Perfect corner reflector

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We numerically investigate the electromagnetic field radiated by a line source within a perfect lens [Phys. Rev. Lett. **85**, 3966 (2000)] consisting of two orthogonal planes delimiting positive and negative index media. Use of a coordinate transformation [J. Phys. Condens Matter **15**, 6345 (2003)] together with a well-adapted transfer-matrix method permits rigorous calculation of the vector field. We find that two negative corners combine to make a cavity that traps light along closed trajectories. Finally, we numerically show that the field presents some spatial oscillations with a period that is proportional to absorption σ inside the negative materials as $1/\ln \sigma$ and that it is associated with an infinite density of states when σ tends toward 0. © 2005 Optical Society of America

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Negative refractive-index materials (NRIMs) have excited the optics community both through the intriguing possibilities they appear to offer and the challenges they present to our understanding of the diffraction process. Most intriguing of all is the possibility of a lens whose resolution is limited not by wavelength, but only by the losses in the constituent materials. The resolution of this lens increases without limit as the losses tend to 0. The cornerstone of the perfect lens[1] is that a negatively refracting slab ($n=-1$) is in some sense complementary to an equal thickness of vacuum and cancels its presence (the optical path vanishes). Moreover, the compensating effect of the slab extends not only to radiative components of the field but also to the evanescent near field that conveys the subwavelength details of the image.

However, the perfect lens is only one member of a whole category of systems, such as intersecting planes, that satisfy a generalized lens theorem.[2] Indeed, any medium can be optically canceled by an equal thickness of material constructed to be an inverted mirror image of the medium, with permittivity ε and permeability μ reversed in sign. As recognized in Refs. 2 and 3, if we combine two square corners of NRIM so that they share the same corner, both propagating and evanescent components of the radiated field should be refocused back onto its original source.

In this Letter we explore resonance effects through negative refraction in such double corner reflectors. We first consider a multilayered metamaterial invariant along the x_1 and x_2 axes and periodic along the x_3 axis. It further contains a periodic arrangement of line current sources along the x_3 axis, which suffer no phase shift between each other. They radiate a periodic field in the overall periodic structure (see Fig. 1). Then we introduce a mapping of coordinates that takes the structure of Fig. 1(a) into the structure shown in Fig. 1(b). The ray diagram suggests that a subset of rays diverging from a source is eventually returned to the source and circulates around the system forever. We numerically check this and investigate the link between loss within the NRIM and spatial oscillations of surface plasmons at the interface separating positive and negative refractive-index media.[4]

We consider the structure in Fig. 1(a), which consists of M stacked homogeneous layers with thickness h_m, permittivity ε_m, and permeability μ_m ($m=1,2,\ldots,M$). Also, some line current sources are located at the planes (x_1,x_2,nh), $n\in\mathbb{Z}$, delimiting the stacked periods. We investigate solutions (whose restriction in every horizontal plane is square integrable and bounded in x_3) of the set of time harmonic Maxwell's equations.

We derive that the two-column vector fields F_ν ($\nu=\varepsilon$ or μ corresponding, respectively, to p or s polarization) that consist of the tangential components of the electromagnetic field should be continuous for all $x_3\in\mathbb{R}$ different from nh and are discontinuous for all $x_3=nh$:

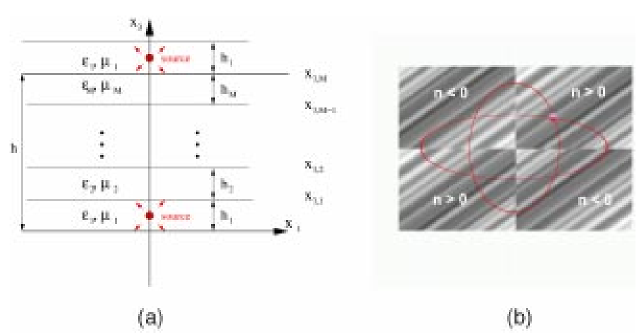

Fig. 1. (a) Geometry of the 1D problem. (b) Ray diagram for the double corner reflector.

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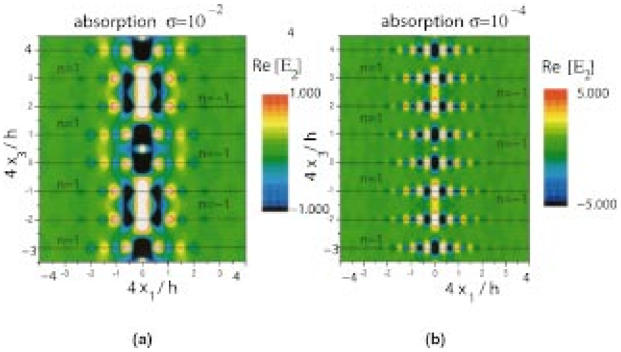

Fig. 2. Real part of the electric field radiated by a periodic set of line sources in a 1D photonic crystal: The number of layers in one period is $M=4$. The four layers have the same thickness, $h/4$. The values of permittivity and permeability are $\epsilon_m=\mu_m=1$ for $m=1,3$ and $\epsilon_m=\mu_m=-1+i\sigma$ for $m=2,4$. The harmonic line sources are located at $(x_1,x_3)=(0,h/8+nh)$, where $n\in\mathbb{Z}$, and have frequency $\omega h/c=16$, where c is the speed of light in vacuum. The value of absorption σ is 10^{-2} in (a) and 10^{-4} in (b).

$$F_\nu(nh+0^+)=F_\nu(nh-0^+)+P_\nu, \quad \nu=\varepsilon,\mu, \tag{1}$$

where $F_\nu(nh\pm 0^+)=\lim_{\eta\downarrow 0}F_\nu(nh\pm\eta)$ and P_ν is a two-column vector that contains the contribution of the line source (its components depend on s or p polarization[5]). Each homogeneous layer m is associated with transfer matrices $T_{\nu,m}$ ($\nu=\varepsilon,\mu$ and $m=1,2,\ldots,M$):

$$T_{\nu,m}=\begin{bmatrix}\cos(\beta_m h_m) & \beta_m^{-1}\nu_m\sin(\beta_m h_m)\\ -\beta_m\nu_m^{-1}\sin(\beta_m h_m) & \cos(\beta_m h_m)\end{bmatrix}, \tag{2}$$

where $\beta_m=(\varepsilon_m\mu_m\omega^2-k_1^2-k_2^2)^{1/2}$ (with k_1 and k_2 being the transverse wave numbers and ω being the frequency). This provides us with the relationship

$$F_\nu(x_{3,m}+nh)=T_{\nu,m}F_\nu(x_{3,m}-h_m+nh), \quad n\in\mathbb{Z}, \tag{3}$$

where $x_{3,m}=\Sigma_{m'=1}^{m}h_{m'}$. Last, combining Eqs. (1) and (3) and using the periodicity of the electromagnetic field, we conclude that

$$F_\nu[(n+1)h]=T_\nu F_\nu(nh)+P_\nu=F_\nu(nh), \tag{4}$$

where T_ν is matrix $T_{\nu,M}T_{\nu,M-1}\ldots T_{\nu,1}$ associated with the period of the structure (stack of M layers). We are eventually led to the following result[5]:

Theorem: If there is at least one number ν_m ($\nu=\varepsilon,\mu$ and $m=1,2,\ldots,M$) with a nonzero imaginary part, then matrix $I-T_\nu$ is invertible for all real frequencies ω. Consequently, there is a unique solution to Eq. (4) that is square integrable in every horizontal plane and bounded in x_3:

$$F_\nu(nh)=[I-T_\nu]^{-1}P_\nu, \quad \nu=\varepsilon,\mu. \tag{5}$$

Figure 2 shows the numerical calculation of the electric field radiated by a periodic set of line sources in a one-dimensional (1D) crystal with a period consisting of $M=4$ layers (data on the system are given in the caption of Fig. 2). These maps clearly show a large amplitude for the electric field at the interfaces delimiting positive and negative media. This large amplitude caused by the enhancement of the evanescent waves is clearly more important when the absorption is smaller in Fig. 2(b).

According to Ref. 4, the field at the interface separating complementary media exhibits some oscillations with a spatial period L proportional to $1/\ln\sigma$ if σ is the absorption within the slabs of negative material. These oscillations are readily observed in Fig. 2 when absorption σ equals 10^{-2} and 10^{-4}. In Fig. 3 we plot the inverse of oscillation period L as a function of the logarithm $\ln\sigma$. These data were obtained from maps of fields such as the ones in Fig. 2 by averaging on a large enough number of oscillations. These results, which are reported in Fig. 3, support those given in Ref. 4. We note that the resolution of the perfect lens is also related to the logarithm of absorption σ.[4,6]

From the knowledge of the field within the 1D crystal containing the periodic set of line sources (see Fig. 2) it is possible to find a solution of the double corner reflector [see Fig. 1(b)] by mapping our system of Cartesian coordinates x_1x_3 (x_2 is left invariant) onto polar coordinates:

$$l=\frac{l_0}{2}\ln\left(\frac{x_1^2+x_3^2}{r_0^2}\right), \quad \Phi=\arctan\left(\frac{x_3}{x_1}\right)[\pi], \tag{6}$$

where $[\pi]$ specifies the branch cut taken for arctan, and r_0 stands for the radial position of the line source in Fig. 4. Here l denotes the radial (logarithmic) coordinate and Φ denotes the azimuthal coordinate. In the new orthogonal frame $lx_2\Phi$ we denote by $\tilde{\varepsilon}_i$ ($\tilde{\mu}_i$) the three nonzero components of the diagonal tensors $\tilde{\varepsilon}$ ($\tilde{\mu}$) that are related to the piecewise constant (complex) function ε (μ) (expressed in the orthogonal frame $x_1x_2x_3$) as per

$$\tilde{\varepsilon}_i=\varepsilon\frac{Q_1Q_2Q_3}{(Q_i)^2}, \quad \tilde{\mu}_i=\mu\frac{Q_1Q_2Q_3}{(Q_i)^2}, \quad i=1,2,3, \tag{7}$$

with $Q_i^2=(\partial l/\partial x_i)^2+(\partial x_2/\partial x_i)^2+(\partial\Phi/\partial x_i)^2$. From Eqs. (6) this can be recast as

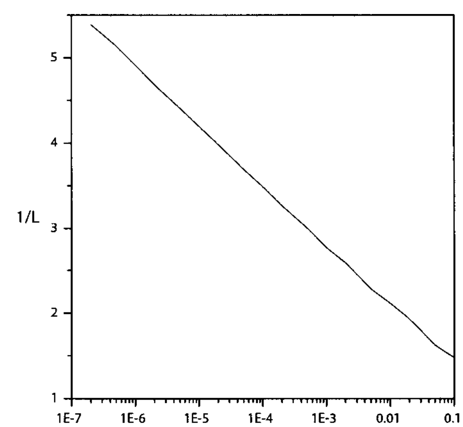

Fig. 3. Inverse spatial period L of field oscillations as a function of absorption σ.

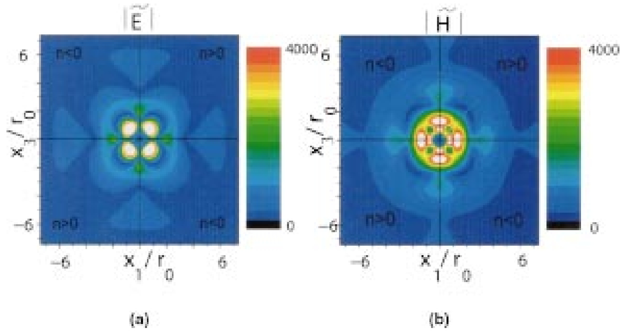

Fig. 4. Modulus of the (a) electric and (b) magnetic fields radiated by a line source in the presence of the double corner reflector. The absorption in negative media is $\sigma=10^{-4}$. The harmonic line source is located at $(x_1,x_3)=(r_0/\sqrt{2},r_0/\sqrt{2})$ and has a frequency of $\omega r_0/c=4$.

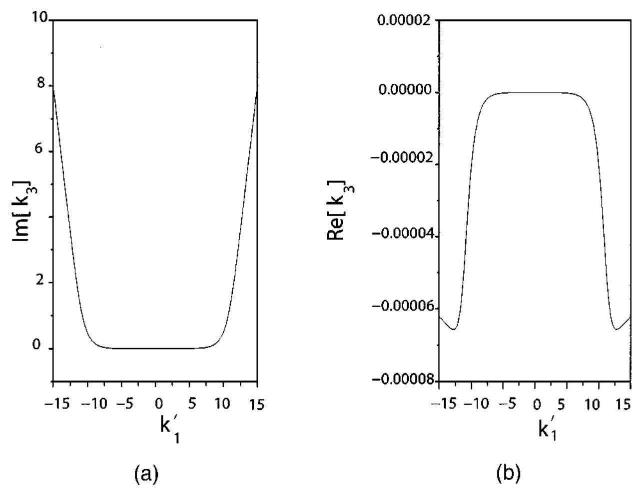

Fig. 5. Constant frequency dispersion diagram at $\omega h/c=16$ in a 1D crystal as described in the caption of Fig. 2, but for $\sigma=10^{-6}$. (a) Imaginary part of k_3 gives attenuation $\exp[-2\pi\,\mathrm{Im}(k_3)]$ of the field through one period. (b) Real part of k_3 provides its phase shift $\exp[2i\pi\,\mathrm{Re}(k_3)]$. If $|k_1'|=(k_1^2+k_2^2)^{1/2}\leq 5$, both $\mathrm{Im}(k_3)$ and $\mathrm{Re}(k_3)$ are close to 0.

$$Q_1=\frac{(x_3^2+l_0^2x_1^2)^{1/2}}{x_1^2+x_3^2},\quad Q_2=1,\quad Q_3=\frac{(x_1^2+l_0^2x_3^2)^{1/2}}{x_1^2+x_3^2}. \tag{8}$$

Upon making the choice of $l_0=1$, we deduce from Eqs. (7) and (8) that the transformed medium consists of four anisotropic heterogeneous regions defined by

$$\tilde{\varepsilon}_1=\tilde{\mu}_1=\tilde{\varepsilon}_3=\tilde{\mu}_3=a,\quad \tilde{\varepsilon}_2=\tilde{\mu}_2=a/(x_1^2+x_3^2), \tag{9}$$

with $a=1$, $\forall(x_1,x_3)\in\mathbb{R}^+\times\mathbb{R}^+\cup\mathbb{R}^-\times\mathbb{R}^-$ (two positive corners), and $a=-1+i\delta$, $\forall(x_1,x_3)\in\mathbb{R}^+\times\mathbb{R}^-\cup\mathbb{R}^-\times\mathbb{R}^+$ (two negative corners with absorption). The transformed electric and magnetic fields take the following forms:

$$\tilde{E}_i=Q_iE_i,\quad \tilde{H}_i=Q_iH_i,\quad i\in\{1,2,3\}, \tag{10}$$

where Q_i is given by Eq. (8). Figure 4 depicts the modulus of the electric [Fig. 4(a)] and magnetic [Fig. 4(b)] fields radiated by a line source in the double corner reflector. This figure clearly shows that the electromagnetic field is confined around the corner: The considered structure is working as a trap for light. It also shows that the line source is well reproduced in the other three quarters. The high amplitude of the field is due to the superposition of all these reproductions of the line sources and is not infinite because of the small absorption of $\sigma=10^{-4}$. We finally propose a result for which one can draw an analogy with the theorem established in Ref. 7: Here it states that, in the static limit and for complementary regions that are the inverted mirror image of each other, surface plasma modes that occur are infinitely degenerate at frequency $\omega=\omega_p/\sqrt{2}$ at which the condition $\varepsilon=-1$ is met {assuming a plasma form of $\varepsilon=1-\omega_p^2/[\omega(\omega+i\sigma)]$}.[2] Figure 5 shows a constant frequency dispersion diagram associated with the 1D crystal described in the caption of Fig. 2 with an absorption of $\sigma=10^{-6}$. Figure 5 shows an infinite number of modes that are x_3 periodic and that correspond to all the values of (k_1,k_2) inside a disk of radius 5 [$(k_1^2+k_2^2)^{1/2}=|k_1'|\leq 5$, i.e., infinite degeneracy]. Indeed, in this domain each Fourier component of the field passes through each period without attenuation [Fig. 5(a)] nor change in phase [Fig. 5(b)]. After the change of coordinates in Eq. (6), these periodic infinitely degenerate modes in the 1D crystal become infinitely degenerate modes in the double corner reflector, leading to an infinite density of states.

In conclusion, we have demonstrated that, using complementary media that optically cancel one another at a specific frequency,[2] it is possible to design a metamaterial that traps states associated with this frequency around infinitely circulating loops. As far as that frequency is concerned, this optical system is invisible to an observer. Such a cavity may allow storage of light at a specific frequency and would therefore have potential applications in the design of all-optical circuits (realization of an optical computer), or it may pave the way toward photon storage for quantum cryptography.

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