

Perfect cylindrical lenses

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Abstract: A slab of negatively refracting material is known to focus light and if $n = -1$ the focussing will be perfect, producing an image which is an exact replica of the object. Magnifying the image requires a new design concept in which the surface of the negatively refracting lens is curved. Here we show how a hollow cylinder of material can be designed to magnify an image but otherwise with the same perfection as the original lens. Curvature requires that ϵ and μ are now a function of position.

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(240.6690) Surface waves

References and links

1. V.G. Veselago, "The electrodynamics of substances with simultaneously negative values of ϵ and μ ," Soviet Physics USPEKHI **10**, 509 (1968).
2. R.A. Shelby, D.R. Smith, S. Schultz, "Experimental verification of negative index of refraction," Science **292**, 79 (2001).
3. J.B. Pendry, "Negative Refraction Makes a Perfect Lens," Phys. Rev. Lett. **85** 3966 (2000).
4. D.F. Sievenpiper, M.E. Sickmiller, and E. Yablonovitch, "3D Wire Mesh Photonic Crystals," Phys Rev Lett **76**, 2480 (1996).
5. J.B. Pendry, A.J. Holden, W.J. Stewart, I. Youngs, "Extremely Low Frequency Plasmons in Metallic Mesostuctures," Phys Rev Lett **76** 4773 (1996)
6. J.B. Pendry, A.J. Holden, D.J. Robbins, and W.J. Stewart, "Low Frequency Plasmons in Thin Wire Structures," J. Phys. [Condensed Matter] **10**, 4785 (1998).
7. J.B. Pendry, A.J. Holden, D.J. Robbins, and W.J. Stewart, "Magnetism from Conductors and Enhanced Non-Linear Phenomena," IEEE Transactions on Microwave Theory and Techniques **47**, 2075 (1999).
8. D.F. Sievenpiper, L. Zhang, R.F.J. Broas, F.J. Alexopoulos, E. Yablonovitch, IEEE Trans. Micr. Theory and Tech. "High impedance electromagnetic surfaces with a forbidden frequency band," **47**, 2059-2074 (1999).
9. R.F.J. Broas, D.F. Sievenpiper, E. Yablonovitch "A high-impedance ground plane applied to a cell phone handset geometry," IEEE Trans. Micr. Theory and Tech. **49**, 1262-1265 (2001).
10. M.C.K. Wiltshire, J.B. Pendry, I.R. Young, D.J. Larkman, D.J. Gilderdale and J.V. Hajnal, "Microstructured Magnetic Materials for RF Flux Guides in Magnetic Resonance Imaging," Science **291** 848-51 (2001).
11. Chiyang Luo, Steven G. Johnson, J.D. Joannopoulos and J.B. Pendry "All-Angle Negative Refraction without Negative Effective Index," Phys. Rev. Rapid Communications **B65**, 201104(R) (2002).
12. J.B. Pendry and S.A. Ramakrishna "Near Field Lenses in Two Dimensions," J. Phys. [Condensed Matter] **14** 1-17 (2002).
13. A.J. Ward and J.B. Pendry "Refraction and Geometry in Maxwell's Equations," Journal of Modern Optics **43** 773-93 (1996).
14. R.H. Ritchie, "Plasma Losses by Fast Electrons in Thin Films," Phys. Rev. **106**, 874 (1957).

1. Introduction

Some time ago Veselago [1] observed that a slab of material with,

$$\epsilon = -1, \mu = -1 \quad (1)$$

would have refractive index $n = -1$ and behave as a lens. The negative refractive index was subsequently confirmed by Shelby, Smith and Schultz in 2001 [2]. A further remarkable property of this new type of material was later pointed out [3]: the property that the focussing is perfect provided that condition (1) is met exactly. Materials that approximate the properties demanded can now be designed both in respect of negative ϵ [4, 5, 6] and negative μ [5, 6, 9, 10]. It has even proved possible to realise these conditions through photonic structures [11].

Building on the perfect lens theory [7] we exploited conformal transformations to bend the shape of the perfect lens into other geometries such as two concentric cylinders or alternatively two touching cylinders. See Figs. 1 and 2. The objective was to make a magnifying glass, and for this purpose a lens must be curved. The conformal transformation,

$$z' = \ln z \quad (2)$$

where $z = x + iy$ resulted in a cylindrical lens which reproduced the contents of the smaller cylinder in magnified but undistorted form outside the larger cylinder. This transformation preserves the solutions of Laplace's equation and leaves the values of ϵ, μ , the electrical permittivity/magnetic permeability, unchanged in their respective domains. However Laplace's equation is a valid description of the fields only in the electrostatic or magnetostatic limit where the electric and magnetic fields separate. Lenses defined by conformal transformations are only valid provided that all dimensions are much less than the wavelength of light which is a somewhat limiting condition. Fortunately there exists a more general theory of arbitrary coordinate transformations [13] which enables us to make an exact transformation of the original planar lens into a perfect lens of almost any geometry we choose, provided that we adjust ϵ, μ accordingly.

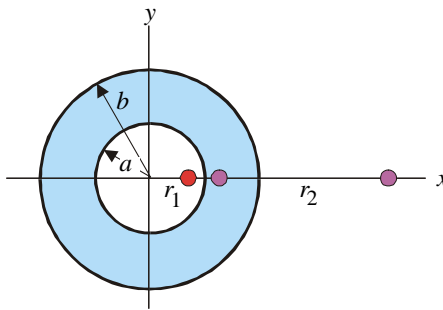


Fig. 1. The annular lens produces a magnified image of internal objects, and a demagnified image of external objects. The magnifying factor is b^2/a^2 . The lens is myopic: only objects closer than $r = b^2/a$ can form an image inside the annulus. Conversely objects within the annulus and closer to the centre than $r = a^2/b$ to the centre will not produce an image outside the annulus.

Consider a cylindrical symmetric system in which,

$$\begin{aligned}
\varepsilon(r) &= \mu(r) > 0, & r < a \\
\varepsilon(r) &= \mu(r) < 0, & a < r < b \\
\varepsilon(r) &= \mu(r) > 0, & r > b
\end{aligned} \tag{3}$$

Let us define a set of cylindrical coordinates as follows,

$$x = r_0 e^{\ell/\ell_0} \cos \phi, \quad y = r_0 e^{\ell/\ell_0} \sin \phi, \quad z = Z \tag{4}$$

From [13] we deduce that in the new frame,

$$\tilde{\varepsilon}_i = \varepsilon_i \frac{Q_1 Q_2 Q_3}{Q_i^2}, \quad \tilde{\mu}_i = \mu_i \frac{Q_1 Q_2 Q_3}{Q_i^2} \tag{5}$$

$$\tilde{E}_i = Q_i E_i, \quad \tilde{H}_i = Q_i H_i \tag{6}$$

where,

$$Q_i^2 = \left(\frac{\partial x}{\partial q_i} \right)^2 + \left(\frac{\partial y}{\partial q_i} \right)^2 + \left(\frac{\partial z}{\partial q_i} \right)^2 \tag{7}$$

so that,

$$\begin{aligned}
Q_\ell &= r_0 / \ell_0 \sqrt{e^{2\ell/\ell_0} \cos^2 \phi + e^{2\ell/\ell_0} \sin^2 \phi} = r_0 / \ell_0 e^{\ell/\ell_0} \\
Q_\phi &= r_0 \sqrt{e^{2\ell/\ell_0} \sin^2 \phi + e^{2\ell/\ell_0} \cos^2 \phi} = r_0 e^{\ell/\ell_0} \\
Q_Z &= 1 \\
Q_\ell Q_\phi Q_Z &= r_0^2 / \ell_0 e^{2\ell/\ell_0}
\end{aligned} \tag{8}$$

and from (5), (8),

$$\begin{aligned}
\tilde{\varepsilon}_\ell &= \ell_0 \varepsilon_\ell, \quad \tilde{\varepsilon}_\phi = \ell_0^{-1} \varepsilon_\phi, \quad \tilde{\varepsilon}_Z = r_0^2 / \ell_0 e^{2\ell/\ell_0} \varepsilon_z \\
\tilde{\mu}_\ell &= \ell_0 \mu_\ell, \quad \tilde{\mu}_\phi = \ell_0^{-1} \mu_\phi, \quad \tilde{\mu}_Z = r_0^2 / \ell_0 e^{2\ell/\ell_0} \mu_z
\end{aligned} \tag{9}$$

where the ℓ coordinate is oriented along the radial direction. Now if we make the choice of $\ell_0 = 1$ and,

$$\begin{aligned}
\varepsilon_\ell = \mu_\ell = +1, \quad \varepsilon_\phi = \mu_\phi = +1, \quad \varepsilon_z = \mu_z = +r_0^{-2} e^{-2\ell} = +r^{-2}, & \quad r < a, \\
\varepsilon_\ell = \mu_\ell = -1, \quad \varepsilon_\phi = \mu_\phi = -1, \quad \varepsilon_z = \mu_z = -r_0^{-2} e^{-2\ell} = -r^{-2}, & \quad b > r > a, \\
\varepsilon_\ell = \mu_\ell = +1, \quad \varepsilon_\phi = \mu_\phi = +1, \quad \varepsilon_z = \mu_z = +r_0^{-2} e^{-2\ell} = +r^{-2}, & \quad r > b
\end{aligned} \tag{10}$$

then in the $\ell\phi Z$ frame,

$$\begin{aligned}
\tilde{\epsilon}_\ell = \tilde{\mu}_\ell = +1, \quad \tilde{\epsilon}_\phi = \tilde{\mu}_\phi = +1, \quad \tilde{\epsilon}_Z = \tilde{\mu}_Z = +1, \quad \ell < \ell_0 \ln(a/r_0), \\
\tilde{\epsilon}_\ell = \tilde{\mu}_\ell = -1, \quad \tilde{\epsilon}_\phi = \tilde{\mu}_\phi = -1, \quad \tilde{\epsilon}_Z = \tilde{\mu}_Z = -1, \quad \ell_0 \ln(a/r_0) < \ell < \ell_0 \ln(b/r_0), \\
\tilde{\epsilon}_\ell = \tilde{\mu}_\ell = +1, \quad \tilde{\epsilon}_\phi = \tilde{\mu}_\phi = +1, \quad \tilde{\epsilon}_Z = \tilde{\mu}_Z = +1, \quad \ell > \ell_0 \ln(b/r_0)
\end{aligned} \quad (11)$$

and hence in the new frame this choice results in specifications for a perfect lens: an object located at,

$$\ell_1 = \ell_0 \ln(r_1/r_0) < \ell_0 \ln(a/r_0) \quad (12)$$

will form an image at,

$$\ell_2 = \ell_1 + 2\ell_0 \ln(b/a) = \ell_0 \ln\left(\frac{r_2}{r_0}\right) \quad (13)$$

where,

$$r_2 = r_1 \frac{b^2}{a^2} \quad (14)$$

This new prescription is almost the same as we arrived at by conformal transformation except that ϵ_z, μ_z now depend on the radius as r^{-2} . The original paper assumed that the electric field was confined to the xy plane and therefore ϵ_z was irrelevant. It also assumed the electrostatic limit so that now magnetic fields were present and hence μ_z was also irrelevant. hence in that limit we retrieve our original result.

2. A perfect crescent lens

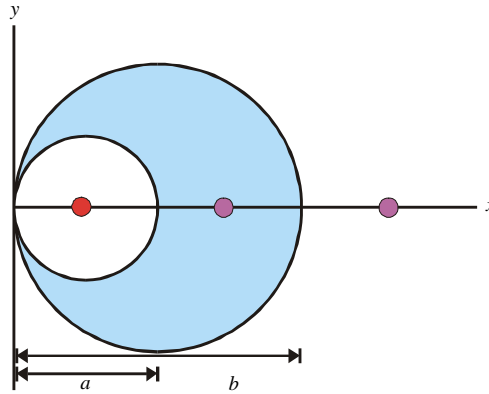


Fig. 2. Other lenses with cylindrical geometry are possible. Here we see the crescent lens in which the inner and outer surfaces touch at the origin.

In the electrostatic approximation the transformation which gave us the crescent lens was,

$$z' = z^{-1} \quad (15)$$

and the corresponding coordinate transformation would be,

$$x = x' / (x'^2 + y'^2), \quad y = -y' / (x'^2 + y'^2), \quad z = z' \quad (16)$$

so that,

$$\begin{aligned} Q_{x'} &= \sqrt{\left[\frac{1}{(x'^2 + y'^2)} - \frac{2x'^2}{(x'^2 + y'^2)^2} \right]^2 + \left[\frac{2x'y'}{(x'^2 + y'^2)^2} \right]^2} \\ &= \frac{\sqrt{[x'^2 + y'^2 - 2x'^2]^2 + 4x'^2 y'^2}}{(x'^2 + y'^2)^2} = \frac{1}{(x'^2 + y'^2)} \end{aligned}$$

$$\begin{aligned} Q_{y'} &= \sqrt{\left[\frac{2x'y'}{(x'^2 + y'^2)^2} \right]^2 + \left[\frac{1}{(x'^2 + y'^2)} - \frac{2y'^2}{(x'^2 + y'^2)^2} \right]^2} \\ &= \frac{\sqrt{4x'^2 y'^2 + [x'^2 + y'^2 - 2y'^2]^2}}{(x'^2 + y'^2)^2} = \frac{1}{(x'^2 + y'^2)} \end{aligned}$$

$$Q_{z'} = 1$$

$$Q_{x'} Q_{y'} Q_{z'} = (x'^2 + y'^2)^{-2} \quad (17)$$

and from (5), (17),

$$\begin{aligned} \tilde{\epsilon}_x = \epsilon_x, \quad \tilde{\epsilon}_y = \epsilon_y, \quad \tilde{\epsilon}_z = (x'^2 + y'^2)^{-2} \epsilon_z = (x^2 + y^2)^2 \epsilon_z \\ \tilde{\mu}_x = \mu_x, \quad \tilde{\mu}_y = \mu_y, \quad \tilde{\mu}_z = (x'^2 + y'^2)^{-2} \mu_z = (x^2 + y^2)^2 \mu_z \end{aligned} \quad (18)$$

Therefore if we choose,

$$\begin{aligned} \epsilon_x = \mu_x = +1, \quad \epsilon_y = \mu_y = +1, \quad \epsilon_z = \mu_z = +r^{-2}, \quad \frac{x}{x^2 + y^2} > a^{-1}, \\ \epsilon_x = \mu_x = -1, \quad \epsilon_y = \mu_y = -1, \quad \epsilon_z = \mu_z = -r^{-2}, \quad b^{-1} < \frac{x}{x^2 + y^2} < a^{-1}, \\ \epsilon_x = \mu_x = +1, \quad \epsilon_y = \mu_y = +1, \quad \epsilon_z = \mu_z = +r^{-2}, \quad \frac{x}{x^2 + y^2} < b^{-1} \end{aligned} \quad (19)$$

$\tilde{\epsilon}$ and $\tilde{\mu}$ revert to the case of the original planar perfect lens so that perfect focussing is again achieved.

3. Conclusions

We have seen that a change of geometry can be expressed as a change in ϵ and μ . In particular a change in geometry can be *compensated for* by a reciprocal change in ϵ and μ . Applying these ideas to the perfect lens generates a whole new class of lens with the same remarkable capability to focus at the sub wavelength level. In the examples we present here curvature in the xy plane is compensated for by changes in ϵ_z, μ_z in the direction normal to the plane of curvature. This ability to transform a concept from one geometry to another can be expected to have applications beyond the present case, for example in the field of photonic crystals.

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