Converging and wave guiding of Gaussian beam by two-layer dielectric rods

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We have shown that a two-layer dielectric structure can give excellent beaming and enhanced transmission simultaneously of a Gaussian source. The front surface of the layer of dielectric rods supports surface states and the rear grading layer couples the surface states to radiation modes. By repeating periodically this two-layer structure, one can obtain excellent beaming and enhanced transmission for very long distances. © *2008 American Institute of Physics*.
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In 2002, Lezec *et al.*[1] demonstrated that a light emerging from a subwavelength aperture can be compressed into a narrow beam. Since then the beaming effect of the metal channel with an extra periodic structure along the output surface is studied experimentally[2,3] and theoretically.[4,5] The periodic structure on the output surface works as a grating. It couples the excited surface plasmons supported by the metallic surface to the radiation modes. When certain resonant conditions are satisfied, radiation from the output surface interferes constructively for a certain angle, resulting in good beaming. Surface plasmon is a kind of surface state that is localized at a surface and decays exponentially away from the surface.[6] Similarly, photonic crystal with a modified surface layer supports surface states when the working frequency is within the photonic band gap.[7–10] Then a grating layer, whose period is several times the lattice constant of the photonic crystal, is added on top of the surface layer. The grating layer will couple the surface states to the radiation modes and good beaming can be achieved too.[11–16]

We have discovered that a single dielectric layer can support surface states too.[17] So theoretically a similar beaming phenomenon will happen for a two-layer structure: one surface layer to support the surface states and one grating layer to couple the surface states to radiation modes. In this paper, we demonstrate the converging property of the two-layer structure. Furthermore, by putting several two-layer structures in series, we can sustain the beaming to long distances.

The two-dimensional (2D) two-layer structure is shown in Fig. 1(a). The top layer is the surface layer. It consists of 41 circular rods with diameter $D=1.83$ mm and lattice constant $a=11$ mm. The bottom layer is the grating layer with 21 square rods with the side length $L=3.15$ mm; the lattice constant is $b=2a=22$ mm. The distance between the two layers is a. All the rods are made of alumina and the permittivity is 9.8.

The commercial finite element method software COMSOL MULTIPHYSICS was used to simulate the system. The simulation area with n structures is shown in Fig. 1(b). A Gaussian beam with E polarization (the electric field parallel to the dielectric rods) is incident normally from the left boundary. The waist of the Gaussian beam is 3λ. The power flow through the red lines given in Fig. 1(b) is calculated by integrating the time average normal component of the Poynting vector along the lines. The power flow through the red line right after the ith structure is defined as T_i and the power flow through the short red line on the right boundary as BEAM [see Fig. 1(b)]. The length of the short red line is $14a$. Suppose the total input power is I. T_n/I describes the transmission after the n structures. Then we define the distribution factor as

$$\text{distribution factor} = \frac{\text{BEAM}}{T_n}.$$

The distribution factor definition is just a simple quantitative index of the directionality of the output field through the n structures. The space between the last long red line and the right boundary, shown in Fig. 1(b), is $99a \times 54a$ for all the simulations. This way we can compare the distribution factors between different simulations.

The converging ability of a single two-layer structure is studied. We compare four cases: empty simulation area, a surface layer only, a grating layer only, and a two-layer structure. The simulation results are shown in Fig. 2 when incident frequency is between 9 and 13 GHz. The transmission

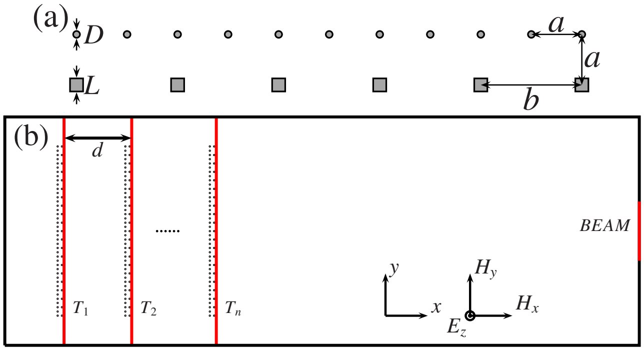

FIG. 1. (Color online) (a) A schematic drawing of the two-layer structure and (b) a schematic drawing of the simulation area.

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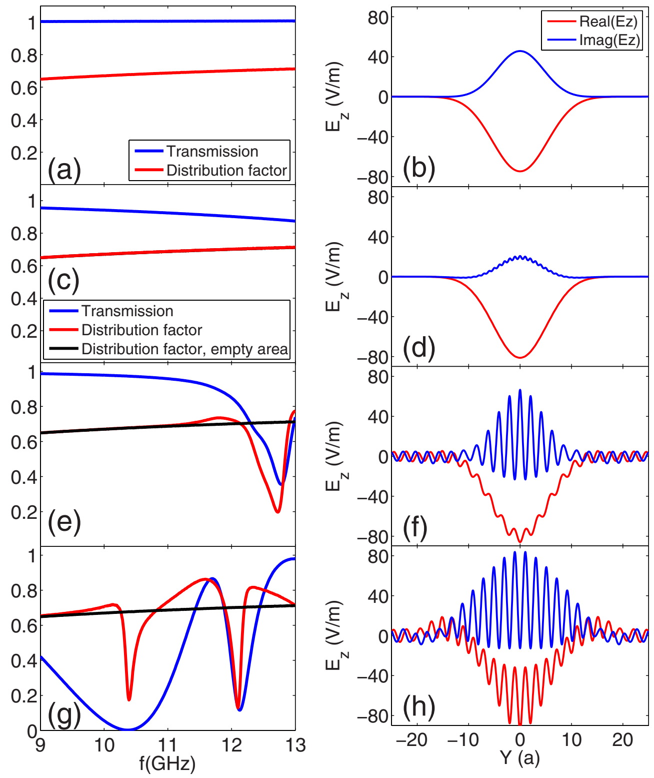

FIG. 2. (Color online) [(a) and (b)] Simulation results of empty area, [(c) and (d)] a surface layer only, [(e) and (f)] a grating layer only, and [(g) and (h)] a two-layer structure. The left column shows how the transmission and the distribution factor change with frequency. The right column shows the electric field along the red line right after the rods when $f=11.59$ GHz.

and the distribution factor curves are shown on the left column. When the frequency is above 13 GHz, the transmission field has more than one beam. When the frequency is below 9 GHz, the distribution factor curves after the rods converge to the empty area curve. In Fig. 2(c), the distribution factor of the surface layer overlaps the empty area curve. In Fig. 2(e), the distribution factor curve of the grating layer is close to the empty area curve when the frequency is small. It has a dip when frequency is around 12.7 GHz. The corresponding wavelength is 23.6 mm, which is close to $b=22$ mm, the period of the grating layer. So the interaction between the field and the rods is strong. However, this interaction does not improve the directionality of the transmission field. The two-layer structure has a much different distribution factor curve [Fig. 2(g)]. The curve has two dips, which are transmission dips too. The interesting frequency region is around 11.5 GHz. The transmission field at this frequency has a big distribution factor. When $f=11.59$ GHz, the distribution factor reaches a maximum value of 0.863. The transmission is acceptable too; it is around 0.87.

To understand how the directionality is improved, the electric field along the z-axis E_z at the red line after the rods [see Fig. 1(b)] at $f=11.59$ GHz is plotted on the right column of Fig. 2. The rods of the surface layer are located at $y=-20a,-19a,\ldots,20a$ and of the grating layer are at $y=-20a,-18a,\ldots,20a$. The field on the right column of Fig. 2 is basically the superposition of the transmission Gaussian beam and some quick oscillations. The field after the surface layer has small oscillation. It means that the interaction is weak. The oscillation is the surface state, which decays exponentially along the x direction. Though the existence of the surface layer reduces the transmission a little bit, it cannot change the distribution factor [see Fig. 2(c)]. Figures 2(f) and 2(h) show the strong interaction between the field and the rods. Comparing the Gaussian envelopes of the fields in the two figures, we can find that the envelope in Fig. 2(f) has the same width as the incident beam and Fig. 2(h) is wider. It is because the surface layer of the two-layer structure supports the surface states. The surface states will propagate along the surface layer and result in wider field distribution. This wider field distribution will bring us better directionality. Our simulations show that the two-layer structure also improves the directionality of oblique incident Gaussian beam when the incident angle is small ($<10°$), though normal incidence gives the best improvement. Our simulations also show that the optimal converging frequencies decrease with the increasing incident angles, which agree with the prediction based on the dispersion relation of the surface state.

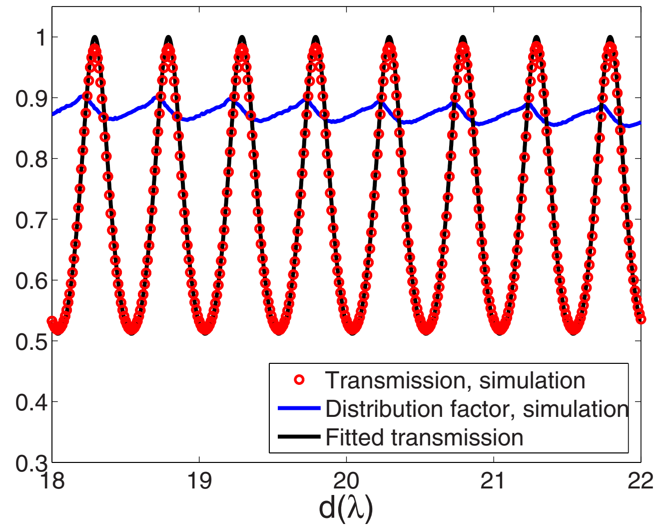

FIG. 3. (Color online) Transmission and distribution factor of the field after two two-layer structures. The black line shows the fitted transmission using the equation of the transmission matrix theory.

Then we examine the two two-layer structures in the simulation area. The working frequency is $f=11.59$ GHz, the optimal converging frequency of the single structure. Figure 3 shows the transmission and the distribution factor of the field as a function of the distance between the two structures d. The transmission curve is periodic with a $\lambda/2$ period. The curve can be explained easily by the one mode assumption. Let us consider the simplest case: the incidence of a Gaussian beam to a single structure. Suppose the incidence, reflection, and transmission electric fields are $\Phi_i(x,y)$, $\Phi_r(x,y)$, and $\Phi_t(x,y)$. The one mode assumption says that $\Phi_i(x,y)\approx\varphi(y)\exp(ikx)$ and $\Phi_r(x,y)\approx A\varphi(y)\times\exp(-ikx)$, $\Phi_t(x,y)\approx B\varphi(y)\exp(ikx)$. Here $k=2\pi f/c$ (c is the speed of light), the reflection A and transmission B coefficients are complex numbers. Only one mode exists during the propagation, reflection, and transmission. In fact, the Gaussian beam will diverge slowly during the propagation. However, if the free length of space is short, such as 20λ, the divergence is small and the lateral profile does not change too much. So $\Phi_i(x,y)\approx\varphi(y)\exp(ikx)$. Then the Gaussian beam hits the structure. The structure converges the beam and cancels the diverging trend during the propagation. So the reflection and transmission fields have a similar profile.

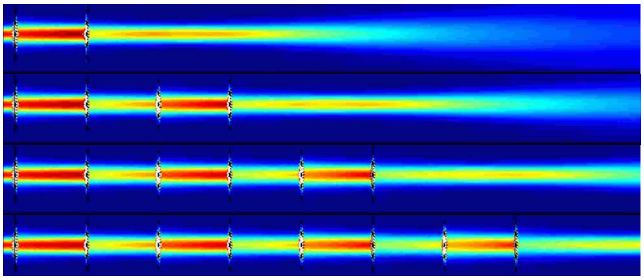

FIG. 4. (Color online) Time average energy density distribution when the simulation area has 2, 4, 6, and 8 two-layer structure. The frequency is $f=11.59$ GHz and the distance between the two adjacent structure is $d=21.29\lambda$.

Under the one mode assumption, the transmission matrix of the system is a 2×2 matrix. Suppose the transmission matrix of one two-layer structure is M and the propagation transmission matrix is P,

$$M=\begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix}, \quad P=\begin{bmatrix} \exp(ikd) & 0 \\ 0 & \exp(-ikd) \end{bmatrix}.$$

Then the transmission T after the two structures is[18]

$$T=\left| \frac{1}{|m_{11}|^2-1+m_{11}^2\exp(2ikd)} \right|^2. \quad (1)$$

Using this equation to fit the simulation transmission, we get $m_{11}=1.058-0.279i$. The fitted transmission is also shown in Fig. 3.

The distribution factor curve in Fig. 3 is a periodic function too. The interesting property of the curve is that its oscillation amplitude is small. This is a natural result of the one mode assumption. The field after the last structure is always proportional to $\varphi(y)\exp(ikx)$. So the distribution factor should not change as we increase the distance d.

When $d=18.29\lambda, 18.79\lambda, 19.29\lambda, \ldots,$ the transmission is close to 1. So the transmission field is $\Phi_t(x,y) \approx B\varphi(y)\exp(ikx)$ and $|B|\approx 1$. The only difference between the transmission field and the incident field is a phase factor. The two structures are transparent to the incident field. Then we can put more structures after the transmission beam to repeat the pattern. A single mode waveguide for the Gaussian beam is formed by the equidistant two-layer structures. Figure 4 shows the time average energy density distributions of the waveguide. We can see clearly that the width of the beam does not change during the propagation. After 8 structures (which correspond to distance of 191λ), the transmission is 0.930 and the distribution factor is 0.874. Some power is lost because of the surface mode but most of the power is guided.

In previous studies, beaming was always referred to the directionality of the output field of a channel. The channel could be a hole in a metal film,[1] or a subwavelength metal slit,[2–5] or a line defect in a 2D photonic crystal.[11–16] In fact, beaming can have a broader meaning based on the source. Our design is a good example of Gaussian beam beaming; others' designs work for channel modes. The beaming light from a subwavelength channel is similar to a Gaussian beam.[4,11,16] The two-layer structure provides a method to extend the beaming for long distances. Finding an effective beaming device for a point source is an interesting problem.

Another problem of the beaming is the transmission. It is difficult to get high transmission and good beaming simultaneously. In our design, a single structure gives us good beaming and the Fabry–Pérot interference between the two structures guarantees the total transmission. The two problems decouple. We can solve them separately. The two-layer structure also has the advantage in theoretical analysis. Comparing with the beaming of the photonic crystal channel, the two-layer structure gives similar results with much fewer rods. The complex channel modes are replaced by the simple Gaussian beam. The theoretical analysis will be much simpler.

In conclusion, we present a numerical analysis of the beaming and transmission of a Gaussian beam through a two-layer structure. This structure does not allow the Gaussian beam to diverge and also gives a high transmission. By arranging several two-layer structures, one can easily sustain the beaming and the high transmission for very large distances of the order of 200λ. This simple design of beaming and transmission has an advantage both in the theoretical analysis and in practical applications.

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