

# Surfaces with holes in them: new plasmonic metamaterials

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## Abstract

In this paper we explore the existence of surface electromagnetic modes in corrugated surfaces of perfect conductors. We analyse two cases: one-dimensional arrays of grooves and two-dimensional arrays of holes. In both cases we find that these structures support surface bound states and that the dispersions of these modes have strong similarities with the dispersion of the surface plasmon polariton bands of real metals. Importantly, the dispersion relation of these surface states is mainly dictated by the geometry of the grooves or holes and these results open the possibility of tailoring the properties of these modes by just tuning the geometrical parameters of the surface.

**Keywords:** surface plasmons, metamaterials, enhanced transmission

(Some figures in this article are in colour only in the electronic version)

## 1. Introduction

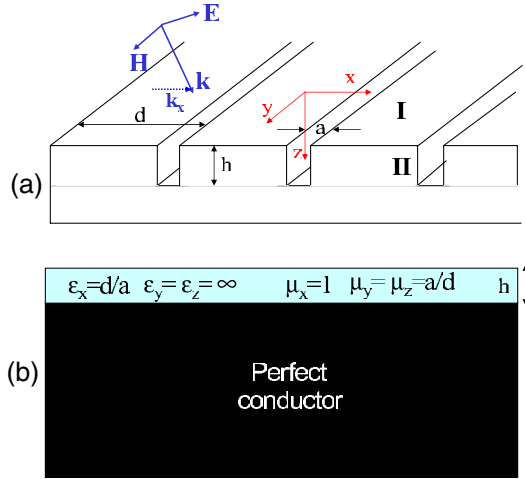
Since the appearance of the paper by Ebbesen *et al* [1] reporting extraordinary optical transmission (EOT) in two-dimensional (2D) arrays of subwavelength holes in metallic films, the study of the optical properties of subwavelength apertures has become one of the most exciting areas in optics research. In this seminal paper [1], the relation between transmission resonances appearing in the spectra and the excitation of the surface plasmon polaritons (SPPs) of the metallic surface was already pointed out. The link between EOT and surface plasmons was corroborated theoretically three years after that [2]. Interestingly, in this last paper we also showed that similar anomalous transmission appears in arrays of subwavelength hole arrays perforated in a perfect conductor. It is well known that the surface of a perfect conductor does not support surface plasmons. This seemed to suggest that the physical origins of EOT in real metals and in perfect conductors were different, leading to discussions as regards the true origin of the EOT phenomenon.

In this paper we solve this paradox by showing that although a flat perfectly conducting surface supports no bound states, the presence of any periodic indentation of the flat

surface (for example, 1D arrays of grooves or 2D hole arrays) provokes the appearance of surface bound states that have strong similarities with the canonical SPPs of a flat metal surface [3, 4]. Importantly, we also show that, as long as the size and spacing of the holes are much smaller than the wavelength, a perforated perfectly conducting surface behaves as an *effective* medium. This medium is characterized by an effective dielectric function that has a plasmon form with a *plasma* frequency dictated by the geometry of the hole or the groove. In other words, the system behaves as a plasmonic metamaterial in which its electromagnetic response is governed by the surface modes that decorate its surface. It is worth commenting that this new class of metamaterials has some links with the metallic metamaterials invented in recent years in connection with the concept of negative refraction [5].

## 2. 1D arrays of grooves

First, we analyse the case of a 1D array of grooves drilled in a perfect conductor (see figure 1(a));  $a$  is the width of the grooves,  $h$  is the depth and  $d$  the period of the array. We are interested in looking at the surface EM modes supported by



**Figure 1.** (a) A one-dimensional array of grooves of width  $a$  and depth  $h$  separated by a distance  $d$ . We are interested in p-polarized surface modes running in the  $x$  direction with  $\mathbf{E}$  lying in the  $x$ - $z$  plane. (b) In the effective medium approximation the structure displayed in (a) behaves as an homogeneous but anisotropic layer of thickness  $h$  on top of a perfect conductor.

this structure. The procedure for calculating the dispersion relation of these surface modes,  $\omega(k_x)$ , is the following. First, we calculate the reflectance of an incident p-polarized incident plane wave with parallel momentum  $k_x$ . As we are interested in a truly surface mode, we will then analyse the expression for the reflectance for the particular case in which the incident plane wave is evanescent,  $k_x > \omega/c_0$  (a truly surface mode has to live outside the light cone). The locations of the divergences in the reflectance will give us the desired dispersion relation for the surface EM modes.

The electromagnetic (EM) fields associated with the incident wave are

$$\begin{aligned} \vec{E}^{\text{inc}} &= \frac{1}{\sqrt{d}} e^{ik_x x} e^{ik_z z} \begin{pmatrix} 1 \\ 0 \\ -k_x/k_z \end{pmatrix} \\ \vec{H}^{\text{inc}} &= \frac{1}{\sqrt{d}} e^{ik_x x} e^{ik_z z} \begin{pmatrix} 0 \\ k_0/k_z \\ 0 \end{pmatrix} \end{aligned} \quad (1)$$

where  $k_0$  is the wavenumber,  $\omega/c_0$ , and  $k_z = \sqrt{k_0^2 - k_x^2}$ . The reflected wave associated with the  $n$ -diffraction order can be written as

$$\begin{aligned} \vec{E}^{\text{ref},n} &= \frac{1}{\sqrt{d}} e^{ik_x^{(n)} x} e^{-ik_z^{(n)} z} \begin{pmatrix} 1 \\ 0 \\ k_x^{(n)}/k_z^{(n)} \end{pmatrix} \\ \vec{H}^{\text{ref},n} &= \frac{1}{\sqrt{d}} e^{ik_x^{(n)} x} e^{-ik_z^{(n)} z} \begin{pmatrix} 0 \\ -k_0/k_z^{(n)} \\ 0 \end{pmatrix} \end{aligned} \quad (2)$$

where  $k_x^{(n)} = k_x + 2\pi n/d$  ( $n = -\infty, \dots, 0, \dots, \infty$ ) and  $k_z^{(n)} = \sqrt{k_0^2 - (k_x^{(n)})^2}$ .

As we assume that the wavelength of light is much larger than the width of the grooves ( $\lambda_0 \gg a$ ), in the modal expansion of the EM fields inside the grooves we only consider the

fundamental TE mode:

$$\begin{aligned} \vec{E}^{\text{TE},\pm} &= \frac{1}{\sqrt{a}} e^{\pm ik_0 z} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \\ \vec{H}^{\text{TE},\pm} &= \frac{1}{\sqrt{a}} e^{\pm ik_0 z} \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}. \end{aligned} \quad (3)$$

Then, the EM fields in region I (vacuum) can be expressed as a sum of the incident plane wave and the reflected ones:

$$\begin{aligned} \vec{E}^{\text{I}} &= \vec{E}^{\text{inc}} + \sum_{n=-\infty}^{\infty} \rho_n \vec{E}^{\text{ref},n} \\ \vec{H}^{\text{I}} &= \vec{H}^{\text{inc}} + \sum_{n=-\infty}^{\infty} \rho_n \vec{H}^{\text{ref},n} \end{aligned} \quad (4)$$

where  $\rho_n$  is the reflection coefficient associated with the diffraction order  $n$ . In region II (inside the grooves), the EM fields can be written as a linear combination of the forward and backward propagating TE modes:

$$\begin{aligned} \vec{E}^{\text{II}} &= C^+ \vec{E}^{\text{TE},+} + C^- \vec{E}^{\text{TE},-} \\ \vec{H}^{\text{II}} &= C^+ \vec{H}^{\text{TE},+} + C^- \vec{H}^{\text{TE},-}. \end{aligned} \quad (5)$$

By applying the standard matching boundary conditions (at  $z = 0$  continuity of  $E_x$  at every point of the unit cell and continuity of  $H_y$  only at the groove's location and at  $z = h$ ;  $E_x$  must be zero), we can easily extract the reflection coefficients,  $\rho_n$ :

$$\rho_n = -\delta_{n0} - \frac{2i \tan(k_0 h) S_0 S_n k_0 / k_z}{1 - i \tan(k_0 h) \sum_{n=-\infty}^{\infty} S_n^2 k_0 / k_z^{(n)}} \quad (6)$$

where  $S_n$  is the overlap integral between the  $n$ th-order plane wave and the TE mode:

$$S_n = \frac{1}{\sqrt{ad}} \int_{-a/2}^{a/2} e^{ik_x^{(n)} x} dx = \sqrt{\frac{a}{d}} \frac{\sin(k_x^{(n)} a/2)}{k_x^{(n)} a/2}. \quad (7)$$

In principle, we could calculate the surface bands of our system by just analysing the zeros of the denominator of equation (6) [6]. The calculation is much simpler if we assume  $\lambda_0 \gg d$ . Then, all the diffraction orders can be safely neglected except the specular one and  $\rho_0$  takes the form

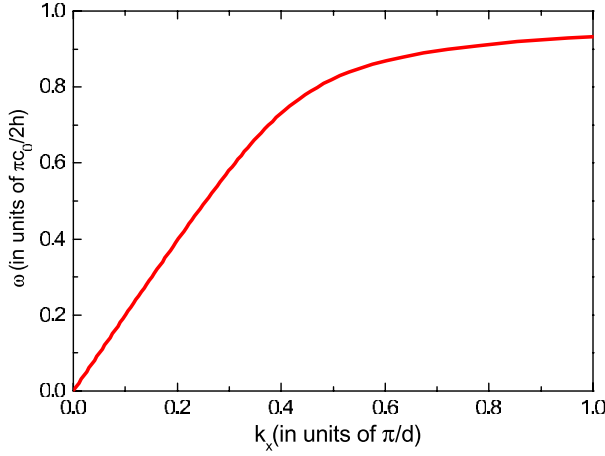
$$\rho_0 = -\frac{1 + i S_0^2 \tan(k_0 h) k_0 / k_z}{1 - i S_0^2 \tan(k_0 h) k_0 / k_z}. \quad (8)$$

For the case  $k_x > k_0$  ( $k_z = i\sqrt{k_x^2 - k_0^2}$ ), we can calculate the dispersion relation of the surface bound state by calculating the location of the divergences of  $\rho_0$ :

$$\frac{\sqrt{k_x^2 - k_0^2}}{k_0} = S_0^2 \tan(k_0 h). \quad (9)$$

This is the dispersion relation of the surface EM modes supported by a 1D array of grooves in the limit  $\lambda_0 \gg d$  and  $\lambda_0 \gg a$ .

It is interesting to note here that the same dispersion relation could be obtained if we replaced the array of grooves



**Figure 2.** The dispersion relation ( $\omega(k_x)$ ) of the surface bound states supported by a 1D array of grooves with geometrical parameters  $a/d = 0.2$  and  $h/d = 1$  as obtained with equation (14).

by a single *homogeneous* but anisotropic layer of thickness  $h$  on top of the surface of a perfect conductor (see the schematic drawing in figure 1(b)). The homogeneous layer would have the following parameters:

$$\epsilon_x = d/a \quad \epsilon_y = \epsilon_z = \infty. \quad (10)$$

As light propagates in the grooves in the  $y$  or  $z$  directions with the velocity of light,

$$\sqrt{\epsilon_x \mu_y} = \sqrt{\epsilon_x \mu_z} = 1 \quad (11)$$

and, hence,

$$\mu_y = \mu_z = \frac{1}{\epsilon_x} \quad \mu_x = 1. \quad (12)$$

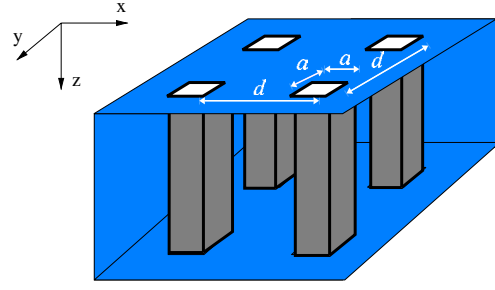
After some straightforward algebra the specular reflection coefficient,  $R$ , for a p-polarized plane wave impinging at the surface of a homogeneous layer of thickness  $h$  with  $\epsilon$  and  $\mu$  given by equations (10) and (12) can be written as

$$R = \frac{(\epsilon_x k_z - k_0) + (k_0 + \epsilon_x k_z) e^{2ik_0 h}}{(\epsilon_x k_z + k_0) - (k_0 - \epsilon_x k_z) e^{2ik_0 h}}. \quad (13)$$

Again, by extending this formula to the case  $k_x > k_0$  and looking at the zeros of the denominator of  $R$  we can calculate the dispersion relation of the surface modes:

$$\frac{\sqrt{k_x^2 - k_0^2}}{k_0} = \frac{a}{d} \tan(k_0 h). \quad (14)$$

Note that this expression coincides with equation (9) in the limit  $k_x a \ll 1$ . In figure 2 we plot the dispersion relation (equation (14)) for the particular case  $a/d = 0.2$  and  $h/d = 1$ . We have checked that this expression (equation (14)) gives accurate results for the range of wavelengths analysed in this case ( $\lambda_0 > 4h$ ) by comparing them with the dispersion relation obtained by calculating the zeros of the denominator of equation (6) in which the approximation  $\lambda_0 \gg d$  is not applied. It is worth commenting on the similarities between this dispersion and the one associated with the bands of SPPs supported by the surfaces of real metals. In a SPP band, at large



**Figure 3.** A two-dimensional square array ( $d \times d$ ) of square holes (side  $a$ ) perforated on a semi-infinite perfect conductor.

$k_x$ ,  $\omega$  approaches  $\omega_p/\sqrt{2}$ , whereas in this case,  $\omega$  approaches  $\pi c_0/2h$ —that is, the frequency location of a cavity waveguide mode inside the groove (in the limit  $a/d \rightarrow 0$ , the locations of the different cavity waveguide modes correspond to the condition  $\cos k_0 h = 0$ ).

### 3. 2D hole array

Now we consider the case of square holes of side  $a$  arranged on a  $d \times d$  lattice perforated on a perfect conductor semi-infinite structure (see figure 3) [7]. We assume that the holes are filled with a material whose dielectric constant is  $\epsilon_h$ . As in the case of the array of grooves, we are interested in looking at the possible surface states supported by this system by looking at divergences of the reflection coefficient of a p-polarized plane wave impinging at the perforated surface. As we are interested in the long wavelength limit ( $\lambda_0 \gg d$ ), now we only take into account the specular reflected wave.

The normalized EM fields associated with the incident and specular reflected waves are

$$\vec{E}^{\text{inc}} = \frac{1}{d} e^{ik_x x} e^{ik_z z} \begin{pmatrix} 1 \\ 0 \\ -k_x/k_z \end{pmatrix} \quad (15)$$

$$\vec{H}^{\text{inc}} = \frac{1}{d} e^{ik_x x} e^{ik_z z} \begin{pmatrix} 0 \\ k_0/k_z \\ 0 \end{pmatrix}$$

$$\vec{E}^{\text{ref}} = \frac{1}{d} e^{ik_x x} e^{-ik_z z} \begin{pmatrix} 1 \\ 0 \\ k_x/k_z \end{pmatrix} \quad (16)$$

$$\vec{H}^{\text{ref}} = \frac{1}{d} e^{ik_x x} e^{-ik_z z} \begin{pmatrix} 0 \\ -k_0/k_z \\ 0 \end{pmatrix}.$$

Inside the holes, as we are interested in the limit  $\lambda_0 \gg a$ , we assume that the fundamental eigenmode will dominate because it is the least strongly decaying. The EM fields are zero inside the perfect metal but inside the holes they take the form

$$\vec{E}^{\text{TE}} = \frac{\sqrt{2}}{a} e^{iq_z z} \sin \frac{\pi y}{a} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \quad (17)$$

$$\vec{H}^{\text{TE}} = -\frac{\sqrt{2}}{a} e^{iq_z z} \sin \frac{\pi y}{a} \begin{pmatrix} 0 \\ q_z/k_0 \\ i\pi/ak_0 \end{pmatrix}$$

where  $q_z = \sqrt{\epsilon_h k_0^2 - \pi^2/a^2}$ .

Again, the EM fields in region I can be expressed as a sum of the incident plane wave and the reflected one:

$$\vec{E}^I = \vec{E}^{\text{inc}} + \rho_0 \vec{E}^{\text{ref}} \quad \vec{H}^I = \vec{H}^{\text{inc}} + \rho_0 \vec{H}^{\text{ref}} \quad (18)$$

where  $\rho_0$  is the specular reflection coefficient, and in region II (inside the holes), as we are dealing with a semi-infinite structure, we only have to consider the decaying mode:

$$\vec{E}^{\text{II}} = \tau \vec{E}^{\text{TE}} \quad \vec{H}^{\text{II}} = \tau \vec{H}^{\text{TE}} \quad (19)$$

where  $\tau$  is the transmission coefficient.

In the matching procedure at  $z = 0$ ,  $E_x$  must be continuous over the entire unit cell ( $x$  and  $y$  ranging from 0 and  $d$ ) whereas  $H_y$  has to be continuous only at the hole. This would yield

$$\rho_0 = \frac{k_0^2 S_0^2 - q_z k_z}{k_0^2 S_0^2 + q_z k_z} \quad (20)$$

where  $S_0$  is the overlap integral of the incident plane wave and the fundamental mode inside the hole:

$$S_0 = \frac{\sqrt{2}}{ad} \int_0^a e^{ik_x x} dx \int_0^a dy \sin \frac{\pi y}{a} dy = \frac{2\sqrt{2}a \sin(k_x a/2)}{\pi d k_x a/2}. \quad (21)$$

By analysing the zeros of the denominator of  $\rho_0$  and extending the expression to  $k_x > k_0$ , we can extract the dispersion relation of the surface states supported by the 2D hole array:

$$\frac{\sqrt{k_x^2 - k_0^2}}{k_0} = \frac{S_0^2 k_0}{\sqrt{\pi^2/a^2 - \epsilon_h k_0^2}}. \quad (22)$$

As in the case of 1D arrays of grooves, we would like to test whether the semi-infinite perfect conductor perforated with holes could be replaced by a semi-infinite *homogeneous* system, characterized by an effective dielectric constant and an effective magnetic permeability. Due to the symmetry of the structure,  $\epsilon_{x\text{eff}} = \epsilon_{y\text{eff}} \equiv \epsilon_{\text{eff}}$  and  $\mu_{x\text{eff}} = \mu_{y\text{eff}} \equiv \mu_{\text{eff}}$ . As the dispersion of the waveguide mode inside the hole is unaffected by parallel momentum,  $\epsilon_{z\text{eff}} = \mu_{z\text{eff}} = \infty$ .

In a homogeneous structure, the reflection coefficient for a normally incident plane wave can be expressed as a function of the impedance of the medium,  $Z = \sqrt{\frac{\mu}{\epsilon}}$ :

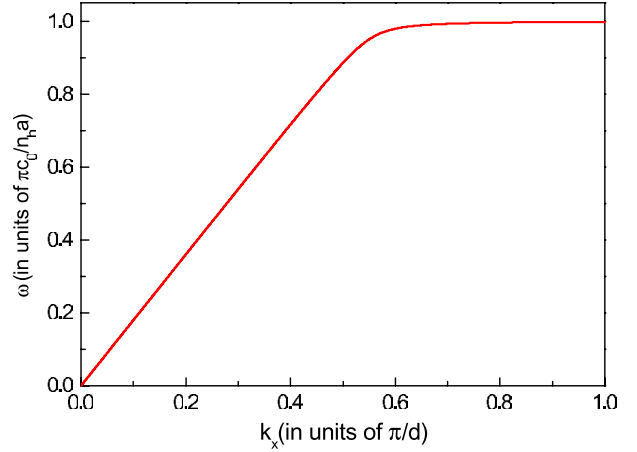
$$R_0 = \frac{Z - 1}{Z + 1}. \quad (23)$$

Then, the *effective* impedance of a 2D hole array perforated on a perfect conductor can be easily calculated by analysing equation (20) in the particular case of a normally incident plane wave ( $k_x = 0$ ,  $k_z = k_0$ ):

$$Z_{\text{eff}} = \sqrt{\frac{\mu_{\text{eff}}}{\epsilon_{\text{eff}}}} = \frac{S^2 k_0}{q_z} \quad (24)$$

where  $S \equiv S_0(k_x = 0) = 2\sqrt{2}a/\pi d$ . The other equation linking  $\epsilon_{\text{eff}}$  and  $\mu_{\text{eff}}$  can be obtained from

$$q_z = k_0 \sqrt{\epsilon_{\text{eff}} \mu_{\text{eff}}} = i \sqrt{\frac{\pi^2}{a^2} - \epsilon_h k_0^2}. \quad (25)$$



**Figure 4.** The dispersion relation ( $\omega(k_x)$ ) of the p-polarized surface bound states supported by a 2D array of holes with  $a/d = 0.6$  and filled with a dielectric material with index of refraction  $n_h = 3$ , as obtained with equation (29).

Combining equations (24) and (25) we can write down the effective magnetic permeability and effective dielectric permittivity of our system:

$$\mu_{x\text{eff}} = \mu_{y\text{eff}} = S^2 \quad (26)$$

$$\epsilon_{x\text{eff}} = \epsilon_{y\text{eff}} = \frac{\epsilon_h}{S^2} \left( 1 - \frac{\pi^2}{a^2 \epsilon_h k_0^2} \right) = \frac{\epsilon_h}{S^2} \left( 1 - \frac{\pi^2 c_0^2}{a^2 \epsilon_h \omega^2} \right) \quad (27)$$

which is the canonical plasmon form with a *plasma frequency*,  $\omega_{\text{pl}} = \pi c_0 / \sqrt{\epsilon_h} a$ . This frequency is just the cut-off frequency of a square waveguide of side  $a$  filled with a material characterized by a dielectric constant  $\epsilon_h$ .

The next step is to calculate the dispersion relation of the surface modes supported by this effective medium and compare it with equation (22). For an interface between vacuum and a semi-infinite structure characterized by  $\epsilon_{\text{eff}}$ , the surface modes have to fulfil the equation

$$k'_z + \frac{q_z}{\epsilon_{\text{eff}}} = 0 \quad (28)$$

where  $k'_z = -i\sqrt{k_x^2 - k_0^2}$  is the inverse of the decaying length of the surface mode inside the vacuum,  $e^{|k'_z|z}$ , and  $q_z$  is the analogue magnitude in the effective medium. By using  $\epsilon_{\text{eff}}$  from equation (27) we obtain

$$\frac{\sqrt{k_x^2 - k_0^2}}{k_0} = \frac{S^2 k_0}{\sqrt{\pi^2/a^2 - \epsilon_h k_0^2}} \quad (29)$$

which coincides with equation (22) in the long wavelength limit (when the effective medium approximation makes sense),  $k_x a \ll 1$ . In figure 4 we plot the dispersion relation of these surface modes for the particular case  $a/d = 0.6$  and  $\epsilon_h = 9$ .

### 3.1. 2D arrays of holes of finite depth (dimples)

It is quite interesting to analyse also the case of a 2D square array ( $d \times d$ ) of square holes (side  $a$ ) of finite depth,  $h$ . The procedure for calculating the dispersion relation of the surface

modes supported by this type of structure is quite similar to the one presented for the previous case. The only difference is that in equation (19) we have to consider not only the decaying mode  $e^{-|q_z|z}$  but also the growing one,  $e^{+|q_z|z}$ :

$$\begin{aligned}\vec{E}^{\text{II}} &= C^+ \vec{E}^{\text{TE},+} + C^- \vec{E}^{\text{TE},-} \\ \vec{H}^{\text{II}} &= C^+ \vec{H}^{\text{TE},+} + C^- \vec{H}^{\text{TE},-}.\end{aligned}\quad (30)$$

Apart from the continuity equations at  $z = 0$ , we have to add the condition  $E_x = 0$  at the bottom of the hole,  $z = h$ . By doing straightforward algebra, we end up with a dispersion relation of the surface modes:

$$\frac{\sqrt{k_x^2 - k_0^2}}{k_0} = \frac{S^2 k_0}{\sqrt{\pi^2/a^2 - \epsilon_h k_0^2}} \frac{1 - e^{-2|q_z|h}}{1 + e^{-2|q_z|h}}. \quad (31)$$

Note that in the limit  $h \rightarrow 0$ ,  $k_x \rightarrow k_0$  (light line) and for  $h \rightarrow \infty$  we recover equation (29), as we should.

#### 4. Conclusions

We have demonstrated that a semi-infinite perfect conductor perforated with a one-dimensional array of grooves or a two-dimensional array of holes can be optically described in the long wavelength limit as an effective medium characterized by a dielectric function of plasmon form in which the *plasma frequency* only depends on the geometry of the indentation (groove or hole). The surface modes supported by this system have close resemblances with the surface plasmon polaritons of a real metal.

In these new plasmonic metamaterials, their electromagnetic response could be engineered by tuning the geometrical parameters defining the corrugated surface. Then, these tailored surface plasmons could be modified at will at almost any frequency because metals are nearly perfect conductors from zero frequency up to the threshold of the THz regime.

Surface electromagnetic modes excited at a metal surface can also be analysed as propagating waves in two dimensions [8, 9]. Our results could be used as an alternative way to control the flow of light in the surface of a metal by just playing with the geometry (size and separation) of the indentations disposed at the surface.

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