Bianisotropic photonic metamaterials

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(Invited Paper)

Abstract

In metamaterials, electric (magnetic) dipoles can be excited by the electric (magnetic) component of the incident light field. Moreover, in the description of bianisotropic metamaterials, cross terms occur, i.e., magnetic dipoles can also be excited by the electric-field component of the incident light and vice versa. For the cross terms, in the general bianisotropic case, the exciting field and dipole vectors include an arbitrary angle. For the special case of chirality, the angle is zero. In the spirit of a brief tutorial, a very simple electric-circuit description of the split-ring resonator is used to give a basic introduction to the cross terms. Mathematical details of the effective parameter retrieval are presented. Furthermore, we briefly review recent experiments on bianisotropic metamaterials operating at optical frequencies.

Index Terms

Bianisotropy, metamaterial, split-ring resonator.
I. INTRODUCTION

Optics has traditionally mainly dealt with dielectric materials. Here, the incident electric-field vector of the light excites microscopic electric dipoles that re-emit electromagnetic waves just like a dipole radio antenna. Other dipoles are excited by this re-emission and so on. This successive excitation and re-emission clearly modifies the phase velocity of light and determines the optical properties of the material, in particular its electric permittivity $\varepsilon$ (or dielectric function).

One of the little revolutions in optics that the concept of artificial effective materials ("metamaterials") has brought about is the fact that, similarly, magnetic dipoles can be excited by the magnetic-field component of the light, which can be cast into the effective magnetic permeability $\mu$ of the material [1]–[8] (or, alternatively, into spatial dispersion). The interplay of $\varepsilon$ and $\mu$ has given rise to interesting new aspects of electromagnetism that have been reviewed several times [9]–[13] and shall not be repeated here. In general, both $\varepsilon$ and $\mu$ are tensors, i.e., the dipole direction is not necessarily identical to the exciting vector direction.

A further set of new possibilities originates from the "cross terms". This means that, in general, magnetic dipoles can not only be excited by the magnetic field but also by the electric field. Similarly, electric dipoles can not only be excited by the electric field but also by the magnetic field. These cross terms are subject of the present review or brief tutorial. We focus on the special case that the dipole vectors are oriented perpendicular to the exciting fields. If the dipole vectors are oriented parallel to the exciting fields, another special case arises, namely chirality.

Bianisotropy and chirality are very well established parts of electromagnetism and a bulk of corresponding theoretical literature does exist [14]–[31]. Yet, based on our own experience, some of that literature is somewhat difficult to digest for an experimentalist. Thus, we start with a simple and intuitive tutorial based on an example, namely on split-ring resonators. The discussion aims at giving an understanding and explaining the physics rather than at a complete and/or a quantitative description. Next, we derive in detail the formulae required for retrieving the effective parameters of bianisotropic metamaterials from transmittance and reflectance data. Finally, we briefly review recent experiments on bianisotropic metamaterials operating at optical frequencies.

II. BIANISOTROPY

A. Split-Ring Resonators

A split-ring resonator (SRR) is a metallic ring with one or several slits [1]–[8], [32]–[34] (see Fig. 1). The incident light field can induce a circulating and oscillating electric current in the metallic wire that gives rise to a local magnetic field (magnetic dipole) normal to the SRR plane. The resonance of the SRR can be thought of as arising from the inductance of an almost closed loop (inductance $L$) and the capacitor formed by the two ends of the wire (capacitance $C$). This leads to an $LC$ eigenfrequency $\omega_{LC} = 1/\sqrt{LC}$. For small SRR, the kinetic inductance can add to $L$ [35], [36]. Also, a more
Fig. 1. Illustration of a split-ring resonator and the parameters used in the calculations. The excitation geometry considered in this section is also depicted.

detailed modeling would have to account for additional surface contributions to the SRR capacitance $C$ [37], [38]. Furthermore, energy can either be dissipated by Ohmic losses or can be radiated into free space, leading to the radiation resistance [39]–[41]. The effect of both can be lumped into an effective resistance $R$ of the circuit.

Using Kirchhoff’s voltage law, the equation of motion of the electric current $I$ in this simple circuit results as

$$U_C + U_R + U_L = \frac{1}{C} \int I dt + RI + L \frac{dI}{dt} = U_{\text{ind}}.$$  \hspace{1cm} (1)

The incident light field induces the source voltage $U_{\text{ind}}$.

The SRR generally has both an electric and a magnetic dipole moment. As usual, the electric dipole moment is given by the charges separated on the capacitor plates, $\int I dt$, times their distance $d$. The macroscopic electric polarization, $P$, is the product of the individual electric dipole moment times the number density of the dipoles $N_{LC}/V = 1/(a_{xy}^2a_z)$, where $a_{xy}$ and $a_z$ are the in-plane and out-of-plane lattice constants of the crystal composed of SRR—provided that we neglect interaction effects among the SRR in the crystal. This leads to

$$P_x(t) = \frac{1}{a_{xy}^2a_z} d \int I dt.$$  \hspace{1cm} (2)

Similarly, the magnetic dipole density, $M$, is the product of the SRR number density and the individual magnetic dipole moment. Within the quasi-static limit (no retardation), the latter is given by the current $I$ times the area of the loop. This leads to

$$M_y(t) = \frac{1}{a_{xy}^2} I(t) l^2.$$  \hspace{1cm} (3)

Note that we have tacitly neglected the displacement current at this point. Hence, our reasoning is only strictly valid provided that the Ohmic current dominates over the displacement current (i.e., the slit in the SRR must not be too large).

(i) Let us start by discussing a current that is solely induced by Faraday’s induction law. In this case, we have $U_{\text{ind}}(t) = -\frac{\partial \phi}{\partial t}$ with the magnetic flux $\phi(t)$ given by $\phi(t) = \mu_0 H_y(t)l^2$. Assuming a harmonically varying magnetic field $H_y(t) = H_y \exp(-i\omega t) + c.c.$, we obtain

$$M_y(t) = M_y \exp(-i\omega t) + c.c. \quad \text{with}$$

$$M_y = \frac{\mathcal{F}\omega^2}{\omega^2_{LC} - \omega^2 - i\gamma\omega} H_y.$$  \hspace{1cm} (4)

Here, we have employed the inductance of a long coil $L = \mu_0 l^2/h$ and have introduced two abbreviations: the damping $\gamma = R/L$ and the SRR volume filling fraction $0 \leq F \leq 1$ with

$$\mathcal{F} = \frac{l^2 h}{(a_{xy}^2a_z)}.$$  \hspace{1cm} (5)

Notably, induction via Faraday’s law also leads to a polarization $P_x(t) = P_x \exp(-i\omega t) + c.c.$ with

$$P_x = \frac{d}{l^2} \frac{i\mathcal{F}\omega}{\omega^2_{LC} - \omega^2 - i\gamma\omega} H_y.$$  \hspace{1cm} (6)
Here, we have employed the capacitance $C = \epsilon_0 wh/d$ for a plate capacitor with large plates. Note that $\vec{B}$ is phase delayed by 90 degrees with respect to the exciting $H$-field. Otherwise it reveals the same resonance behavior around the $LC$ eigenfrequency as the magnetization $\vec{M}$. Also note that the induced polarization goes to zero as the slit width $d$ of the SRR is made smaller and smaller.

(ii) Next, we discuss a current that is induced by a voltage drop over the plate capacitor that arises from the electric-field component, $E_x(t)$, of the light field. In this case, we have $U_{\text{ind}}(t) = E_x(t) d$. Assuming a harmonically varying electric field $E_x(t) = E_x \exp(-i \omega t) + \text{c.c.}$, we obtain

$$M_y = \frac{1}{\mu_0} \frac{d}{l^2} \frac{-i \mathcal{F} \omega}{\omega^2_{LC} - \omega^2 - i \gamma \omega} E_x \quad (7)$$

and

$$P_x = \frac{1}{\mu_0} \left( \frac{d}{l^2} \right)^2 \frac{\mathcal{F} \omega}{\omega^2_{LC} - \omega^2 - i \gamma \omega} E_x \quad (8)$$

In this case, the 90 degrees phase delay (see imaginary unit in numerator) occurs for the magnetization $M_y$.

Provided the displacement current is negligible [27] (which is usually fulfilled in the vicinity of the resonance), we can identify the macroscopic magnetization with the magnetic dipole density $\vec{M}$ discussed above. Hence, we get the macroscopic material equations

$$\vec{B} = \epsilon_0 \vec{E} + \vec{\vec{P}} \quad (9)$$

and

$$\vec{B} = \mu_0 (\vec{H} + \vec{\vec{M}}) \quad (10)$$

We can summarize our findings in (i) and (ii) for the SRR by

$$\begin{pmatrix} D_x \\ B_y \end{pmatrix} = \begin{pmatrix} \epsilon_0 & -i \epsilon_0^{-1} \xi \\ +i \epsilon_0^{-1} \xi & \mu_0 \mu \end{pmatrix} \begin{pmatrix} E_x \\ H_y \end{pmatrix} \quad (11)$$

Here we have introduced the (dimensionless) electric permittivity

$$\epsilon(\omega) = 1 + \left( \frac{d \epsilon_0}{l^2} \right)^2 \frac{\mathcal{F} \omega}{\omega^2_{LC} - \omega^2 - i \gamma \omega} \quad (12)$$

the (dimensionless) magnetic permeability

$$\mu(\omega) = 1 + \frac{\mathcal{F} \omega^2}{\omega^2_{LC} - \omega^2 - i \gamma \omega} \quad (13)$$

and the (dimensionless) bianisotropy parameter

$$\xi(\omega) = - \frac{d \epsilon_0}{l^2} \frac{\mathcal{F} \omega}{\omega^2_{LC} - \omega^2 - i \gamma \omega} \quad (14)$$

and the vacuum speed of light $c_0 = 1/\sqrt{\epsilon_0 \mu_0}$. $\epsilon$ describes the excitation of electric dipoles by the electric field of the light, $\mu$ the excitation of magnetic dipoles by the magnetic field, and $\xi$ the excitation of magnetic dipoles by the electric field and vice versa. Fig. 2 (a)–(d) illustrates these quantities.

If the slit in the SRR in Fig. 1 is at the lower part rather than at the top, the current induced by the electric field flows into the other direction and $+\xi \rightarrow -\xi$ (whereas $+\epsilon \rightarrow +\epsilon$ and $+\mu \rightarrow +\mu$).

Clearly, for the excitation geometry considered in Fig. 1, the single-slit SRR has no center of inversion along the propagation direction of light. If we introduce a second slit at the bottom of the SRR (i.e., opposite to the first slit), inversion symmetry is recovered. Provided we neglect retardation effects, the voltage drop over the second slit induced by the electric field is opposite in sign to that of the first slit, i.e., $I = 0$. Thus, neither a magnetization nor a polarization is induced by the electric field. A magnetic field component normal to the SRR plane can still induce a circulating and oscillating current, leading to a magnetic dipole moment. However, the electric dipole moment of the second slit is opposite to that of the first slit. Hence, no electric polarization results from the magnetic field of the incident light. Indeed, it is straightforward to show within our model that—under these conditions—the bianisotropy parameter $\xi$ is
strictly zero and $\epsilon = 1$, while $\mu \neq 1$. This finding, i.e., that a non-zero bianisotropy parameter requires breaking of inversion symmetry, is also valid beyond our simple SRR example.

B. Bianisotropic Parameter Retrieval

Light impinging under normal incidence onto such a slab of effective bianisotropic material will be partially reflected and partially transmitted according to the generalized version of the Fresnel equations. Owing to the lack of inversion symmetry, the reflectance does depend on from which side of the slab light impinges. In contrast, due to reciprocity, the transmittance does not depend on from which side light impinges. The dependence of the complex field transmittance and the two complex field reflectances on $\epsilon$, $\mu$, and $\xi$ can be inverted, which forms an important ingredient for retrieving these effective parameters from numerical calculations and/or from experimental data. We have previously published the closed formulae for this retrieval for normal incidence of light in [42] (also see [19], [23]). In this subsection, we present the (somewhat lengthy) derivation that we have not published previously. These formulae are important when actually working with bianisotropy.

We consider a monochromatic, linearly polarized field $\vec{E}^i = E^i_x e^{i(k_1 z - \omega t)} \hat{e}_x$ and $\vec{H}^i = H^i_y e^{i(k_1 z - \omega t)} \hat{e}_y$ which impinges under normal incidence from an isotropic material of relative impedance $z_1$ (e.g., air or vacuum) onto a bianisotropic metamaterial slab of thickness $d_s$ and which is transmitted into another isotropic material of relative impedance $z_2$ (e.g., a glass substrate). The geometry and the nomenclature used are illustrated in Fig. 3.

Considering the constitutive relations of the bianisotropic material (11) and introducing the following plane-wave ansatz $\vec{E}^\pm = E^\pm e^{i(k_\pm z - \omega t)} \hat{e}_x$ and $\vec{H}^\pm =...
where the refractive index $n$ is given by

$$n^2 = \varepsilon \mu - \xi^2.$$  

For a passive material, the root has to be chosen such that $\text{Im}(n) \geq 0$. Otherwise, exponentially growing solutions occur, violating energy conservation.

The bulk impedance of the bianisotropic material is $Z_+$ for propagation in the (+)-direction and $-Z_-$ for propagation in the (-)-direction. These quantities are given by $Z_+ = E^+ / H^+$ and $Z_- = E^- / H^-$. We derive from Maxwell’s equations

$$z_{\pm} \equiv Z_{\pm} / Z_0 = \mu (\pm n - i\xi)^{-1}$$  

where $Z_0 = \sqrt{\mu_0 / \varepsilon_0}$ is the vacuum impedance. We note that $z_+ \neq -z_-$.  

Using the boundary condition that the tangential components $E$ and $H$ are continuous and the fact that $HZ_0 = E/z_0$ and writing the complex reflectance and transmittance for a wave incident in the (+)-direction $r_+ = E^t / E^i$ and $t_+ = E^t / E^i$, we get the following equations at $z = 0$:

$$(1 + r_+) E^i = E^+ + E^-, \quad (17)$$  
$$(1 - r_+) E^i / z_1 = E^+ / z_+ + E^- / z_- \quad (18)$$

and at $z = d_0$:

$$E^+ e^{i k_0 d_0} + E^- e^{-i k_0 d_0} = t_+ E^i, \quad (19)$$  
$$E^+ e^{i k_0 d_0} / z_+ + E^- e^{-i k_0 d_0} / z_- = t_+ E^i / z_2 (20)$$

With (17)–(18) (respectively (19)–(20)) we express $E^+ / E^i$ and $E^- / E^i$ as linear functions of $r_+$ (respectively $t_+$):

$$E^+ / E^i = a_+ + b_+ r_+ \quad \text{and} \quad E^+ / E^i = c_+ + d_+ t_+,$$

$$E^- / E^i = a_- + b_- r_+ \quad \text{and} \quad E^- / E^i = c_- + d_- t_+.$$  

This yields two linear relationships between $r_+$ and $t_+$

$$t_+ = \alpha + \beta r_+ \quad \text{and} \quad (21)$$  
$$t_+ = \gamma + \delta r_+ \quad (22)$$

where

$$\alpha = e^{i k_0 d_0} (1 - z_- / z_1) (1 - z_- / z_2)^{-1},$$  
$$\beta = e^{i k_0 d_0} (1 + z_- / z_1) (1 - z_- / z_2)^{-1},$$  
$$\gamma = e^{-i k_0 d_0} (1 - z_+ / z_1) (1 - z_+ / z_2)^{-1},$$  
$$\delta = e^{-i k_0 d_0} (1 + z_+ / z_1) (1 - z_+ / z_2)^{-1}.$$  

We want to deduce the three complex parameters $\varepsilon$, $\mu$ and $\xi$, which depend directly on $n$, $z_+$ and $z_-$.  

---

Fig. 3. Illustration of field components for the generalized version of Fresnel’s equations for retrieving effective parameters including bianisotropy. The metamaterial of interest is clad between an isotropic material #1 (e.g., air) and another isotropic material #2 (e.g., a glass substrate). A substrate occurs in most metamaterial experiments at optical frequencies.
from the complex transmittance and reflectance of the material. Therefore, (21) and (22) alone are not sufficient to solve the problem and we need to consider the case of propagation in the (−)-direction, too. In this case, (17)–(20) take the same form as previously, except that we have to make the following substitutions (see Fig. 3)

\[
\begin{align*}
\text{(+)-direction:} & & z_1 & z_2 & z_+ & z_- \\
& & \downarrow & \downarrow & \downarrow & \downarrow \\
\text{(-)-direction:} & & -z_2 & -z_1 & z_- & z_+ \\
\end{align*}
\]

As a consequence, we obtain the following equations, corresponding to (21) and (22), for the (−)-direction

\[
\begin{align*}
t_- &= \alpha' + \beta' r_- \quad \text{and} \quad (23) \\
t_- &= \gamma' + \delta' r_- \quad \text{(24)}
\end{align*}
\]

where

\[
\begin{align*}
\alpha' &= e^{inkd} (1 + z_+/z_2) (1 + z_+/z_1)^{-1}, \\
\beta' &= e^{inkd} (1 - z_+/z_2) (1 + z_+/z_1)^{-1}, \\
\gamma' &= e^{-inkd} (1 + z_-/z_2) (1 + z_-/z_1)^{-1}, \\
\delta' &= e^{-inkd} (1 - z_-/z_2) (1 + z_-/z_1)^{-1}.
\end{align*}
\]

We note that \( t_+ / z_2 = t_- / z_1 \) (calculation not detailed here), which results in \( T = T_+ = T_- \). i.e., the intensity transmittance \( T \) does not depend on from which side of the slab light impinges onto the slab.

We now need to invert (21)–(24) in order to calculate \( z_+, z_- \) and \( n \) for known \( t_+, r_+, t_- \) and \( r_- \). Multiplying (21) by (24) and (22) by (23) yields

\[
\begin{align*}
t_+ t_- &= \alpha' \gamma' + \beta' r_+ + \alpha \delta' r_- + \beta \delta' r_+ r_- \quad \text{(25)} \\
t_+ t_- &= \gamma' \alpha' + \delta' r_+ + \gamma \beta' r_- + \delta \beta' r_+ r_- \quad \text{(26)}
\end{align*}
\]

It follows that (25) and (26) are the same equation for \( z_+ \) and \( z_- \). It can be rewritten as a second degree polynomial equation for \( z_{\pm} \): \( a z_{\pm}^2 + b z_{\pm} + c = 0 \), which means that

\[
z_{\pm} = \left(-b \mp \sqrt{b^2 - 4ac}\right)/(2a) \quad \text{(27)}
\]

Again assuming a passive medium, the sign in (27) must be chosen in order to have a positive real part of the medium impedance. We have already noted that \( z_+ \) is the relative impedance of the bianisotropic medium in the (−)-direction and \( z_- \) is the opposite of the relative impedance in the (−)-direction, which yields \( \text{Re}(z_+) > 0 \) and \( \text{Re}(z_-) > 0 \).

To find the refractive index, we can rewrite (21) and (22) as

\[
\begin{align*}
t_+ &= e^{inkd} [1 + r_+ - (1 - r_+) z_- / z_1] (1 - z_- / z_2)^{-1}, \\
t_+ &= e^{-inkd} [1 + r_+ - (1 - r_+) z_+ / z_1] (1 - z_+ / z_2)^{-1}.
\end{align*}
\]
from (27) and (28) via

\[ \epsilon = (n + i \xi)/z_+ , \quad (29) \]
\[ \mu = (n - i \xi) z_+ , \quad \text{and} \quad (30) \]
\[ \xi = i n (z_+ - z_z) (z_+ - z_-)^{-1} . \quad (31) \]

Let us illustrate this retrieval by the simple example shown in Fig. 4. The three dielectric layers shown there can be viewed as one (\( N = 1 \)) unit cell of a periodic structure that has no center of inversion along the propagation direction of light (just like the SRR depicted in Fig. 1). Due to the broken inversion symmetry, the field reflectance clearly depends on from which side light impinges onto the stack under normal incidence. Applying the above bianisotropic retrieval to this case at, e.g., \( \lambda = 1 \mu m \ (\gg \ d_1 = d_2 = d_3 = 10 \text{nm}) \) wavelength, leads to the effective parameters \( \epsilon = 6.72, \mu = 1.00, \) and \( \xi = -0.21 \) that refer to a fictitious single homogeneous effective slab with total thickness \( d_e = d_1 + d_2 + d_3 = 30 \text{ nm} \). We have explicitly verified that the same parameters are retrieved if \( N = 2, 3, 4, ..., 20 \) unit cells of the identical three-layer structure are considered (i.e., the total slab thickness is \( N \times 30 \text{ nm} \)). Thus, the retrieved quantities \( \epsilon, \mu, \) and \( \xi \) can indeed be interpreted as effective material parameters. As the damping is strictly zero in this example, no absorption occurs. Hence, the sum of transmittance and reflectance is unity – for each propagation direction. Thus, the two intensity reflectances \( R_+ \) and \( R_- \) are identical in this case and differences occur only in the phases of the field reflectances \( r_+ \) and \( r_- \).

This simple example clearly shows that one should be somewhat cautious with using the well known Maxwell Garnett approximation at this point, as it would cast the effective behavior of the three subwavelength dielectric layers in Fig. 4 into just an effective dielectric function

\[ \epsilon = 1 + \frac{\epsilon_1 - 1) d_1 + (\epsilon_2 - 1) d_2 + (\epsilon_3 - 1) d_3}{d_1 + d_2 + d_3} = 6.67 , \quad (32) \]

assuming \( \mu = 1 \) and \( \xi = 0 \), leading to a single impedance \( Z = \sqrt{\mu/\epsilon} \times Z_0 \). The Maxwell–Garnett approximation obviously ignores that the field reflectance depends on from which side of the slab it is measured. This may or may not be important, depending on the problem.
C. Experiments

Experiments on bianisotropic metamaterials—including retrieval of the bianisotropy parameter—have been published for microwave [19], [23], [43], far-infrared [44], and optical frequencies [42], [45], [46]. Related structures have been fabricated previously [47], [48] but bianisotropy has not been mentioned. Fig. 5 shows electron micrographs of three different optical-regime samples that have been fabricated in our group by direct laser writing (see, e.g., review in [13]) of a polymeric template and subsequent metallization. Metallization is accomplished either by chemical vapor deposition of silver [42], [45] (Fig. 5 (a), (c), and (d)) or by high-vacuum shadow evaporation of silver [46] (Fig. 5 (b)). The structures in Figs. 5 (c) and (d) are derived from (a) via post-processing using focused-ion-beam (FIB) milling [45]. The structures in (a) and (b) have also been FIB cut to reveal their interior. Obviously, all four samples in Fig. 5 are variations of the SRR geometry shown in Fig. 1. In Fig. 5 (a), the SRR are connected in two directions, in (b) they are connected along one direction only and an additional orthogonal set of intentionally elevated metallic wires has been introduced. The corresponding increased design freedom has led to a negative phase velocity [46] (i.e., to \( \text{Re}(n) < 0 \)). In (c), the SRR are also only connected along one direction, whereas (d) is a two-dimensional array of disconnected SRR similar to Fig. 1.

As an example, Fig. 7 (a)–(d) shows the results of the parameter retrieval (see previous subsection) for the structure corresponding to Fig. 5 (a), the measured intensity transmittance and reflectance spectra of which are depicted in Fig. 6. For computational details see [45]. This structure is an improved version [45] of the one
Fig. 6. Measured normal-incidence intensity transmittance ($T$) and reflectances ($R_+, R_-$) corresponding to the calculated spectrum shown in Fig. 7(e). The polarization of the incident electromagnetic field is illustrated on the upper left-hand-side corner of Fig. 5(a). The rapid oscillations of $R_-$ are due to Fabry-Perot interferences in the 170-$\mu$m-thick glass substrate. Artifacts at around 70 THz are caused by absorption lines of CO$_2$.

Published in [42]. Again (compare Fig. 2 (a)), the real part of the refractive index is positive ($\text{Re}(n) > 0$) despite the fact that both $\text{Re}(\varepsilon) < 0$ and, at the same time, $\text{Re}(\mu) < 0$ due to the very significant influence of the bianisotropy parameter $\xi$. The fact that $\xi \neq 0$ in Fig. 7 (c) is intimately connected to $R_+ \neq R_-$ in Fig. 7 (e).

III. CHIRALITY

In the previous section and in particular in (11), we have used a scalar formulation for $\xi$ and the other quantities ($\varepsilon$ and $\mu$). Equation (11) has been a special example of the more general form for reciprocal media

$$
\begin{pmatrix}
\vec{D} \\
\vec{B}
\end{pmatrix} =
\begin{pmatrix}
\varepsilon_0 \varepsilon - i \varepsilon_0^{-1} \xi \\
+ i \varepsilon_0^{-1} \xi^t \\
\mu_0 \mu
\end{pmatrix}
\begin{pmatrix}
\vec{E} \\
\vec{H}
\end{pmatrix},
$$

where $\varepsilon$, $\mu$, and $\xi$ are tensors. $\xi^t$ is the transposed of $\xi$. In this general bianisotropic case, regarding the cross terms, the electric polarization is no longer necessarily perpendicular to the exciting magnetic-field vector and, similarly, the magnetization is no longer necessarily perpendicular to the exciting electric-field vector.

Fig. 7. (a)–(d) Retrieved effective parameters from numerical calculation of the complex transmittance and reflectances of the structure shown in Fig. 5 (a). (e) Calculated normal-incidence intensity transmittance ($T$) and reflectances ($R_+, R_-$) of this structure.
Another special example of the general bianisotropic case (33) is

\[
\begin{pmatrix}
\vec{D} \\
\vec{B}
\end{pmatrix} =
\begin{pmatrix}
\epsilon_0 \epsilon & -i \epsilon_0^{-1} \xi \\
+i \epsilon_0^{-1} \xi & \mu_0 \mu
\end{pmatrix}
\begin{pmatrix}
\vec{E} \\
\vec{H}
\end{pmatrix},
\]

(34)

where \(\epsilon, \mu\), and \(\xi\) are again scalars. Note that (34) looks rather similar to (11) at first sight. However, it has a totally different meaning and leads to completely different behavior, namely to chirality. Chiral metamaterials are a subclass of bianisotropic metamaterials. Let us briefly elaborate on the differences with respect to the previous section.

For a plane wave propagating through a medium, the incident electric and magnetic vector components are perpendicular to each other. If, as the wave passes through the chiral medium following (34), the magnetic component induces an electric dipole parallel to the magnetic field vector, the resulting net local electric field vector will clearly be rotated a bit. Likewise, magnetic dipoles are excited by the electric vector component. Hence, the magnetic field vector rotates as well—regardless of the incident polarization of light. Thus, the eigenstates no longer correspond to linear polarization of light (as in the preceding section) but rather to circular polarization of light. In this basis, the refractive index can be expressed [49] via

\[
n_{\pm} = \sqrt{\epsilon \mu \mp \xi},
\]

(35)

As usual, the sign of the complex root has to be chosen appropriately. The other signs in (35) refer to right-handed (+) and left-handed (−) circular polarization of light, respectively. The difference of the two refractive indices becomes \(n_+ - n_- = -2\xi\). The behavior of (35) is very different from that for the other special case of bianisotropy in (15). For example, in principle, (35) allows for a negative index of refraction (precisely, a negative phase velocity of light) for one handedness of light if both \(\epsilon\) and \(\mu\) are mainly real and positive—if only the modulus of \(\xi\) is sufficiently large. In this sense, a large real value of \(\xi\) is helpful for pure chirality, whereas a large real value of \(\xi\) works against a negative phase velocity of light for pure bianisotropy, because a large \(\xi^2\) in (15) leads to a negative \(n^2\), i.e., to evanescent waves.

Retrieval of the effective parameters of purely chiral metamaterials (the analogue of our discussion in Sect. II. B.) has been published [49]. First chiral negative-index metamaterials [50] have recently been realized at microwave [51] and far-infrared [52] frequencies.

IV. SUMMARY AND OUTLOOK

The advance of man-made metamaterials has significantly increased our possibilities regarding manipulating light via optical materials. Light is an electromagnetic wave with an electric and a magnetic vector component. Either of them can excite both electric and magnetic dipoles inside the material. These dipoles can be parallel or orthogonal to the exciting field component—leading to a rich variety of cases. If, for example, the magnetic (electric) dipoles excited by the electric (magnetic) field vector are perpendicular to each other, the reflectance of a slab of such material becomes asymmetric. A negative phase velocity of linearly polarized light (“negative-index metamaterials”) can still be achieved in this case, however bianisotropy is not usually helpful. In particular, the phase velocity of light can be positive even if both electric permittivity and magnetic permeability are negative. In contrast, if, for example, the magnetic (electric) dipoles excited by the electric (magnetic) field vector are parallel to each other, the medium becomes chiral. Chirality tends to enable negative phase velocities. In particular, for strong chirality, the phase velocity of circularly polarized light can be negative even if both permittivity and permeability are positive. At optical
frequencies, special bianisotropic negative-index structures have been realized experimentally, while chiral negative-index structures have not. However, interesting and encouraging results have recently been published at GHz (microwave) and THz (far-infrared) frequencies. Results on chiral photonic metamaterials exhibiting a positive phase velocity of light have been published previously [53]–[58].

It is rather likely that chiral negative-index metamaterials operating at optical frequencies will be realized experimentally in the near future. However, all chiral photonic metamaterials presented so far are uniaxial and, hence, highly anisotropic. The design and experimental realization of isotropic chiral artificial materials operating at optical frequencies pose a major future challenge, especially regarding three-dimensional nanofabrication. For such materials, negative reflection of light has been predicted theoretically [30]. Finally, further possibilities arise if Faraday active ingredients are incorporated into the metamaterial. In this case, not only the reflectance but also the transmittance can become asymmetric [22], [31], which might, e.g., give rise to very compact optical isolators.

ACKNOWLEDGMENT

The authors thank Costas M. Soukoulis for discussions. They acknowledge financial support via the project PHOME of the Future and Emerging Technologies (FET) program within the Seventh Framework Programme for Research of the European Commission, under FET-Open grant number 213390 and via the META-MAT project by the Bundesministerium für Bildung und Forschung (BMBF). The authors further acknowledge support of the Deutsche Forschungsgemeinschaft (DFG) and the State of Baden–Württemberg through the DFG-Center for Functional Nanostructures (CFN) within subprojects A 1.4 and A 1.5. The research of S. L. is further supported through a Helmholtz-Hochschul-Nachwuchsgruppe (VH-NG-232), the PhD education of C. E. K. and M. S. R. by the Karlsruhe School of Optics and Photonics (KSOP). C. E. K. is also supported by the Université de Bourgogne (Dijon, France).

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