plasma frequency of the wires, ω_p . An easy to apply criterion was proposed¹³ to identify if an experimental transmission peak is left-handed (LH) or right-handed (RH): If the closing of the gaps of the SRRs in a given LH structure removes only a single peak from the T data (in the low frequency regime), this is strong evidence that the T peak is indeed LH. This criterion is valuable in experimental studies, where one cannot easily obtain the effective ϵ and μ . It was applied experimentally and it was found that some T peaks that were thought to be LH turn out to be RH.¹⁴ It seems that a careful study of the SRR behavior, both electric and magnetic, is necessary for the design and realization of LH structures. Marqués *et al.* considered bianisotropy in SRR structures and developed an analytical model to evaluate the magnitude of cross-polarization effects.¹⁵

In this letter, we report numerical and experimental results for the transmission coefficient of a lattice of SRRs alone for different orientations of the SRR with respect to the external electric field, \mathbf{E} , and the direction of propagation. Incidence is always normal to some face of the orthorhombic unit cell of this metamaterial, which implies six distinct orientations (Fig. 1). It was considered an obvious fact that an incident EM wave excites the magnetic resonance of the SRR only through its magnetic field; hence one could conclude that this magnetic resonance appears only if the external magnetic field \mathbf{H} is perpendicular to the SRR plane,

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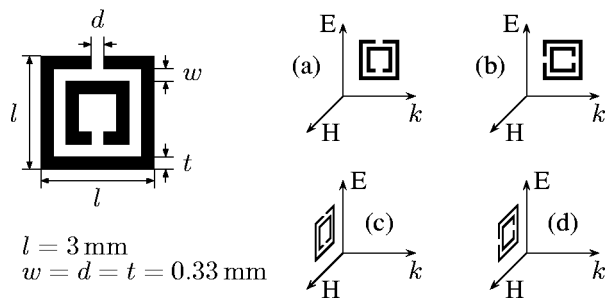


FIG. 1. Left-hand side: SRR geometry studied. Right-hand side: The four studied orientations of the SRR with respect to the triad \mathbf{k} , \mathbf{E} , \mathbf{H} of the incident EM field. The two additional orientations, where the SRR are on the \mathbf{H} - \mathbf{k} plane, produce no electric or magnetic response.

which in turn implies a direction of propagation parallel to the SRR [Figs. 1(a), and 1(b)]. If \mathbf{H} is parallel to the SRR [Figs. 1(c), and 1(d)] no coupling to the magnetic resonance was expected. In this article we show that this is not always the case. If the direction of propagation is perpendicular to the SRR plane and the incident \mathbf{E} is parallel to the gap-bearing sides of the SRR [Fig. 1(d)], an electric coupling of the incident EM wave to the magnetic resonance of the SRR occurs. This means that the electric field excites the resonant oscillation of the circular current inside the SRR, influencing either the behavior of $\epsilon(\omega)$ only [as in Fig. 1(d)] or $\epsilon(\omega)$ and $\mu(\omega)$ [as in Fig. 1(b)]. Experiments as well as numerical results based on the transfer matrix (TMM), and on the finite difference time domain (FDTD) method reveal that for propagation perpendicular to the SRR plane a dip in the transmission spectrum close to the magnetic resonance ω_m of the SRR appears whenever the mirror symmetry of the SRR with respect to the direction of the electric field is broken by the gaps of its rings [Fig. 1(d)]. As we point out below the possibility of such electric coupling to the magnetic resonance does also affect the conventional orientations [Figs. 1(a), and 1(b)], that have the direction of propagation along the SRR plane. A simple analytic model is given that provides an explanation for the phenomenon.

For the experimental study, a CMM consisting of SRRs was fabricated using a conventional printed circuit board process with $30\text{ }\mu\text{m}$ thick copper patterns on one side of a 1.6 mm thick FR-4 dielectric substrate. The FR-4 board has a dielectric constant of 4.8 and a dissipation factor of 0.017 at 1.5 GHz . The design and dimensions of the SRR, which are the same as those of Ref. 5, are described in Fig. 1. The CMM was then constructed by stacking together the SRR structures in a periodic arrangement. The unit cell contains one SRR and has the dimensions 5 mm (parallel to the cut sides), 3.63 mm (parallel to the continuous sides), and 5.6 mm (perpendicular to the SRR plane). The transmission measurements were performed in free space on a CMM block consisting of $25 \times 25 \times 25$ unit cells, using a Hewlett-Packard 8722 ES network analyzer and microwave standard-gain horn antennas.

Additionally, numerical simulations using TMM and FDTD method were performed to understand the couplings to the SRR. Both methods use a discretized model of the SRR, similar to the one shown in the inset of Fig. 3, and periodic boundary conditions perpendicular to the direction of propagation. The TMM directly computes the complex

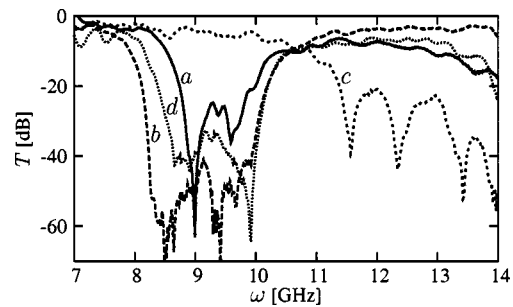


FIG. 2. Measured transmission spectra of a lattice of SRRs for the four different orientations shown in Fig. 1.

transmission and reflection amplitudes and thus allows us to obtain the effective medium $\epsilon(\omega)$ and $\mu(\omega)$ via a retrieval procedure.¹² In addition, the FDTD allows visualization of the spatial distribution of the fields and currents inside the system, as a function of time.

We considered the four nontrivial orientations of the SRR, which are shown in Fig. 1. Figure 2 presents the measured transmission spectra, T , of the CMM. The continuous line (line *a*) corresponds to the conventional case shown in Fig. 1(a), with \mathbf{H} perpendicular to the SRR plane and \mathbf{E} parallel to the symmetry axis of the SRR. Notice that T exhibits a stop band at $8.5\text{--}10.0\text{ GHz}$, due to the magnetic resonance. The dashed line (line *b*) shows T for the orientation of Fig. 1(b); here \mathbf{E} is no longer parallel to the symmetry axis of the SRR and thus there is no longer a mirror symmetry of the combined system of SRR plus EM field. Notice that now T exhibits a much wider stop band at $8\text{--}10.5\text{ GHz}$, starting at lower frequency. Very interesting results are obtained by comparing T for the two cases shown in Figs. 1(c) and 1(d), for which there is no coupling to the magnetic field since \mathbf{H} is parallel to the SRR plane. For the case of Fig 1(c), where \mathbf{E} is parallel to the symmetry axis, no structure is observed around the magnetic resonance frequency (line *c* in Fig. 2), as expected. However in the case of Fig. 1(d), where the SRR plus EM field exhibit no mirror symmetry, a strong stop band in T around ω_m is observed (line *d*), similar to that of the conventional case [Fig. 1(a)]. This strongly suggests that the magnetic resonance can be excited by the electric field provided that there is no mirror symmetry.

These observations are in good agreement with the numerical results, presented in Fig. 3. For the propagation perpendicular to the SRR we observe a stop band only if \mathbf{E} is parallel to the cut-bearing sides of the SRR and “sees” its

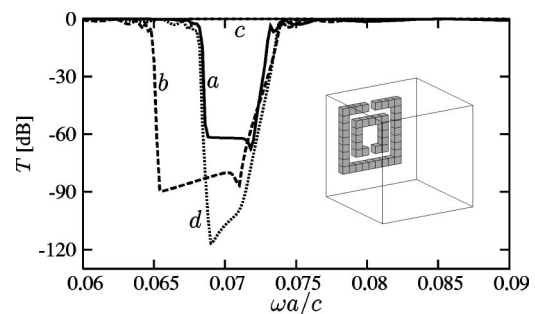


FIG. 3. Calculated transmission spectra of a lattice of SRRs for the four different orientations shown in Fig. 1. The curve *c* practically coincides with the axis. The discretization of one particular SRR is shown in the inset.

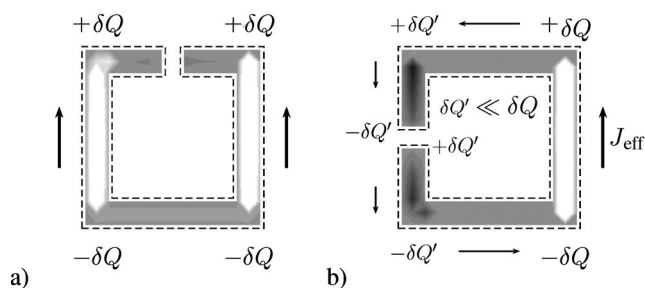


FIG. 4. Simple drawing for the polarization in two different orientations of a single ring SRR. The external electric field points upward. Only in case of broken symmetry (b) a circular current will appear which excites the magnetic resonance of the SRR. The interior of the ring shows FDTD data for the polarization current component $J_{\parallel E}$ at a fixed time for normal incidence [Figs. 1(c) and 1(d)] as a gray scale plot.

asymmetry (line *d*); otherwise we have transparency (line *c*). At low frequencies, the SRR can basically be represented only by its outer ring. As shown in Fig. 4, the SRR ring will experience different spatial distributions of the induced polarization, depending on the relative orientation of \mathbf{E} with the SRR gap. If \mathbf{E} is parallel to the no gap sides of the SRR its polarization will be symmetric and the polarization current is only flowing up and down the sides of the SRR, as shown in Fig. 4(a). If the SRR is turned by 90° shown in Fig. 4(b), the broken symmetry leads to a different configuration of surface charges on both sides of the SRR, connected with a compensating current flowing between the sides. This current contributes to the circulating current inside the SRR and hence couples to the magnetic resonance. We directly observed both types of currents in the FDTD simulations; as an example, the component of the polarization current parallel to the external electric field is shown in Fig. 4. The retrieval procedure for ϵ and μ indicates that the electric coupling leads to a resonant electric response in ϵ near ω_m . Also the experimentally observed broadening of the conventional SRR dip for the turned SRR was found numerically as well (line *b* in Fig. 3). The reason is the *additional* electric coupling which adds an electric resonant response [in $\epsilon(\omega)$] directly below the resonant magnetic response. Closing the gaps of the SRR¹³ we observed both in the experiment and in the simulations that the dips disappeared.

In summary, we have studied experimentally and theoretically the propagation of EM waves for different orientations of the SRR. It is found that the incident electric field couples to the magnetic resonance of the SRR, provided its direction is such as to break the mirror symmetry. This unexpected electric coupling to the magnetic resonance of the SRR is of fundamental importance in understanding the refraction properties of SRRs in the low frequency region of the EM spectrum. Also this new finding is very important for the design of LHMs in higher dimensions.

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