Reply to Comments on "Resonant and antiresonant frequency dependence of the effective parameters of metamaterials"

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We argue, in agreement with our previous work, that periodicity of the metamaterial is responsible for antiresonant behavior of the effective permittivity as well as for the negative sign of the imaginary part of an effective permittivity or permeability. This agrees with the Comment of Efros.

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In our recent paper [1], we have discussed resonant and antiresonant behavior of the effective permittivity ϵ and permeability μ of metamaterials. We have observed that the *imaginary* part of either ϵ or μ is negative. Similar results were obtained by Markoš and Soukoulis [2] and by O'Brien and Pendry [3]. Our results were obtained by the use of the transfer-matrix method and subsequent use of the retrieval procedure described in Refs. [2,4].

The fact that our retrieval procedure also indicated that the product of the imaginary parts of ϵ and μ is negative was criticized in two Comments [5,6].

In the retrieval procedure, we have assumed that the metamaterial can be described as a continuous medium with effective $\epsilon(\omega)$ and $\mu(\omega)$. We have clearly stated in our recent paper [1] that the antiresonant frequency dependence of the effective permittivity is due to the periodicity of the metamaterial. As was argued in Ref. [1], the effective refractive index possesses a maximum close to the resonance frequency. The periodicity of the metamaterial is also responsible for negative values of the imaginary parts of ϵ or μ [1].

We want to assure readers that the results that were obtained for the retrieved values of ϵ and μ are numerically correct. Using the retrieved $\epsilon(\omega)$ and $\mu(\omega)$ for a continuous, homogeneous slab of finite length in the propagation direction with the periodic boundary condition in the perpendicular directions, we analytically reproduce the scattering amplitudes obtained from the simulation of the metamaterial almost independent of the system length. In particular, we can numerically demonstrate that there are effective media with Im $\epsilon<0$ but Im $\epsilon/|\epsilon|+\text{Im}\,\mu/|\mu|>0$ that do not violate the positiveness of losses, i.e., $1-|t|^2-|r|^2>0$ for the scattering on a slab of arbitrary thickness [1,2]. We believe, therefore, that there is nothing wrong mathematically in describing the scattering problem with this effective medium, and our retrieval results describe very well the scattering problem. This suggests that the effective parameters ϵ and μ in a scattering setup, as was considered in Ref. [1], have more freedom than in a real material.

The main difference between real homogeneous material and artificial metamaterials consists in the additional k dependence of ϵ and μ, introduced by the reduced translation symmetry of the metamaterial. The effective homogeneous medium that we obtained by assuming a k-independent but only frequency-dependent ϵ and μ is *in general* not related with a real homogeneous material. So, as Efros has argued in his Comment, there is no real homogeneous material that gives the behavior obtained by the scattering setup. Also, we do not expect that our effective parameters are relevant for the theoretical description of small samples, discussed in [5,6].

Finally, we want to stress again that the numerical results obtained in Ref. [1] are correct, the retrieval procedure is also correct, but the effective parameters obtained for an effective medium that omits the k dependence of ϵ and μ introduced by the periodicity might not correspond to a real material under arbitrary conditions. We agree with Efros [6] that the antiresonant frequency dependence of the effective permittivity is due to the periodicity of the metamaterial. It is therefore appropriate to reformulate the retrieval procedure with an effective medium that has both k and ω dependence for its ϵ and μ.

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