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Project Co-ordinator: Costas M. Soukoulis (FORTH)
Partners:
1) Foundation for Research and Technology, Hellas (FORTH)
2) Bilkent University (Bilkent)
3) Imperial College of Science Technology and Medicine –
   Physics Department (ICSTM1)
4) Imperial College of Science Technology and Medicine –
   Imaging Sciences Department (ICSTM2)

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Technical Point of Contact: E. N. Economou and C. M. Soukoulis
IESL, FORTH
Box 1527, Vassilika Vouton,
71110 Heraklion, Crete, Greece
TEL: +30 2810 391560; FAX: +30 2810 391505
E-mail: economou@admin.forth.gr
E-mail: soukouli@iesl.forth.gr
E-mail: kafesaki@iesl.forth.gr

Administrative point of Contact: Evangelia Hatzidaki
Contracts & Finance Division - Central
Administration FORTH
Vassilika Vouton, PO Box. 1527
71110 Heraklion, Crete, GREECE
Tel: +30810 391512; Fax: +30810 391555
E-mail: vxatzida@admin.forth.gr

Project URL:  http://gate.iesl.forth.gr/~cond-mat/photonics/DALHM/
1. EXECUTIVE SUMMARY

During this reporting period we focused on a number of issues. In the theory/simulation domain one of the most important issue was the development of a periodic effective medium description for the scattering behavior of a metamaterial. This description enabled us to understand better the metamaterial transmission characteristics and to eliminate all the spurious features that the homogeneous effective medium approach gives for the effective metamaterial parameters ($\varepsilon$ and $\mu$).

Another important issue studied in this period was the electric coupling to the magnetic SRR resonance ($2^{nd}$ electric coupling, i.e. the excitation of the resonant circulating currents around SRR by the external electric field). This coupling is a result of the SRR asymmetry relative to the external electric field and can have strong destructive influence on the LH behavior. The only way to avoid it in 2D and 3D LH materials is the use of fully symmetric SRR designs (e.g. multigap SRRs). Thus, a large part of our theoretical effort in this period was focused on the investigation of fully symmetric SRR structures and on the study of their behavior in SRRs+wires 2D or 3D metamaterials. We studied many symmetric SRRs and we isolated some designs that can be good candidates for the construction of 2D and 3D LH materials.

Apart of the symmetric SRR designs, we also continued the study of our conventional SRR, examining in detail the dependence of the SRR characteristic frequencies - magnetic, $\omega_m$, and electric cut-wire, $\omega_0$ - on various of the SRR parameters, as well as on the SRR shape, orientation, and the presence or not of a second ring. Moreover, we examined the influence of the continuous wires on the SRRs magnetic resonance, as well as the influence of the dielectric boards. Finally, we studied various experimentally investigated structures (SRRs+wires structures operating in GHz and THz, photonic crystals and swiss-roll systems), trying to analyze and to explain the experimental data, and also to guide the current and the future experimental effort.

On the modeling tools development it is worth mentioning a major improvement in the discretization scheme used in transfer matrix (TMM) technique. A more symmetric discretization that was implemented in this period led to the elimination of the spurious non-zero cross-polarization terms in the scattering parameters obtained by the TMM in its previous implementation.

Finally, trying to explore the novel and fascinating phenomena which can arise from the employment of LH media or thin metallic elements, we studied in this year a corner made of a LH material (with $n=-1$), showing its ability to act as a perfect corner reflector, and we demonstrated the possibility of evanescent Bessel beam generation using transmission through thin metallic films.

On the characterization domain, we performed a very large number of free-space transmission measurements in 1D and 2D composite metamaterials (CMMs) operating at $\sim$3-10 GHz. In CMMs of SRRs and wires we managed to achieve the highest up to now LH transmission peaks (the transmitted intensity was $-5$ dB in the 1D sample and $-10$ dB in the 2D - both of 5 unit cells along propagation direction). The negative refraction at these peaks was clearly demonstrated, using prism experiments, and using phase measurements (recording the reduction of phase with the increase of the sample thickness) the effective refractive index was determined. Other important tasks of this reporting period were the demonstration of the electric coupling to the magnetic resonance ($2^{nd}$ electric coupling), the experimental
verification of the electric response of the SRR, the demonstration of the influence of the wires on the SRRs magnetic resonance, and the examination of the effects of disorder on this resonance. In this year we also managed to demonstrate the existence of negative magnetic permeability in the THz regime; measuring 1D μm-size SRR systems we found a broad negative μ regime around 5 THz.

Apart of the metamaterials composed of SRRs and wires, we also continued our experimental work on photonic crystals (photonic crystals are the most promising candidates for extending the negative refraction regime to very high frequencies, e.g. visible): we demonstrated negative refraction and focusing capabilities in a 2D photonic crystal of alumina rods in air, at frequencies around 40 GHz; moreover we examined the dependence of the focusing properties on the crystal thickness. Negative refraction and focusing, though, are not the only fascinating effects that can arise from the peculiar dispersion characteristics of the photonic crystals. Another effect, which we demonstrated during this period, is an enhancement of the emitted radiation and the directivity of sources if placed in 2D and 3D photonic crystals – see demonstrators (WP3) part summary.

As was mentioned in the year 1 report, our aim in DALHM project is to study and to exploit the properties of metamaterials also in the MHz regime, where the MRI applications constitute the most attractive target. In this framework, we constructed and characterized a square array of swiss-rolls, demonstrating its capability to transfer an input RF signal almost unaltered. Moreover, we continued the characterization (started in the previous year) of the hexagonal swiss-roll array, demonstrating also its lensing capabilities around the magnetic resonance frequency.

On the fabrication activities (WP2), the most worthmentioning tasks are (a) the development of double-side metamaterial structures operating at around 100 GHz, on glass substrate, using UV photolithography and thermal metal deposition, (b) the development of SRR structures operating around 5 THz, on polyimide, and (c) the optimization of an etching procedure for use in the fabrication of 2D metamaterial structures operating around 30-100 GHz.

In what concerns demonstrators (WP3) one important achievement of this period was the realization of antennas with enhanced radiation rate and high directivity. This was done by putting a monopole source inside a 2D or a 3D photonic crystal, exploiting either the high density of states close to the band edge of such a crystal or the presence of surface states in a finite PC.

In the MHz regime, we focused on the development of components that can be used for the manipulation of the RF signal in MRI machines. We developed a RF yoke, which can be used as a low reluctance path for the transferring of the RF signal, as well as an optimized flux compressor, composed of a series of resonant loops of progressively smaller diameter.

2. MAJOR ACCOMPLISHMENTS DURING THIS PERIOD

2.1 Theory and Simulation (WP1 – Task 1.1)
1. Development of a “periodic effective medium” approach for the description of periodic composite metamaterials.
2. Detailed examination of the 2nd electric coupling and its destructive effects on the LH transmission of 2D and 3D LH materials.
3. Study of symmetric SRR structures, which do not suffer from the 2nd electric coupling; study of 3D metamaterials made up of symmetric SRRs and wires, and identification of some optimum designs for LH transmission.
4. Understanding of the various characteristic frequencies appearing in the SRR spectrum and of their dependence on the various system parameters.
5. Understanding of the influence of the continuous wires on the SRR’s magnetic response in a metamaterial and identification of optimum wires positions.
6. Reproduction of the experimental data and explanation of the majority of these data (including data on the structures operating in THz – these structures were constructed after the guidance of the simulations).
7. Improvement of the transfer matrix technique through the use of a more symmetric discretization scheme, which led to the elimination of the spurious non-zero cross-polarization terms in the scattering parameters.
8. Demonstration of the LH and focusing properties of 2D photonic crystals.
9. Demonstration of the ability of materials with n=-1 to act as perfect corner reflectors.
10. Demonstration of the generation of evanescent Bessel beams which can be maintained over a large propagation distance, using EM wave transmission through thin metallic films.

2.2 Characterization (WP1 – Task 1.2)

1. Demonstration of the highest reported up to now transmission peaks in 1D CMMs of SRRs+wires, in the GHz regime.
2. Achievement of our first LH transmission peaks in 2D systems of SRRs+wires.
3. Demonstration of negative magnetic permeability in SRRs operating in the THz regime.
4. Demonstration of the electric response of the SRR.
5. Verification of the electric coupling to the SRR resonance.
6. Realization of wedge experiments to demonstrate the negative refraction of the LH transmission band of 2D SRRs+wires metamaterials.
7. Realization of phase measurements for the effective refractive index in metamaterials – verification of negative refractive index at the LH transmission band in SRRs+wires metamaterials.
8. Demonstration of the left-handed and focusing capabilities of 2D photonic crystals
9. Achievement of highly directional and of enhanced radiation sources, using monopoles embedded in 2D and 3D photonic crystals. The 3D PC gave full angular confinement of the emitted radiation (see also WP3).
10. Utilization of the surface modes of a 2D PC for highly directive, larger bandwidth antenna systems (see also WP3)
11. First transmission measurements on CMMs of SRRs+wires, operating around 100 GHz. Demonstration of the negative SRR response in this regime.
12. Demonstration of the lensing properties of swiss-roll systems (using a square and a hexagonal swiss-roll array).
2.3 Fabrication (WP2)

1. Fabrication of CMMs operating around 100 GHz, on glass substrates.
2. Fabrication of SRR structures operating in THz, on polyimide substrate.
3. Optimization of the etching procedure for the fabrication of 2D metamaterials operating in 30-100 GHz.
4. Fabrication of a “lens”, consisting of swiss-rolls packed into a tetragonal prism, operating at MHz (presented in WP1, together with its characterization).
5. Fabrication of devices (based on resonant µ elements) useful for efficient signal transferring in MRI systems (presented in WP3 - as demonstrators)

2.4 LH-based demonstrators (WP3)

1. Demonstration of enhancement of radiation and high-directionality of sources placed inside photonic crystals.
2. Demonstration of the efficiency of a swiss-roll yoke in transferring the signal in MRI systems.
3. Demonstration of an efficient flux compressor composed of a sequence of resonant loops with progressively smaller diameter.

3. ACCOMPLISHMENT OF THE DELIVERABLES

D8: First 2D LH test structures
   Pictures of these structures with their properties are given in the progress report on WP1-Task.1.2

D10: Testing of LH structures and comparison between theoretical results and experimental data
   See the separate report on D10.

D11: Theoretical study of the extension of the LH behavior towards optical frequencies
   See the separate report on D11.
4. DISSEMINATION AND USE PLAN

The dissemination of the DALHM results is done mainly through publications, conference/workshop presentations, and seminars in individual Universities and Institutions. Moreover, two of the participating in DALHM organizations (FORTH, Bilkent) are members of a newly formed Network of Excellence (NoE) on metamaterials (METAMORPHOSE), which includes almost all groups in Europe working on metamaterials; this participation will help to the dissemination and the use of the DALHM results, as it will give the chance to share these results with the other members of the NoE (through participation to the NoE meetings and discussion forums, connection of the DALHM web-page with the NoE web page, and participation to all the dissemination activities organized by the NoE - schools, conferences etc.).

Below we give a list of publication and conference presentations realized within this reporting period.

Publications


Conference presentations
5. Ekmel Ozbay, 323. WE-Heraeus-Seminar, Physikzentrum Bad Honnef, Germany, April 2004.

**Colloquia/Seminars**

C. M. Soukoulis
1) University of Karlsruhe, EE Department, Germany, July 2003
2) Iowa State University, Physics Department, September 2003
3) Michigan State University, Physics Department, December 2003
4) Nuclear Research Center, Democritos, Athens, Greece, December 2003
5) Boston College, Physics Department, April 2004
6) University of Karlsruhe, Physics Department, Germany, July 2004

E. N. Economou
7) Meeting of the topical Greek Physical Society, Heraklion, Greece, October 2003
8) Physics Department, University of Crete, March 2004
9) Meeting of the Greek Physical Society, Loutraki, Greece, January 2004

J. B. Pendry
10) Paderborn, Germany, January 2004
11) Salford, UK, January 2004
12) Bad Honnef, Germany, April 2004
13) San Sebastian, Spain, May 2004
14) Durham UK, June 2004

M. C. K. Whiltshire
15) University of California, Los Angeles, October 2003
16) University of California, San Diego, October 2003
17) University of Oxford, January 2004

6. **MANAGEMENT**

We held two meetings during the second project year:

i) A general meeting (Crete, 19/3 & 20/3, 2004) after the first 18 months of the project.

ii) A general meeting (Crete, 29/7 & 30/7, 2004) close to the completion of the second project year.

We wrote a report on the work done within the project months 12-18, and we submitted it to the EU.

The reports on the deliverables concerning the second year, as well as the cost information are submitted together with this progress report.

7. **FUTURE PLANS**
1. Examination of the possibility and ways to obtain tunable LH materials. Introduction of magnetic ferrites and ferroelectrics.
2. Examination of the possibility to obtain LH materials that match the free space impedance.
3. Investigation of ways to further reduce losses.
4. Fabrication of additional 2D structures, characterization of them and attempts to optimize them.
5. Fabrication, simulations and measurements on additional structures operating at 100 GHz.
6. Fabrication and measurement of 30 GHz true left-handed composite meta-materials.
7. Fabrication, simulations and measurements on additional structures operating at THz frequencies.
8. Continuation of the work on 2D PCs with negative refractive properties. Experiments and simulations. Attempts to achieve $n=-1$. Point sources in photonic crystals.
9. Demonstration of negative refraction and focusing in 1D and 2D LHMs.
10. Emission measurements in 1D and 2D LH structures.
11. Studies of the emission properties of 1D and 2D LHMs. Possible enhancement of the emitted radiation and possible high directivity.
12. Continuation of work on Bessel beam generation using metamaterials.
13. Continuation of the study of swiss-roll systems and demonstration of the validity of the effective medium approach for the description of those systems.
15. Characterization of a 2D isotropic material (Great Wall) and comparison with theory to investigate sub-wavelength imaging behavior.
16. Implementation and characterization of improved metamaterial yoke for MRI applications.
17. Investigation of tuning in RF metamaterials.
The following is a list of our responses to the conclusions drawn during the evaluation of 1st year report and the recommendations given:

1. **A closer cooperation between the demonstrator part on medical imaging and the theoretical simulations is recommended.** [FORTH, ICSTM]

   A closer collaboration has been already established. A number of simulations have been curried out in FORTH in order to test the validity of the effective medium approach for the description of the Swiss-Role systems. The simulations are in progress and the data up to now are very promising (see also the WP1 report of the second year).

2. **Comparison of the different numerical modeling tools with respect to dispersive media (one test structure for all tools with error analysis).** [FORTH]

   A general comparison of the available to the consortium methods has been given in the report of the deliverable D2 (“Assessment of the consortium modeling tools”), which was attached to the first annual DALHM report. There, we described in detail the methods (transfer matrix method (TMM), finite difference time domain (FDTD) method, and Finite Integration method – the last is used through the commercial program Microwave Studio (MWS)), and we gave their advantages and disadvantages, together with their regions of applicability.

   Below we make a precise comparison of the different methods, applying them on one/same system, and we comment on the errors of each method. The system we use is an SRR system operating in THz (the parameters are the ones of the THz structure fabricated in FORTH).
In Fig. 1 we present the structure as well as the transmission coefficient obtained through MWS, FDTD and TMM. The structure consists of 1 unit cell along the propagation direction, while in the other directions periodic boundary conditions are used for the TMM and FDTD; for the MWS the boundary conditions are magnetic at the $x$-boundaries ($H_t=0$ at $x_{\text{min}}$ and $x_{\text{max}}$) and electric at the $y$-boundaries ($E_t=0$ at $y_{\text{min}}$ and $y_{\text{max}}$).

The MWS calculations have been performed considering the metal both as Drude material ($\varepsilon = \varepsilon_0 [1 - \omega_p^2/(\omega^2 + i\omega\gamma)]$, $\omega_p=15\times10^{15}$ sec$^{-1}$ and $\gamma=3.3\times10^{13}$ sec$^{-1}$) and as lossy metal (with conductivity the static value for the cooper, $\sigma = \sigma(0)=5.88\times10^7$ S/m). In the FDTD the metal has been treated also as Drude material, while in the TMM a constant value for the $\varepsilon$ of the metal has been employed, and it is the one obtained by the Drude model at frequency 6 THz.

As can be seen from Fig. 1, the agreement between the three methods is extremely good.

The error in each method is strongly related with the discretization level which is used. The coarser discretization that can be selected for a satisfactory representation of a specific structure is of grid cell equal or close to the minimum size of the metallic elements of the structure. Extensive error analysis showed that this discretization level (i.e. only one grid cell for the cross-section of the metals) usually gives satisfactory accuracy; the characteristic frequencies of the transmission curves are shifted from their actual positions (obtained with a much finer discretization) less than 10%.

(Note that in the frequency regimes that we study the skin depth of the metal is much smaller than the minimum metal size. Note also that we work with wavelengths much larger than the lattice constant of the system, thus we have high accuracy in the treatment of the dielectrics even with a coarse discretization (e.g. 7 grid cells per lattice constant, which usually gives a grid cell larger than the minimum metallic size).)

Fig. 1.: SRR structure operating in THz. Unit cell: $7\times7\times5$ $\mu$m$^3$
SRR size: $5\times5$ $\mu$m$^2$. All other lengths: 1 $\mu$m
Substrate’s $\varepsilon=2.8$ (polyimide). Structure fully embedded in dielectric.
The discretization in the FDTD and the TMM calculations presented here is 28 grid-cells per lattice constant (i.e. grid cell = 0.25 \( \mu m \)), a value which requires a not extremely large computational effort; the free space wavelength is around 50 \( \mu m \) which means 200 grids per wavelength. The accuracy of the calculations in this regime is very high, as a 20\% reduction of the grid cell (to 0.2 \( \mu m \)) leads to only 1.5\% shift of the transmission dip around 6 THz, for both FDTD and TMM.

The MWS calculations of Fig. 1 have been performed with a non-uniform grid with minimum grid cell 0.25 \( \mu m \). Repeating the calculations using a uniform grid with grid cell 0.5 \( \mu m \) we obtained only a 3.3\% shift of the transmission dip around 6 THz, which is quite satisfactory.

3. Clarifying the effect of the small gap (capacitor) in split ring resonators compared to closed ring used in current numerical simulations. [FORTH]

The existence of the gap in the SRR-ring is the basis for the magnetic resonance of the SRR which arises from the circulating current in the ring.

Due to the capacitance provided by the gap and the self-inductance of the ring, there is an eigen-oscillation of the current around the ring -- much similar to an ordinary LC oscillator circuit -- which, driven by the ring voltage induced by the external magnetic field, exposes a resonant behavior that can lead to the typical negative magnetic response function, known as the key-feature of the SRR.

If we close the gap making the ring a continuous loop, there is no eigen-oscillation of the circular current, and hence no resonant magnetic response function any more, and the induced current just follows the phase of the external field. Around the magnetic resonance frequency, \( \omega_m \), of the SRR the closed SRR (cSRR) shows almost no magnetic activity, such that the magnetic response function \( \mu(\omega) \) is smooth and close to unity.

The scattering properties of a SRR or cSRR meta-material around \( \omega_m \) are determined by the magnetic and electric response functions, \( \mu(\omega) \) and \( \varepsilon(\omega) \), respectively.

The combined electric response of SRR/cSRR and board in this region (far away from the resonant electric cut-wire response of the SRR/cSRR, which in our SRR systems is at much higher frequency) exposes a smooth, slowly varying, essentially positive \( \varepsilon(\omega) \). Together with \( \mu(\omega) \approx 1 \) for the cSRR, this explains the smooth, featureless transmission spectrum of the cSRR around the magnetic resonance of the SRR.

As long as the gap is small and we are far away from the electric cut-wire resonance (and the edge of the Brillouin zone), opening and closing the gap has very little influence on \( \varepsilon(\omega) \) but switches the resonance in \( \mu(\omega) \) on and off. We use this feature to identify the magnetic resonance in experiments and to study the isolated electric response of the SRRs meta-material.

Below we illustrate the effect of the gap of the SRR by presenting specific transmission and field plot results for a single SRR of only one ring (for simplicity) with and without gap. The model system that we use is the one described in Fig.1, with only difference the width of the SRR gap and the incident
field propagation and polarization. The propagation here is in the SRR plane (x-y plane, see Fig. 3) and the external electric field is parallel to the continuous sides of the SRR (along x-axis, see Fig. 3).

In Fig. 2 we show the transmission for a SRR with gap width 0.1 µm (left panel) and for a cSRR (right panel). In the left-panel, the dip around 3.2 THz corresponds to the magnetic SRR resonance. In the right panel, the removing of the gap has as a result the disappearance of the dip, according to what we described above.

To illustrate that the origin of the magnetic resonance at around 3.2 THz is resonant circular currents around SRR we plotted also (see Fig.3) the magnetic field component $H_z$ (perpendicular to the SRR plane) for the SRR with the gap (left panel) and for the cSRR (right panel). For the SRR, the presence of resonant circular currents is indicated by the strong magnetic field in all the SRR inside, which is of opposite direction to the outside field. There is no such a picture for the cSRR.
If we consider the SRR as a LC circuit, then, as we discussed above, the role of the gap is to introduce the capacitance (C) in the system. The resonance frequency will be $\omega_m^2 = 1/LC$ (L is the inductance). As one decreases the gap, the capacitance C is increased and thus $\omega_m$ goes to lower frequencies. Simultaneously, the depth of the transmission dip is decreased. The cSRR can be considered as the limit $C \rightarrow \infty$ and depth $\rightarrow 0$.

In the following table we give the approximate $\omega_m$ and the approximate depth of the magnetic dip, for the SRR of Fig. 3, as one changes the gap width.

<table>
<thead>
<tr>
<th>Gap width (in µm)</th>
<th>Dip strength (-dB)</th>
<th>$\omega_m$ (in THz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01</td>
<td>-2.0</td>
<td>1.20</td>
</tr>
<tr>
<td>0.02</td>
<td>-2.6</td>
<td>1.65</td>
</tr>
<tr>
<td>0.1</td>
<td>-8.8</td>
<td>3.2</td>
</tr>
<tr>
<td>0.2</td>
<td>-12</td>
<td>4.0</td>
</tr>
<tr>
<td>1</td>
<td>-18</td>
<td>5.6</td>
</tr>
<tr>
<td>2</td>
<td>-20</td>
<td>6.4</td>
</tr>
<tr>
<td>3</td>
<td>-21</td>
<td>6.8</td>
</tr>
</tbody>
</table>

The fact that the cSRR preserves the complete spectrum of the SRR with the exception of the features associated with magnetic resonances (this is a crucial point for the identification of LH peaks) is illustrated also in Fig. 4, where we present in the same graph the transmission for a SRR (black line) and for the cSRR (red line). (The SRR here is similar with the one of Fig. 2 but of the order of mm. The propagation is in the SRR plane ($E$-$k$ plane, see Fig. 5) and the external electric field is parallel to the continuous sides of the SRR.)
One can see that the only difference between the two curves of Fig. 4 is the dip at around $\omega_m = 8$ GHz. In Fig. 5, left panel, demonstrates again that the origin of this dip is the resonant circular currents around the SRR. The origin of the other transmission dip, at $\omega_0 = 21$ GHz, which appears in both the SRR and the cSRR spectrum is illustrated in the right panel of Fig. 4. This picture, which is the same for both open and closed SRR, indicates the presence of a cut-wire-like resonance, i.e. resonant linear currents along the parallel to the electric field sides of the SRR.

4. The issue of absorption by the substrate and the metals (different type) should be addressed. What is the contribution of free carriers in the 10, 35, 100 GHz range? [FORTH, BILKENT]

The absorption by free carriers in a silicon substrate was addressed by Ozbay et al. [Appl. Phys. Lett. 64, 2059 (1994)]. The bulk silicon attenuation can be calculated as

$$\alpha = \omega \sqrt{\mu \varepsilon, \varepsilon_0} \left(\frac{1}{2} \left[\left(1 + \frac{1}{\rho \omega \varepsilon_0 \varepsilon_r} \right)^{2} - 1\right]\right)^{1/2},$$

where, $\alpha$ is the free-carrier absorption, $\rho$ is the resistivity of the silicon wafer, $\varepsilon_r$ is the silicon dielectric constant (11.6 at W-band frequencies), and $\omega = 2\pi f$ is the angular frequency. $\mu$ and $\varepsilon_0$ are the free-space permeability and permittivities, respectively. The following table lists the attenuation rates for typical resistivity and frequency regimes we are interested in.

<table>
<thead>
<tr>
<th>Resistivity ((\Omega \cdot \text{cm}))</th>
<th>10</th>
<th>30</th>
<th>100</th>
</tr>
</thead>
<tbody>
<tr>
<td>Absorption (dB/cm)</td>
<td>19.0</td>
<td>21.3</td>
<td>23.8</td>
</tr>
<tr>
<td>Frequency (GHz)</td>
<td>13.8</td>
<td>15.7</td>
<td>17.0</td>
</tr>
</tbody>
</table>
According to this table, the absorption depends strongly on the resistivity of the substrate, and decreases with increasing resistivity. The frequency dependence is more conspicuous at low resistivities.

In our metamaterials the influence of the substrate resistivity is stronger than the one indicated in the above table, especially close to the resonances, as (i) in thin and high index substrates there is a high field concentration in the substrate and (ii) the field concentration in the substrate (also close to the metallic elements) becomes more pronounced close to the resonances, which makes the effect of the substrate conductivity more pronounced on resonances than off-resonances. All these show the necessity of a high resistive material as substrate, for obtaining satisfactory transmission levels.

On the losses of the metals: According to our simulations, the metals at the regimes 10, 30 and 100 GHz behave almost as perfect conductors, as the skin depth ($\delta^2=2/\mu_0\sigma\omega=6.6$ $\mu$m (10 GHz), 0.2 $\mu$m (100 GHz)) is much smaller than the minimum metal size, and the plasma and collision frequencies of the metal are larger than $10^4$ GHz. In general, in these frequency regimes and for the materials we use, we have found that the contribution of the metals in the total loss is smaller than that of the substrate.

In addition, we have to mention that in our systems the material loss is not always the main factor of the reduction of the transmitted intensity, as there is also a certain amount of reflectance loss, arising from system-air interfaces, which also contribute to the reduction of the transmitted power, in some cases considerably.

5. Try to analyze theoretically as well as experimentally alignment errors when stacking tens of slit-ring layers. [BILKENT, FORTH]

We have investigated the effect of misalignment of SRR structures on their transmission spectrum both theoretically and experimentally. Overall, the misalignment does not impose restrictions on the stacking of SRR structures, provided that it remains smaller than the wavelength of the EM field propagating in the medium. We refer the reader to WP1-Task 1.2 section of this report for details of the experimental work.

6. Compare phase measurements with theory and try to interpret them. [BILKENT, FORTH]

In view of the phase measurements presented in WP1-Task 1.2 section of this report, an agreement between the theoretical predictions for homogeneous left-handed materials is obtained regarding the presence of negative phase velocity. Nevertheless, the wave propagation within a CMM acting as a left-handed medium is expected to be quite complicated, and identifying the phase velocity by the wavefronts may be difficult. We can only use the transmission coefficient and its phase to obtain the index of refraction, provided that the impedance is close to one.
7. **Try to understand the effect of the thickness of a photonic crystal with negative refraction on the lensing and the imaging properties.**
   *BILKENT, FORTH*

Evidently, an immediate effect is that the overall amplitude of the signal in the focal plane reduces when a thicker photonic crystal is used. This can be clearly seen in WP1-Task 1.2 part of this report. The loss within the photonic crystal occurs mainly in the off-plane direction by diffraction, and this is inherent to the dimensionality of the photonic crystal.

We found that with increasing photonic crystal thickness, the focusing pattern shifts away from the crystal. This behavior is anticipated by naïve geometric optic arguments, which actually hold for homogeneous media only. In a thicker photonic crystal, the beam expands to a larger lateral span within the crystal before reaching the second interface. It then requires a longer way to focus along the optic axis. Other than that we do not see a profound effect on the field pattern in the focal plane. For the details, we refer the reader to the WP1-Task 1.2 section of the report.

The thinnest photonic crystal we tested has 5 layers, and it exhibits a similar focusing pattern, this time closer to the interface, consistent with the arguments above. Below that, the field pattern starts to impair. However, we note that the negative refraction and the focusing effects associated with that originate from the underlying band dispersion characteristics of the photonic crystal. To this end, the band structures are calculated assuming an infinite size photonic crystal. Therefore the bands will be altered substantially when the size is reduced in a crystal direction. Consequently the focus pattern will change. From this point of view, a lower limit for the thickness of the photonic crystal may be applicable.

8. **Try to develop together with theory designs (symmetry, filling factor) which are more suitable for the fabrication of 2D and 3D LHM.** *FORTH*

There are many SRR designs that have been examined theoretically, and some promising ones have been isolated. The main condition for these designs was the highest possible symmetry. One of these designs is currently under experimental investigation (in 1D metamaterial structures). The theoretical investigations are presented in WP1-Task.1.1 section of this report and the experimental one, which is in progress, in WP1-Task.1.2 section.

9. **Develop further the new ideas towards applications while considering what is experimentally feasible.** *FORTH, ICSTM, BILKENT*

We consider employing the composite metamaterials for antenna applications. By proper choice of the metamaterial parameters, the permittivity $\varepsilon$ and permeability $\mu$ can be made sufficiently small, so that a low-index metamaterial is obtained. It has been shown that a dielectric photonic crystal, when the frequency is chosen at the band edge, can also act as a low-index material [1,2]. The idea is extended and successfully applied to a 3D layered metallic wire planes [21]. In the case of left-handed metamaterials, we have the flexibility to approach low index values both from negative and positive values. One issue that has to be addressed here is that $\varepsilon$ and $\mu$ should have
proper values so that the impedance $Z \sim (\mu/\varepsilon)^{1/2}$ remains close to unity, otherwise, large impedance mismatch may degrade antenna efficiency considerably.

Next, we investigate the feasibility of tunable composite-metamaterial structures. An elaborate discussion of tunable materials, and how they can be implemented into composite metamaterials is presented in the appendix of this part.

Apart of the above mentioned ideas, we have commissioned Dr Tony Holden to carry out a critical review of the potential applications of LHMs and write a report on their feasibility. He was the author of the report on “Inside the wavelength: electromagnetics in the near field” for the UK Department of Trade & Industry “Foresight Exploring the Electromagnetic Spectrum” (EEMS) initiative (see http://www.foresight.gov.uk/emspec.html and follow the links Reports & Publications and EEMS: State of Science Reviews.). He was previously at Marconi (manager of the group that did the work on metamaterials), and has the ideal background and experience to carry out this task.

10. Clarify resolution of swiss rolls in MRI machine together with theory. [ICSTM]

In our study last year of the resolution of the face-plate, we found that the transmitted flux tends to be confined to the individual rolls, but the overall faceplate resolution was accounted for by the loss in the material. However, if we consider rather shorter rolls, we expect the resolution to be dominated by the size of the individual elements, with each roll producing its own field distribution. This effect was seen in our MRI experiments on the faceplate, and is clear in the data from the square prism sample. This limits the resolution in an imaging sense, and restricts the field uniformity achievable in a yoke. Therefore, devices need to be constructed of rolls with the smallest possible diameter.

However, using very small source and detector elements along with very large rolls, there is evidence (yet to be confirmed) to suggest that some details of the flux patterns may be transmitted through a single roll. More detailed work is being carried out to determine whether this is a real effect or an artifact.

11. The work on the tapered solenoid concentrator looks like a promising application and should be continued. [ICSTM]

Following our initial feasibility study of the solenoid flux compressor, we have investigated a more flexible approach using resonant loop elements. We established last year that uniform arrays of such loops could transfer flux; we have now investigated whether an array of capacitively loaded loops whose diameters are progressively reduced could be used to concentrate flux.

A compressor was built from a sequence of resonant loops, all tuned to the same frequency, and wound on different diameter formers. The device was tested in two configurations: first with all the elements pushed close together, and then with the inter-element spacing increased to 5.2 mm. We have
compared the measured and calculated spectra and found quite good agreement. Although the bandwidths of the devices and the details of the resonant peaks depend on the configuration, their overall transmission levels are little affected, so that the design of these devices seems to be quite robust. We plan to include these devices in the RF yoke that is currently under construction.
Deliverable 10: Testing of LH structures and comparison between theoretical results and experimental data

In the progress report concerning the 2nd DALHM year we made an extensive presentation of results arising from the testing of our LH structures and we demonstrated the existence of LH transmission peaks in both 1D and 2D composites. Here we will not present again all these results but we will isolate few of them and we will be focused on their comparison with the corresponding computational data.

The computational method that we usually use for the reproduction and the analysis of the experimental data is the Finite Integration Technique, applied through the commercial program MicroWaveStudio (MWS). The reason that we prefer to reproduce the experimental data through MWS is its possibility to employ a non-uniform discretization scheme, and thus to treat thin metallic elements with small additional computational effort. The advantage of a non-uniform grid is offered neither from the Transfer Matrix Method (TMM) nor from the Finite Difference Time Domain Method (FDTD) – in their current implementation – and it essential in our case, where the depth of the metallic elements is much smaller than their other characteristic lengths. (FDTD has the additional disadvantage that can be applied only in systems of length scales smaller than 10 µm.)

![Fig. 1(left): Measured (a) and calculated (b) transmission as a function of frequency for a metamaterial composed of circular SRRs and continuous wires (solid-thick line), of only SRRs (solid-thin line) and of only wires (dashed line). The structure is shown in the figures below. The metal is patterned on FR4 boards of thickness 1.6 mm; the wires are at the back side of the boards; the propagation is along x axis (see figure below) and the electric field polarization along y. We have 5 unit cells along propagation direction, with unit cell size: $a_x = a_y = 8.8$ mm, $a_z = 6.5$ mm. The SRR parameters are $d = t = 0.2$ mm, $w = 0.9$ mm and $r = 1.6$ mm (as shown in the figure below).]
In Fig. 1 we show the transmission for a composite metamaterial of circular SRRs and wires, for a metamaterial of only SRRs and for one of only wires. Panel (a) shows the experimental data and panel (b) the corresponding computational results, obtained through MWS. One can see that the agreement between theory and experiment is extremely good. We want to stress that the experimental LH transmission peak shown in Fig. 1(a) has the lowest losses of all the fabricated LH structures all over the world.

Fig. 2: (a) A metamaterial of square SRRs on FR4 dielectric boards. The system parameters are: SRR size: 3x3 mm$^2$, metal width, rings distance and gap 0.33 mm, metal depth 0.03 mm, unit cell: $a_x=5$ mm, $a_y=3.63$ mm, board thickness 1.5 mm. (b) The experimental transmission for the system presented in (a), for boards distance $a_z=3.1$ mm (dashed line) and 5.6 mm (solid line). The propagation is along $x$ and the $E$ polarization along $y$.

Fig. 3: Calculated S parameter vs frequency for the system presented in Fig. 2(a). The left panel is for boards distance 5.6 mm and the right panel for boards distance 3.1 mm.

The next example that we present here concerns a system of square SRRs. The system is shown in Fig. 2(a), while Fig. 2(b) shows the experimental transmission (for boards
distance 5.6 mm (solid line) and 3.1 mm (dashed line)). The corresponding theoretical curves are shown in Fig. 3.

Fig. 3 shows the corresponding theoretical data, obtained with $\varepsilon_b=2.8$. Here, although the qualitative agreement between theory and experiment is very good, the quantitative agreement is not as good as in the case of Fig. 1, especially close to the electric cut-wire resonance frequency (~13-14 GHz in the experimental curves). This discrepancy can be attributed to the fact that the dielectric constant of the PCB substrate is not the same in all the frequency range that we perform our experiments, as has been shown from transmission measurements on only PCB boards.

The third and last example that we present concerns the SRR structure operating in THz frequency region which has been fabricated in the framework of the project. A picture of the structure is shown in Fig. 4(a), where the structure parameters are also given. In this system we consider propagation perpendicular to the SRRs plane, locating the magnetic SRR resonance through the second electric coupling. Fig. 4(b) shows the measured transmission spectra for the two possible wave polarizations: $E//y$ (red line), where 2nd electric coupling is present and $E//x$ (blue line). For $E//y$ the asymmetry of the SRRs relative to the external electric field has a result the excitation of a circular current around SRR, which becomes resonant around 6.5 THz, giving the observed in Fig. 4(b) transmission dip around 6.5 THz.

The corresponding to Fig. 4(b) theoretical results for the polarization $E//y$ are shown in Fig. 5. One can see that the agreement between theory and experiment is again very good. The small discrepancies are attributed to the difference between the actual and the used values of the substrate dielectric constant and loss factor.

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**Fig. 4:**

- (a) Single-ring SRR structure operating in THz. Unit cell: $a_x \times a_y \times a_z = 7 \times 7 \times 5$ µm$^3$, SRR size: 5×5 µm$^2$. All other lengths: 1 µm. Board’s $\varepsilon=2.8$ (polyimide). Structure fully embedded in board.
- (b) Measured transmission spectra for the SRRs system described in (a). In both cases the propagation of EM waves is perpendicular to the SRRs plane (along z) and we have considered 5 unit cells along propagation direction. The red curve shows the transmission (divided by 2, as the reference wave is unpolarized) for a polarization with $E$ along $y$, while the blue curve shows the transmission for $E$ polarization along $x$. 

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**Fig. 5:** The corresponding theoretical results for the polarization $E//y$ are shown in Fig. 5. One can see that the agreement between theory and experiment is again very good. The small discrepancies are attributed to the difference between the actual and the used values of the substrate dielectric constant and loss factor.
In general, as was shown also in the results presented here, the agreement between theoretical results and experimental data is in most of the cases extremely good.

Fig. 5: Calculated transmission (divided by 2) vs frequency for the system described in Fig. 4(a). The propagation is perpendicular to the SRRs plane and we have considered 5 unit cells along propagation direction. $E$ along $y$ (see Fig. 4(a)).
1. LH behavior using photonic crystals: As we have discussed in the DALHM reports, one way to obtain negative refraction is by using photonic crystals (PCs) and by exploiting their peculiar dispersion characteristics. These PCs can be composed of only dielectric materials. Since the properties of the dielectrics do not change considerably going from microwaves to the optical regime, we expect all the effects connected with negative refraction in PCs to be scalable and to remain unchanged as one goes from microwaves to the optical regime. Thus photonic crystals offer a way to extend the LH behavior towards optical frequencies. Of course technical difficulties exist in the fabrication of PCs at optical wavelengths; in a recent Physical Review Letters (PRL 93, 073902 (2004)) A. Berrier et. al. reported the first experimental evidence of negative refraction at telecommunication wavelengths (λ=1480 nm). They used a PC that the DALHM partners had made experiments and simulations at microwave frequencies. There are still a lot of experimental difficulties on how one can couple the incident light from a fiber to the photonic crystal. The DALHM partners have a lot of experience in negative refraction and subwavelength focusing in PCs but it is another, completely different project to achieve these two goals at optical wavelengths. This might be a new future direction for another FET grand for the DALHM and possibly more partners.

2. LH behavior in systems of SRRs and wires: In systems of SRRs and wires, since they contain metallic elements, there is an upper frequency limit for the LH behavior. This is the plasma frequency of the metal ω_pm, which is of the order of 15×10^{15} sec^{-1}, thus f_pm=ω_pm/2π=2.4×10^{15} Hz=2400 THz (values for Cooper). Above f_pm metal is transparent to the EM radiation and all the phenomena connected with negative ε stop to exist. The question is how close to f_pm one can go, continuing having LH behavior. We study this question by exploring separately the magnetic response of the SRRs (which gives the μ(ω)) and the electric response of the wires.

Magnetic SRR response: We examine the frequency limits of the magnetic SRR response in two different examples:
(a) The first example concerns a single-ring SRRs with dimensions those of the experimentally constructed DALHM structure operating in THz (in-plane lattice constant 7 μm), which was scaled down progressively. Calculating the transmission through this structure, we located the magnetic resonance frequency and we examined up to which scale this magnetic resonance is maintained. Our SRR is made of Cu (embedded in polyimide environment, with ε=2.8) and initially it has a linear dimension of 5 μm and all other characteristic lengths equal to 1 μm. For the dielectric function of the SRR, ε(ω), we used the Drude formula, ε=1-(ω^2/ω^2_{pm}+iωγ), with ω_pm=15×10^{15} sec^{-1} and γ=3.3×10^{13} sec^{-1}. In the following table we list the values of the magnetic resonance frequency for each one of the structures that came out from the scaling of the initial SRR, as well as the minimum metal size and the skin depth around magnetic resonance (the Drude parameter γ and the skin depth have been calculated using the static conductivity).
We have to mention that for the smallest of the above structures the transmission dip at the magnetic resonance frequency it was hardly seen (it becomes more clear if we try to locate it using the second electric coupling), and it disappears as we go to smaller scales.

(b) Our second example concerns an SRR made of Au ($\omega_{pm}=13.7 \times 10^{15}$ sec$^{-1}$ and $\gamma=4 \times 10^{13}$ sec$^{-1}$), in substrate of $\varepsilon=2.14$. Again we reduce progressively the size of the structure (not uniformly) but here we not only calculate the transmission but we also invert the transmission data and obtain the effective magnetic permeability. We find again that the magnetic resonance disappears in the regime between 130 and 200 THz. In the following table we present plots showing the real part (solid lines) and the imaginary part (dashed lines) of the magnetic permeability for the various structure sizes (the structure characteristic lengths are listed next to the plots (uc denotes the lattice constant of the unit cell)).


**Wires electric response:** Starting from a periodic system of wires, of lattice constant 7 µm and wire cross-section 1 µm, and scaling it down progressively, we examine the change in the electric cut-off frequency of the system, \( \omega_p \), (where \( \varepsilon_{\text{eff}}(\omega) \) goes form negative to positive). The lattice constants of the resulting systems and the corresponding cut-off frequencies are listed in the table below.

<table>
<thead>
<tr>
<th>Lattice constant (in-plane)</th>
<th>Approx. magnetic resonance freq., ( f_p = \omega_p/2\pi )</th>
<th>Metal size</th>
<th>Skin depth</th>
</tr>
</thead>
<tbody>
<tr>
<td>7 µm</td>
<td>10.5 THz</td>
<td>1 µm</td>
<td>0.02 µm</td>
</tr>
<tr>
<td>0.7 µm</td>
<td>100 THz</td>
<td>0.1 µm</td>
<td>0.006 µm</td>
</tr>
<tr>
<td>0.07 µm</td>
<td>1000 THz</td>
<td>0.01 µm</td>
<td>0.002 µm</td>
</tr>
<tr>
<td>0.007 µm</td>
<td>10000 THz</td>
<td>0.001 µm</td>
<td>0.0006 µm</td>
</tr>
</tbody>
</table>

For the smallest systems the transmission after \( \omega_p \) is very low, and the difference in the transmission for \( \omega < \omega_p \) and \( \omega > \omega_p \) becomes less and less.

In summary, we have seen that by using a single cut SRR one can obtain a magnetic resonant response, and therefore left-handed behavior up to 150 THz. At present, we are using two-cut and four-cut SRRs, and it looks very promising that we can have magnetic resonant response and LH behavior for frequencies higher than 200 THz, with relatively small losses.
WP1: Modelling and Characterization

Connection with year 1

To connect with what was reported in the year 1 report and to remind the notation, we shall mention here that one of our most important theoretical achievements in year one was the new effective medium theory of composite metamaterials (CMMs) composed of SRRs and wires, and the resulting new criterion for the identification of transmission peaks in such CMMs.

What was commonly believed before our contribution is that the electric response of a CMM is provided only by the continuous wires. Our investigation showed that this is not true and that there is a dominant contribution to this response coming from the SRRs. The SRR electric response is of cut-wire type, providing a resonant effective epsilon around a frequency $\omega_0$. The total electric response of the metamaterial was found to be a sum of the electric responses of the SRRs and of the continuous wires. This summation leads to a new cut-off frequency for the metamaterial, $\omega_p'$, always smaller than the cut-off frequency of the continuous wires, $\omega_p$. This cut-off frequency can be demonstrated by closing the SRRs (removing their gaps), switching thus off their magnetic response.

According to the above, in order to have possibility for left-handed (LH) behaviour the magnetic resonance frequency of the SRRs ($\omega_m$) has to lie below the new cut-off frequency, $\omega_p'$. Moreover, the fact that closing the SRR gaps the full electric response of the system is maintained while the magnetic is switched off leads to an easy and extremely useful in experimental studies criterion for identification of transmission peaks as LH or right-handed (RH). According to that criterion a peak around the SRRs magnetic resonance will be LH if it disappears by closing the SRR gaps.

Task 1.1: Modelling

Concerning modelling, the issues that have been addressed in the second DALHM year are:

- Improvement of the transfer matrix code through the incorporation of a more symmetric discretization scheme. This scheme leads to the elimination of the cross-polarization terms in the scattering amplitudes, having as a result more correct effective material parameters in the retrieval procedure.
- Device of a periodic effective medium description of composite metamaterials. This periodic effective medium description leads to a more accurate and broadband reproduction of the metamaterials transmission features than the homogeneous effective medium description; moreover, it eliminates all the spurious features that the homogeneous effective medium approach produces in the effective material parameters.
- Study of the electric coupling to the magnetic resonance ($2^{\text{nd}}$ electric coupling), i.e. the coupling of the external electric field to the resonant circulating currents around SRR. This coupling occurs due to the SRR asymmetry relative to the electric field, and can play a significant role in the
LH behaviour of multidimensional metamaterial structures (e.g. complete destroy of LH behaviour).

- Examination of highly symmetric SRR structures (e.g. multigap structures), which do not experience the 2nd electric coupling and therefore could be ideal for multidimensional LH structures.
- Study of two-dimensional (2D) and three-dimensional (3D) metamaterial structures composed of highly symmetric SRRs and continuous wires.
- Detailed study of the SRR. It includes parametric study of the characteristic frequencies of single-ring SRR as well as comparison between single and double-ring SRRs. It also includes comparison of circular and rectangular SRRs.
- Study of structures operating in THz and mainly of the experimentally constructed THz structure.
- Simulations concerning the experimentally studied structures and explanation of the various observed transmission characteristics.
- Study of the influence of the wires on the magnetic response of the SRRs. This was done mainly by comparing in-plane and off-plane wire cases and various wire positions.
- Analytic investigation of the effective material constants. The attempt was to conclude about the sign of the effective $\varepsilon$ and $\mu$ in a LH material using energy conservation and causality arguments.
- Study of systems of wires, with 2 wires per unit cell.
- Other issues concerning metamaterials of SRRs and wires or of only SRRs: influence of the distance from the board on the electric and the magnetic response of the SRRs; approximation of the electric response of a SRRs+wires metamaterial with the electric response of a continuous plus a cut- wire per unit cell; effect of the orientation of the SRR (relative to the electric field) on the electric cut-wire SRR response; comparison of the LH transmission level in metamaterials composed of circular and of rectangular SRRs (plus wires); comparison of the LH transmission level for metamaterials of square vs metamaterials of non-square unit cells; etc.
- Simulations on 2D photonic crystals, demonstrating LH behavior and focusing capability.
- Theoretical study of a system of Swiss-rolls and demonstration of its lensing capabilities (using both an analytic theory and numerical modeling).
- Study of a corner reflector made of a LH material and demonstration of its ability to act as perfect mirror.
- Evanescent Bessel beam generation using transmission through thin metallic films.

1.1.1 Material discretization in the TMM

In previous transfer matrix simulations of real meta-materials we always obtained non-zero cross-polarization scattering amplitudes, e.g. transmitted field of different polarization than the incident one. These cross-polarization terms are usually of the order of $10^{-5}$-$10^{-3}$ but can amount to the order of one in some 3D designs. They disturb the simulation results in two ways:

(a) For designs where one polarization experiences a transmission gap while the orthogonal polarization is transmitted nearly unaffected (e.g. the conventional
1D SRR or LHM) those cross-polarization terms can cut-off the exponential spatial decay of the transmission inside the gap, giving thus a wrong transmission vs system-length dependence in the gap and therefore no correct effective material constants in the gap.

(b) In more-dimensional meta-materials those terms may lead to effects associated with different than the incident one polarizations, which might not be present in real experimental samples.

Studies performed with increasingly finer discretizations indicated that those cross-polarization terms may be, at least partially, discretization artifacts. A thorough investigation revealed that indeed the standard-discretization of the material (meaning the way that material constants are assigned to the various discretization cells) used in the previous TMM simulations artificially introduces an anisotropy in the discretized Maxwell equations, which causes the aforementioned cross-polarization scattering amplitudes, even for perfectly symmetric samples. We devised an alternative isotropic material discretization which fixes that problem. This allows us to obtain nearly perfect length-independent effective medium behavior for some meta-materials over a range of at least 1-11 unit cells in propagation direction.

Additionally, we derived a criterion for the occurrence of real, physical cross-polarization scattering amplitudes: They occur if and only if the inversion-symmetry of the discretized unit cell in both directions perpendicular to the direction of propagation is simultaneously broken. This will be helpful in the design of more-dimensional materials where the coupling introduced by such cross-polarization terms is usually undesirable.

1.1.2 Periodic effective medium

We have investigated the influence of the inherent periodic structure, which is always present in our artificial meta-materials on the effective medium approximation.

In previous work it has always been assumed that the scattering behavior of the meta-materials can be described in terms of a homogeneous effective medium, because the vacuum wave-length of the incident radiation is much larger than the size of the unit cell and hence the typical size of the internal structure of the meta-material. Although we (and others) could previously demonstrate that the homogeneous effective medium picture is a reasonable approximation of the meta-materials behavior exposing a clear magnetic resonance of the SRR and left-handed behavior of the LHM with simultaneously negative $\varepsilon$ and $\mu$, the retrieved effective material constants show details which cannot be explained by the homogeneous effective medium (HEM) picture.

These aberrations include:

(a) The resonant $\text{Re}[n(\omega)]$ near the magnetic resonance, $\omega_m$, is cut-off at values corresponding to the upper (SRR) or lower (LHM) edge of the first Brillouin zone of the unit cell.

(b) We find resonance/anti-resonance coupling in $\varepsilon$ and $\mu$, both, at the magnetic resonance for SRR and LHM as well as for the electric resonance of the cut-wire.
(c) Simultaneously, we observe unexpected negative imaginary parts in the effective material constants $\varepsilon(\omega)$ (SRR, LHM) or $\mu(\omega)$ (cutwire), which should be positive as absorption-peaks.

(d) The retrieved resonances of the effective parameters are misshaped, non-symmetric, and $n$ and $z$ disagree in details about the position of the resonance frequency.

(e) A lot of additional structure in the scattering spectrum not accounted for by the assumed effective response (magnetic, cut-wire and plasmonic), especially at higher frequencies.

We have shown analytically that all the above “paradoxs” can be explained in terms of the periodic structure: A very simple stratified periodic medium model involving slabs of vacuum alternating with slabs of a homogeneous material with simple resonant $\varepsilon(\omega)$ and $\mu(\omega)$ is able to reproduce all the artifacts found in the HEM approximation of numerically simulated SRR arrays and LHMs, but also meta-materials build of continuous wires and of cut-wires. Whenever the effective wavelength inside the meta-material becomes comparable to the size of the unit cell the periodicity causes additional band-gaps, which account for the aberrations from the expected HEM behavior. We found for our meta-materials that in good approximation the scattering behavior can be decomposed into an effective behavior of the constituents of the meta-material and an explicit contribution of the periodicity. Remarkably, the average contribution of the constituents like the single-ring SRR behaves much like expected from the assumed homogeneous medium picture, up to frequencies where the vacuum wavelength becomes comparable to the size of the unit cell. This allows a more reliable effective description and interpretation of real meta-materials in terms of a periodic effective medium (PEM) instead of the awkward conventional homogeneous effective medium (HEM) with all the hard to understand artifacts.

The effects caused by the periodicity are generic; they do qualitatively not depend on the particular geometry chosen for the meta-material and universally apply to SRR, LHM, continuous wires and cut-wires materials. Obviously, the impact of the periodicity is noticeable throughout the range of the $\lambda /L$ ratio $\sim$6-10 (i.e. $\sim$vacuum wave length /unit cell length) that is actually found in published simulations and experiments for left-handed and related meta-materials. Our simulations indicate that an unencumbered homogeneous effective medium behavior, though reachable in the low-frequency limit, would require a $\lambda/L$ ratio of the order of 30 or larger, which is geometrically not easy to obtain in real samples.

We investigated the difference between the continuum and the lattice formulation of the PEM approximation and found the latter to be better suited for application to our numerically simulated scattering data for real meta-materials obtained with a lattice-TMM implementation.
1.1.3 Second electric coupling

Working towards more dimensional LHM structures we investigated different orientations of the SRR with respect to the polarization and propagation direction of the incoming wave. We found that due to the asymmetry of the conventional SRR, not only the magnetic field but also the electric field of the incoming wave may couple to the resonance of the circulating currents in the SRR. This coupling, for a single ring SRR is demonstrated in Fig. 1.

![Fig. 1: Single-ring SRR in two different orientations relative to electric field (E). The shadow in the ring shows the current (black: negative, white: positive). The asymmetry in case (b) leads to different charge quantities at the ends of the SRR.](image)

The most important effect of this coupling is an electric resonant response around the SRR magnetic response frequency, $\omega_m$, which may add a right-handed transmission peak that may diminish or even destroy the LH behaviour. This coupling has been demonstrated also experimentally. The problem of the asymmetry-mediated electric coupling will be present in all higher dimensional LHM apart from some special cases that only work for a particular orientation or a constrained direction of propagation. We have tried to examine and we are still working on exactly symmetric SRR designs for use in 2D and 3D metamaterials, which will not suffer from these problems.

1.1.4 Symmetric SRR structures

The attempts to go to multi-dimensional LH materials avoiding the 2\textsuperscript{nd} electric coupling led to the necessity of highly symmetric SRRs. One choice, which was studied here, is SRRs with more than one gaps. Some of the structures which were studied are shown in Fig. 2. The problem with the multigap SRR structures is that the magnetic resonance is shifted to higher frequencies (the gaps act like capacitors in series and the magnetic frequency for a ring with n equal in size gaps goes as $\omega_m^2 (n \text{ gaps}) \sim n \omega_m^2 (1 \text{ gap})$) and thus the probability $\omega_m$ to lie below the cut-off frequency

![Fig. 2. Multigap SRRs studied: Comparing the five cases shown here, structure (a) shows a wider transmission dip at $\omega_m$ than structure (b), (c) shows a stronger dip than (d); and (e) could be promising if the two rings go close enough, although due to technological limitations this is not an easy task. For the two rings not close together (e) behaves like the conventional double-ring SRR but exhibiting a weaker magnetic dip.](image)
of the electric response of the system becomes considerably smaller. A solution will be a decrease in the size of the gaps. This though in many cases is limited by technology. Another solution could be a modification of the structures as the capacitance of the gaps to be increased. Such modifications for the structures of Figs 1.2(a) and 1.2(d) are shown in Fig. 3. The result is usually a lowering of $\omega_m$ but not always an improvement in the transmission picture, as the strength of the magnetic resonance and the width of the $\mu<0$ regime are affected as well.

![Fig. 3: Modifications of the structures of Fig. 2(a) and 1.2(d) that lead to increase of the capacitance of the rings and thus decrease of $\omega_m$. The modification shown in (a) leads to an improvement in the transmission if the additional wire parts are not long. From the modifications shown in (b) only the left constitutes an](image)

In Fig. 4 we show three of the most promising SRR designs which came out from the above mentioned study and which are currently under further investigation.

![Fig. 4: “Promising” SRRs. (a): Detailed study on progress (in 1d); (b): Not studied in detail yet; (c): Good LH transmission but difficulty in fabrication.](image)

### 1.1.5 Multidimensional metamaterials

Next step after the study of the multigap SRRs was to combine them with continuous wires as to form a 2D or a 3D metamaterial structure (see, e.g., Fig. 5). We studied the transmission through several of those structures and we found in some cases peaks close to the magnetic frequency $\omega_m$. However, most of them seem to yield RH instead of LH. Partially this can be attributed to the presence of the second electric coupling. The study of 3D CMM structures is still in progress.

![Fig. 5: Symmetrized SRRs (a) and (b) and wires (c) in a 3D unit cell. Combination](image)
1.1.6 Detailed SRR study

A. Single-ring SRR

We demonstrated that a single-ring SRR acts also as a magnetic resonator, providing a negative $\mu$ regime. The inductance in such a resonator is provided by the loop while the capacitance by the gap of the ring. Since the ring exhibits also an electric cut-wire-like resonance, theoretically there might be a possibility to achieve a LH transmission regime using only SRR (i.e. no additional continuous wires), by tuning the magnetic resonance into the negative epsilon band provided by its own electric cut-wire response.

B. Controlling $\omega_m$ and $\omega_0$ for a single-ring SRR

One of the targets in LHM study is to shift $\omega_m$ below the cut-off frequency of the electric response of the system and to keep the electric cut-wire resonance frequency as high as possible. This leads to the necessity to control these two frequencies and thus to examine their dependence on the system parameters. One parameter that we examined is the SRR size. This investigation showed that $\omega_m$ for a single-ring SRR is inversely proportional to the square root of the SRR area, i.e. $\omega_m^2 \propto 1/l_1 l_2$ (see Fig. 6) (in agreement with the $\omega_m^2$~LC picture that we accept for the SRR), while $\omega_0$ depends only on the parallel to the electric field sides of the SRR, following the approximate relation $\omega_0 \propto a/l_1$, with $a$ being the corresponding lattice constant.

C. Single- vs double-ring SRR
Comparing the transmission through our common double ring SRR with the transmission from its outer and inner ring we found that the lower magnetic frequency of the double ring (see first dip in Fig. 7(a)) is essentially the one of the outer ring, but with a small downwards shift due to the presence of the inner ring (in the double ring case in the capacitance provided by the gap is added the capacitance between the two rings). Moreover, the lower cut-wire response frequency of the double-ring (third dip in Fig. 7(a)) is also that of the outer ring, while in its spectrum are also presented the magnetic and the electric responses of the inner ring (in some cases though it seems that the outer ring screens the magnetic response of the inner one). A demonstration of the above is given in Fig. 7. We have to mention also that at the lower magnetic frequency of the double ring the currents in the two rings flow in the same direction (giving a capacitance between the rings) while in the second magnetic frequency (that of the inner ring) they flow in opposite directions.

D. Circular vs rectangular SRRs

Comparing circular and rectangular SRR (see Fig. 8) we found no any qualitative difference between the two cases. For SRRs of the same linear dimension, metal
characteristics and gaps, the circular one shows higher $\omega_m$ and $\omega_0$, in accordance with what was discussed in part (B) of the present section.

1.1.7 THz structures

Fig. 9: Structure operating in THz. Unit cell: 7x7x5 $\mu$m$^3$
SRR size: 5x5 $\mu$m$^2$
All other lengths: 1 $\mu$m
Board’s $\varepsilon$=2.8 (polyamide). Structure fully embedded in

Another issue that we examined in the second DALHM year was SRR structures operating in THz. This study provided material and structure parameters which are offered for easy experimental realization and lead to magnetic response in the THz regime. Such a structure is shown in Fig. 9 and it has been fabricated and measured. The computational transmission results are in very good agreement with the experimental data, while inversion results (using the homogeneous effective medium approach) certify the negative $\mu$ in the regime of the transmission dip (around 6 THz). Detailed results on this structure are in the separate report on D10: “Comparison between theoretical results and experimental data”.

1.1.8 Theoretical interpretation of experimental data

In the attempts to help and guide the experimental effort, we performed calculations

Fig. 10: SRRs structure studied in Bilkent. After detailed investigation of this structure it appeared that the only region with $\mu<0$ is around 3 GHz. The dip around 7 GHz is due to an electric resonance ($\varepsilon<0$, $\mu>0$) while the dip at around 11 GHz is a periodicity-related band gap.
on various of the experimentally studied systems. A specific examples concerns a system studied experimentally by Bilkent. We managed to interpret the experimental data obtained by Bilkent and to identify the observed transmission characteristics. It came out that metamaterial peaks initially considered as left-handed were in fact right-handed. This study identified the region where a LH peak should be expected (see Fig. 10) and became a guide to subsequent experiments which finally gave a left-handed pass-band with very high transmittance.

1.1.9 Continuous wires influence on SRRs resonance

Studying in-plane metamaterial structures (wires besides SRRs) we found in many cases a displacement of the magnetic response compared to that of the only-SRRs systems. This led us to examine in detail the influence of the wires on the SRRs resonance. The main conclusion from this study is that the in-plane wires (see Fig. 11(A)) modify the magnetic flux in the SRR, leading in most of the cases to a decrease of this flux and thus to a small upwards shift of the magnetic response. This does not occur in the off-plane case (Fig. 11(B)), where the wires are more symmetric in respect to SRRs. Other things noticed are that (a) the in-plane wires weaken the magnetic response of the SRRs and (b) in the off-plane case when the wires go very close to the SRRs the electric cut-off frequency of the system (ωp) moves upwards. These things though are subject to further investigation.

![Fig. 11: A: In-plane wire geometry; B: off-plane geometry; C: Transmission through the geometry of A (red line) and of B (blue line).](image)

1.1.10 Analytic investigation of the effective material constants

Using the retrieval procedure for the effective material constants we found around the magnetic resonance and the electric cut-wire resonance regions where either μ or ε have negative imaginary parts. Textbook formulae through for the energy inside arbitrary media suggest that both imaginary parts of μ and ε should be independently positive. Since it is not clear at the moment whether this has to apply to the scattering results of our metamaterials, we attempted an analytical investigation of those parameters.

To avoid the problem of defining an energy current inside an arbitrary medium we just considered the energy transported (in the vacuum before and after the sample) into and out of the slab of metamaterial. As a passive material our sample must expose a positive absorption independent on the size of the slab. By this approach we
derived just from the scattering formulae for the homogeneous slab in the continuum the required conditions \( \text{Im}(\mu) + \text{Im}(\varepsilon) \geq 0 \) and \( \text{Im}(\mu)/|\mu| + \text{Im}(\varepsilon)/|\varepsilon| \geq 0 \). This seems to allow for a certain negative imaginary part of one material constant if the imaginary part of the other is sufficiently positive. Whether the conditions are sufficient is not yet clear. Neither is clear whether the observed negative imaginary parts are in conflict with the effective material interpretation.

The common sign of refractive index \((n)\) and impedance \((z)\) do not enter the Maxwell equations nor the scattering formulae and may be chosen by convention. It is only the relative sign that matters. We adopt \( \text{Re}(z) \geq 0 \); as a consequence a passive material will have \( \text{Im}(n) \geq 0 \).

1.1.11 Adding a second wire in the unit cell

The attempts to shift upwards the electric cut-off frequency of a metamaterial leads to the idea of adding a second wire in the unit cell. This idea led us to examine how the addition of a secondary wire in the unit cell affects the plasma frequency, \( \omega_p \), of the initial (primary) wires and which is the optimum (for high \( \omega_p \)) position of this secondary wire (here we remind that the cut-off frequency of the total metamaterial depends in a large degree on \( \omega_p \)). What we found is that the secondary wire always leads to a considerable increase of \( \omega_p \), while the largest \( \omega_p \) is obtained in the case where the secondary wire is located in the middle between two successive primary wires in the perpendicular to the propagation direction. The component of the secondary wire position in the propagation direction seems to play a less significant role.

1.1.12 Other issues on SRRs+wires metamaterials

Other issues examined within the second DALHM year, concerning metamaterials of SRRs plus wires or only SRRs are:

a) **System of continuous plus cut-wire per unit cell:** We demonstrated that the combined electric response of SRR and continuous wire in a LHM with its typical signature in the transmission spectrum, including the new characteristic frequency \( \omega'_p \), can be modelled by a combination of a single cut-wire and a single continuous wire in the unit cell. We found that \( \omega'_p \) increases with increasing \( \omega_p \) of the continuous wire but it is essentially dominated by the cut-wire resonance frequency.

b) **Examination of the influence of the SRR distance from the board on \( \omega_m \) and \( \omega_0 \).** We found that \( \omega_m \) is more strongly affected by the local dielectric environment, in particular in the neighborhood of the SRR gaps, while \( \omega_0 \) sees a more “averaged” environment.

c) **LH materials of circular vs. LH materials of rectangular SRRs.** Comparing the LH transmission level of metamaterials composed of circular SRRs and wires with the LH transmission level of metamaterials composed of rectangular SRRs (of the same linear size as the circular ones), we found no difference between the two cases.

d) **\( \omega_0 \) for SRRs of different orientations.** Comparing the electric cut off-frequency of the SRR \( (\omega_0) \) for the SRR orientations presented in Fig. 1 and propagation
in the SRR plane we found that $\omega_0$ for the orientation (a) is always lower than that for the orientation (b). The corresponding field patterns at $\omega_0$ for the two orientations are shown in Fig. 12.

![Fig. 12: Electric field component parallel to the external E, at the electric cut-wire response frequency of the SRR, for two orientations.](image)

e) **Square vs non-square metamaterial unit cells.** Comparing the LH transmission level for SRRs+wires metamaterials of square and of non-square unit cells (in the SRRs plane) we find better LH transmission (higher and broader peak) in the case where the parallel to the electric field unit cell side is smaller than the perpendicular side (“reduced side” case). This is a feature found also in the experiments and it might be due to the better impedance mismatch between the metamaterial and the free space in the “reduce side” case. The better impedance mismatch can be attributed either to the fact that the SRRs of the neighboring unit cells interact more strongly in the “reduced side” case and thus the negative $\mu$ regime is wider and deeper, or to the fact that the electric cut-wire response of the SRRs is reduced and thus cut-off frequency of the total system ($\omega_p'$) is also reduced, going closer to the magnetic resonance frequency of the system.

### 1.1.13 Simulations on 2D photonic crystals

Using the Plane Wave technique we calculated the band structure and the equifrequency contours (EFCs) for a 2D photonic crystal which has been fabricated experimentally (hexagonal pattern of cylindrical alumina rods ($\varepsilon=9.6$) in air; the rods radius is 1.56 mm and the lattice constant 4.79 mm). The aim was to locate frequency regimes where negative refraction and focusing effects can be achieved. Fig. 13 shows TM band structure and EFCs. To obtain negative refraction the EFCs have to be both convex and larger than the EFCs for air, which are circles with radii proportional to the frequency. This is obeyed in the 5th band (shaded in Fig. 13a), thus the experimental effort was focused on this band. We note that the EFCs are not isotropic at the low frequency edge of the band but become more isotropic (circular) towards the high frequency edge.
1.1.14 Theoretical analysis and lensing properties of Swiss-Roll structures

In this part of the work we examine theoretically the lensing properties of Swiss-Roll structures, (a) by developing and using an analytic theory and (b) by applying numerical modelling, using the MicroWave Studio program.

A. Analytic theory
The system that we study is a 2D array of swiss-rolls, arranged periodically in a square lattice, with the total slab having a square cross-section in the x-y plane (perpendicular to the rods axes). (The system is identical to that studied experimentally by ICSTM2 partner.) The fact that the individual rolls are extremely small compared to the wavelength of the EM field allowed us to assess the magnetic response of the total metamaterial by treating it as a homogeneous (effective) medium, and more specifically as a short uniaxial waveguide (see Fig. 14). The magnetic permeability component parallel to the rolls exhibits resonant-type behaviour with resonance frequency close to that of the individual roll. More specifically, the magnetic response of the waveguide medium is given by the magnetic permeability tensor

\[
\hat{\mu} = \begin{pmatrix}
\mu_{||} & 0 & 0 \\
0 & \mu_{||} & 0 \\
0 & 0 & \mu_{\perp}
\end{pmatrix}
\]

with

\[
\mu_{\perp}(\omega) = 1 - \frac{F}{\left(1 - \frac{\omega_0^2}{\omega^2}\right) + \frac{\Gamma}{\omega}}, \quad \mu_{||} = 1
\]

(1)

The parameters resonance frequency, \(\omega_0\), loss factor, \(\Gamma\), and filling fraction occupied by the Swiss Rolls, \(F\), were obtained from the experiment.

The propagation within the guide is governed by the dispersion relation of the effective medium. The normal modes of the guide are determined by its shape and the boundary conditions. The modes in the neighbouring space are defined within a fictitious waveguide with the same lateral boundary conditions as assumed for the medium. For each normal mode, the transmission and reflection coefficients are derived for the air - material interfaces. We then calculate the coupling of a small
external dipole source to the normal modes of the entrance space, and hence calculate the transmitted field distribution above the output surface.

\[
\sum_{n,m} a_{nm}^\perp h_{nm}^{\perp} \exp(-k_{nm}z) + \sum_{n,m} a_{nm}^\parallel h_{nm}^{\parallel} \exp(+k_{nm}z)
\]

Fig. 14: The effective medium wave guide model. The field distributions in each region are shown. The overall transmission coefficient for each mode \((n,m)\) is given by \((c_{nm}/\alpha_{nm})\).

We investigated both Dirichlet and von Neumann boundary conditions for the “sides” of the guide: Dirichlet conditions require \(H_z = 0\) at the boundary, whereas the von Neumann conditions require \(\partial H_z/\partial n = 0\). In fact neither condition is fully satisfied, but the von Neumann conditions appear to be the better approximation.

On resonance, the model predicts that the input field pattern is reproduced on the output face (Fig. 15), as observed experimentally. This result is independent of the boundary conditions adopted, because the field is localised to the centre of the prism.

Fig. 15: Input (left) and transmitted (right) distribution for \(\log(|H_z|^2)\) from a magnetic dipole source as calculated by the effective medium waveguide model.

We note that all the measurements contain a central peak, a feature that is not predicted theoretically above the resonance frequency. We believe that this arises because the flux is partially trapped within the central roll, thus leading to extra intensity at the centre of the distribution, so our comparisons are made with this feature being truncated in the experimental data. Moreover, to avoid difficulties with scaling, we compare the calculated and measured field patterns by considering the characteristic minima in the field distribution, rather than the maxima. On that basis, we can identify several patterns for which the match is quite good, two of which are shown in Fig. 16.
Fig. 16: Comparison of calculated (left) and measured (right) $H_z$ field distributions near 22.9 MHz (top) and 23.7 MHz (bottom). The correspondence of the minima in the patterns is highlighted.

A more quantitative comparison of the field distributions can be made by calculating their Fourier components and plotting these as a function of frequency. This has been done using a discrete transform for each data series, both measured and calculated, and the coefficients in the irreducible $1/8^{\text{th}}$ of the Fourier space have been compared. Fig. 17 shows the first three of these. We see that, although some aspects of the match are excellent (e.g. the second peak in the $k_{11}$ and $k_{20}$ data), the calculation does not reproduce the two peak structure that we observe. This suggests that the assumed

Fig. 17: Comparison of the Fourier components $k_{10}$, $k_{11}$, and $k_{20}$ for the measured (red) and calculated (blue) $H_z$ field distributions. The peak near 25 MHz in $k_{11}$ and $k_{20}$ is well matched, but the presence of the lower frequency
normal modes in the calculation (determined by the choice of boundary conditions, and the artificial extension of the waveguide into the vacuum region) are not correct. We have therefore attempted to calculate the field distributions using numerical simulations.

B. Numerical modeling
The numerical modelling of the Swiss Roll structures has been carried out using MicroWave Studio (MWS), in collaboration with the group at FORTH. The intended approach was to treat the Swiss Roll array as a uniform prism of effective medium described with the measured permeability. However, this met with two problems. First, MWS did not permit the use of $\mu_{\infty} < 1$ in its Lorentzian dispersion formula, as is required in our model, and second for the cases of negative $\mu$ it seems that the results obtained by the magnetic Lorentz model are not correct.

We tried to bypass these problems using different dispersion models (since we treat a linear system, wherein the field at any given frequency is independent of the field at other frequencies, the result for any particular $\mu$ should be independent of the dispersion relation used to obtain that $\mu$), like Drude model (i.e. the magnetic equivalent of a plasma) but this required a large number of calculations (using non-physical parameters) to describe the behaviour near resonance.

We therefore considered another approach, in which we exploited the fact that, in the very near field regime, there is an equivalence between the electric and magnetic problems: a calculation of the E-field for a dielectric should be the same as that for the H-field for a magnet. This was validated for the Drude model, and we then found that the dielectric calculation did give the correct result in the negative epsilon regime of the Lorentz model, and converged to a stable result. Based on this finding, we carried out simulations using a dielectric prism to mimic the Swiss Roll material, and an electric dipole source to replace the current loop magnetic dipole source. This has been done for both the square and hexagonal prisms.

A detailed comparison between the measured and simulated results is still being carried out, but the preliminary results for the hexagonal prism are reported here.

On resonance
As predicted by effective medium theory, on resonance the fields are tightly localised throughout the prism, and the “input distribution” is transported through the prism, which acts as a “faceplate” (see Fig. 18).

Plasma frequency
Here $\varepsilon = 0$, and the simulation shows only a very weak transmitted field, because of the huge impedance mismatch between the medium and the surrounding vacuum. In the experiments, however, we observe a strong, uniform field distribution, whose strength arises in part from uneveness and
granularity in the surfaces, and in part from the initial channelling of the flux by a single Swiss Roll, before it is coupled internally to other rolls. This cannot be reproduced in the simulation. However, such field as does penetrate the interface spreads out and propagates in quite a uniform mode (see Fig. 19).

**First resonance**

This occurs in the model when the maximum intensity of the $H_z$ field distribution extends into the corners of the prism (see Fig. 20) and is at 25.0 MHz. At this frequency, the field direction at the exit face rotates across the surface, giving rise to a ring of high $H_z$ in the outer part of the prism. This is shown in Fig. 21, along with the distribution of $H_z$. In the measured data, there is a strong central peak, which, as previously discussed, is not reproduced in the simulation.

![Fig. 20: $H_z$ amplitude distribution in the plane $x = 0$ of the hexagonal prism at 25 MHz showing the measured pattern at 24.7 MHz (right).](image)

Detailed analysis of the full data sets for both the hexagon and the square calculations is still in progress.

**1.1.15 Corner reflectors**

Material with refractive index $\lim n \to -1$ has some things in common with a mirror: the angle of refraction inside the medium is equal to the incident angle, just as a ray is reflected from a mirror with equal incident and reflected angles. One of the tricks to play with mirrors is the corner reflector which has the property that all incident rays emerge from the corner with the $xy$ components of their wave vectors reversed, assuming the corner is bent in the $xy$ plane. A ray construction, Fig. 22, shows that this is a
Fig. 22: A corner reflector constructed from negatively refracting material, \( n = -1 \). Note that for each incident wave, the direction of the wave vector is reversed. This is not equivalent to time reversal because the rays do not return to the point of origin, but appear to radiate from point C, rotated by 180\( \text{o} \) about the corner from the true origin at A. We show that this corner reflector is ‘perfect’ in the sense that the images at B and C include the correct near field components.

good analogy: a corner made of \( \lim n \to -1 \) material also reverses the \( xy \) components of the wave vectors. This wave vector reversal is quite distinct from time reversal as can be seen from the figure: the rays are not returned to their point of origin but have a different focus. In contrast to the corner mirror, this second focus is in free space external to the corner.

The ray argument establishes the principles of the negative corner for propagating rays, but does the corner share the ‘perfect’ property of some other negatively refracting lenses? We use the powerful technique of coordinate transformation to prove that it does.

We start from the properties of the material in Fig. 22:

\[
\begin{align*}
\varepsilon(\phi) &= \mu(\phi) = +1, \quad -\frac{\pi}{2} < \phi < \pi \\
\varepsilon(\phi) &= \mu(\phi) = -1, \quad -\pi < \phi < \frac{\pi}{2}
\end{align*}
\]

(1)

then introduce a mapping of coordinates that takes the structure of figure 9 into the structure shown in Fig. 23.

\[
\begin{align*}
x &= r_0 \cos \phi e^{i\ell_0} , \\
y &= r_0 \sin \phi e^{i\ell_0} , \\
z &= Z
\end{align*}
\]

(2)
Fig. 23: The previous figure can be mapped into a stack of slabs, where every fourth slab in the stack is complementary to the other 3. A point source is included in every fourth layer as shown so that the whole system is periodic in the $\phi$ direction.

In the new frame,

$$\tilde{\varepsilon}_i = \varepsilon_i \frac{Q_\ell Q_\phi Q_Z}{Q_i^2}, \quad \tilde{\mu}_i = \mu_i \frac{Q_\ell Q_\phi Q_Z}{Q_i^2}$$  \hspace{1cm} (3)

where,

$$Q_\ell = r_0 / \ell_0 \sqrt{e^{2\ell/\ell_0} \cos^2 \phi + e^{2\ell/\ell_0} \sin^2 \phi} = r_0 / \ell_0 e^{\ell/\ell_0}$$

$$Q_\phi = r_0 \sqrt{e^{2\ell/\ell_0} \sin^2 \phi + e^{2\ell/\ell_0} \cos^2 \phi} = r_0 e^{\ell/\ell_0}$$

$$Q_Z = 1$$

$$Q_\ell Q_\phi Q_Z = r_0^2 / \ell_0 e^{2\ell/\ell_0}$$

and hence,

$$\tilde{\varepsilon}_\ell = \ell_0 \varepsilon_\ell, \quad \tilde{\varepsilon}_\phi = \ell_0^{-1} \varepsilon_\phi, \quad \tilde{\varepsilon}_Z = r_0^2 / \ell_0 e^{2\ell/\ell_0} \varepsilon_Z$$

$$\tilde{\mu}_\ell = \ell_0 \mu_\ell, \quad \tilde{\mu}_\phi = \ell_0^{-1} \mu_\phi, \quad \tilde{\mu}_Z = r_0^2 / \ell_0 e^{2\ell/\ell_0} \mu_Z$$  \hspace{1cm} (5)

Substituting from equation (48) and setting $\ell_0 = 1$ gives,

$$\tilde{\varepsilon}_\ell = \tilde{\mu}_\ell = \tilde{\varepsilon}_\phi = \tilde{\mu}_\phi = +1; \quad \tilde{\varepsilon}_Z = \tilde{\mu}_Z = +r_0^2 e^{2\ell} \quad -\frac{\pi}{2} < \phi < \pi$$

$$\tilde{\varepsilon}_\ell = \tilde{\mu}_\ell = \tilde{\varepsilon}_\phi = \tilde{\mu}_\phi = -1; \quad \tilde{\varepsilon}_Z = \tilde{\mu}_Z = -r_0^2 e^{2\ell} \quad -\pi < \phi < \frac{\pi}{2}$$  \hspace{1cm} (6)

Therefore the transformed medium satisfies the conditions of complementary media and the sources shown in Fig. 23 will each form two perfect images: one in the negative medium, and the other on the far side.

Hence it follows that the images in the corner reflector will also be perfect.
1.1.16 Surface plasmons and Bessel beams

Bessel beams are solutions of the Helmholtz equation which have well defined beam radii. For such beams, the central spot radius can be extremely narrow without diffractive spreading and physical realisations of these beams exhibit a rapid intensity drop after a fixed propagation distance. Such properties suggest applications for microwave-region Bessel beams, in covert communications and radar. In the optical region, Bessel beam applications include their use as so-called ‘optical tweezers’ and in high density recording.

Motivated by the goal of superlensing, amplification of evanescent fields using a silver film has been experimentally demonstrated. Physically, this requires exciting surface plasmons on both sides of the silver film. Whereas such attempts seek amplification over all the components of the wavevector which are parallel to the film, our research is concerned with regimes where only a narrow band of such components is transmitted. Since the transmission as a function of the wavevector is isotropic, a narrow band of transmission leads to a transmitted field which can be described by a superposition of wavevectors lying on the surface of a cone. This has been shown to be the defining characteristic of a Bessel beam.

Our simulations show that an evanescent Bessel beam with a tiny central spot size of approximately 300nm can be maintained for 750nm (Fig. 24) which may have applications in high density recording. Furthermore, this technique for Bessel beam generation can be extended into low frequency regimes by the use of metamaterials replacing the silver film. Because of the greater tunability of metamaterials, it is hoped that both evanescent and propagating Bessel beams can be generated in such regimes which meet specific technical challenges.

![Transmitted intensity versus radial distance (nm)](image)

Fig. 24: Transmitted intensity versus radial distance (nm) for transmission through a silver film in air, where film thickness is 100nm and the wavelength of light used is 827nm. The z direction is that perpendicular to the plane of the silver film.
Task 1.2: Characterization

Concerning characterization, the issues that have been addressed in the second DALHM year are:

a) Transmission studies of 1D composite metamaterials (CMMs) of SRRs and wires, and demonstration of i) true left-handed transmission peaks with high transmitted intensity, ii) the contribution of the SRRs in the electric response of the total CMM and iii) the effect of the continuous wires on the magnetic SRR resonance in a CMM.

b) Detailed study of 1D Split Ring Resonators (SRRs), including comparison between single and double ring SRRs, as well as demonstration of an electric coupling to the magnetic SRR resonance.

c) Free space refraction and phase measurements in 2D metamaterials of SRRs and wires; demonstration of negative refraction and determination of the negative refractive index (through the phase measurements).

d) Negative refraction and focusing in a 2D photonic crystal.

e) Free space transmission measurements of 100 GHz CMM structures.

f) Free space transmission measurements of 6 THz SRR structures.

g) Characterization of hexagonal and square Swiss Roll structures.

a) Free-space experimental measurements of 1D composite metamaterials (CMMs)

a.1) CMMs consisting of square SRRs and wires with a LH passband at ~ 9GHz

We have first studied 1D CMMs consisting of square SRRs and continuous wires in detail. They were fabricated using a conventional printed circuit board process, with copper patterns on 1.6 mm thick FR-4 dielectric substrates. The FR-4 board has a dielectric constant of ~4 and a dissipation factor of ~0.017 at 1.5 GHz.

![Figure 1: Schematic drawing of an in-plane CMM composed of SRRs and continuous wires on the same side of the dielectric board.](image-url)
First, CMMs with SRRs and wire structures on the same side of the boards were fabricated (in-plane CMMs, see Fig. 1). The u.c. dimensions are $a_k=5$ mm, $a_E=3.63$ mm and $a_H=5.6$ mm and 6.3 mm, and it contains one copper wire per SRR. The width of the copper wire is 0.5 mm while its thickness is 0.03 mm. The periodicity along the $H$-direction was obtained by stacking the boards. The total system consisted of $25 \times 25 \times 25$ u.c. Furthermore, variations of the above CMMs were fabricated, containing closed SRRs (SRRs without the gaps) in the place of SRRs, as well as only SRRs and only wires.

The transmission of EM waves through in-plane CMMs, SRR-only and wire-only structures, and finally CMMs containing SRRs with no gaps was measured. The transmission measurements were performed in free space, using a Hewlett-Packard 8722 ES network analyzer and microwave standard-gain horn antennas. For all measurements the wave propagation was parallel to the boards with the $E$ polarization parallel to the wires and the continuous sides of the SRRs.

Figure 2: (a) Measured transmission spectra for in-plane CMMs composed of SRRs and continuous wires (red line), SRR-only structures (green line) and wire-only structures (blue line). The latter two identify $\omega_m$ and $\omega_p$. The width of the continuous wires is 0.5 mm. (b) In-plane CMM solid red curve redrawn to show that the same threshold is exhibited (at $\omega_p'$) as the non-magnetic structure consisting of closed-SRRs and wires (magenta line). Thus the $T$ peak in the CMM curve is actually RH and not LH; the dip in the CMM $T$ spectrum corresponds to the SRR stop band as shifted due to the presence of the wires.

Figure 2(a) shows the measured $T$ spectra of SRRs-only structure (green line), wire-only structure (blue line) and of the in-plane CMM (red line), for $a_H=5.6$ mm. The SRRs-only structure shows the expected $T$ dip at $\sim 8.5$-10 GHz, corresponding to the magnetic resonance of the SRR, while the wires-only structure shows a cutoff frequency at $\sim 10.5$ GHz that corresponds to its plasma frequency, $\omega_p$. The CMM (red line) shows a $T$ peak between 8.5 and 9.5 GHz, i.e., within the frequency region of the SRRs dip. The occurrence of a CMM transmission peak within the stop bands of the
SRRs-only and wires-only structures was originally taken as clear evidence for the appearance of LH behavior. The fault with this reasoning is that it ignores the effect of the SRRs on the electric response of the CMM.

We demonstrate here experimentally this effect by closing the gaps of the SRRs, switching thus off the magnetic SRR resonance, and thus monitor only the electric response of the CMM. (The changes in the electric response of the CMM from the closing of the gaps are expected to be weak, since only a small amount of metal is added (in the gaps of the SRRs), while the symmetry is preserved.) As it can be clearly seen from Fig. 2(b), the $T$ spectrum for the metamaterial with closed SRRs (magenta line) is almost the same with that of the ordinary CMM (red line), with the main difference being the absence of the dip at $\sim 10$ GHz, which indicates that this dip is due to the SRRs magnetic response. Moreover, it is observed that the combination of the wires with the SRRs shifts the effective plasma frequency, $\omega_p \,'$, of the CMM to $\sim 8.5$ GHz, which is lower than the plasma frequency of the wires-only array ($\omega_p \sim 10.5$ GHz). Hence, we must conclude that in the CMM $\varepsilon>0$ for $f>8.5$ GHz; consequently, there should be no LH peak for $f>8.5$ GHz. Therefore, the peak observed at $\sim 9$ GHz ought to be RH. On the other hand, based on the same arguments one should expect a dip and not a peak at $\sim 9$ GHz, since in this region $\varepsilon>0$ and $\mu<0$ (due to the SRRs resonance). To explain this apparent discrepancy we examined the influence of the wires on the magnetic resonance frequency, $\omega_m$, of the SRRs. We revealed numerically, using Finite Difference Time Domain calculations, that the currents in the wires tend, on the average, to diminish the magnetic flux on the SRRs and thus to decrease their effective inductance ($L_{\text{eff}}$); the net result is an increase of the magnetic resonance frequency, $\omega_m \sim 1/(L_{\text{eff}} C_{\text{eff}})^{1/2}$ ($C_{\text{eff}}$ is the effective capacitance). In our case, this led to a shift of the resonance frequency from $\omega_m \sim 8.5$ GHz to $\omega_m \,' \sim 9.5$ GHz, explaining the peak at $\sim 9$ GHz as RH and accounting for the dip in the CMM $T$ spectrum at $\sim 10$ GHz (as $\varepsilon>0$ and $\mu<0$). To further check this interpretation we placed the wires off-plane and symmetrically with respect to the SRRs. In this case the currents in the wires have no net effect on the magnetic flux in the SRRs, and consequently we expect $\omega_m$ not to be influenced by the wires. Our microwave measurements for off-plane CMM structures confirmed this expectation.
Figure 3: (a) Measured transmission spectra for in-plane CMMs (red line), SRR-only structures (green line) and wire-only structures (blue line). The width of the continuous wires has been increased to 1 mm. (b) The coincidence of the CMM solid curve with the closed-SRRs plus wires (magenta) curve determines the effective plasma frequency, $\omega_p'$, which is now well above the CMM $T$ peak, identified as left-handed. For obtaining a real LH peak it is necessary $\omega_p'$ to be larger than $\omega_m'$. To achieve this we doubled the width of the copper wires to 1 mm. The $T$ data for the new-developed in-plane CMM are shown in Figs. 3a and 3b. It is shown that $\omega_p'$ is now at ~10.4 GHz (magenta line in Fig. 3b). In this case $\omega_p'$ is clearly higher than $\omega_m'$ and thus the observed CMM peak (see red line in Figs. 3(a) and 3(b), centered at ~9.5 GHz, is really LH. Notice also that the presence of the in-plane wires leads indeed to a shift of the magnetic resonant frequency from $\omega_m' \approx 8.5$ GHz to $\omega_m' \approx 9.2$ GHz.

We finally tried to improve the transmittance of the LH peak by considering off-plane CMMs (theoretical results indicate higher transmittance for off-plane than for in-plane cases). In this case, the wires are printed on the backside of the boards opposite to the gaps of the SRRs. The dimensions of the unit cell are $\alpha_k = 5$ mm, $\alpha_k' = 3.63$ mm and $\alpha_H = 5.6$ mm, 3.1 mm, and 2.6 mm. The total system consists again of 25 25 25 unit cells. The $T$ spectrum for the off-plane CMM with $\alpha_H = 3.1$ mm is shown in Fig. 4. A well-defined peak can be clearly observed at ~9.5 GHz. The closing of the gaps of the SRRs demonstrates that $\omega_p'$ is at ~10.5 GHz. Since the SRRs-only $T$ dip is at 8.5-10 GHz, it is apparent that the observed peak is LH.

Figure 4: Measured transmission spectra for off-plane CMMs composed of SRRs and continuous wires (red line), SRR-only structures (green line) and structures consisting of closed-SRRs and wires (magenta line). The width of the continuous wires is 1 mm. The frequencies $\omega_m' = \omega_m'$ and $\omega_p'$ are well separated and the CMM peak is clearly LH.

To further increase the transmittance of the LH peak of Fig. 4 we varied several parameters. We first varied the width of the wires in order to shift $\omega_p'$ to higher frequencies and get a pronounced LH peak. Thus, we present data for a CMM with 10 u.c. in the propagation direction and wires’ width of 1, 1.2 and 1.5 mm (Fig. 5).
Furthermore, we decrease the number of unit cells along propagation direction. We studied the transmittance through the CMM for 3, 6, 10 and 25 u.c. (see Figs. 6(a), (b)). We finally show the transmittance spectra taken for CMMs with a distance between the rings of 0.33 and 0.2 mm (Fig. 7). We see that increasing the width of the wires leads to a shift of $\omega_p$ to higher frequencies but simultaneously to a decrease of the transmission due to losses introduced by the metal. However, fine-tuning of the wires’ width results in optimising the transmittance of the LH peak. Decreasing the number of u.c. in the propagation direction leads to a noticeable increase of the LH band transmittance due to diminishing the losses caused by the boards. Finally, a decrease of the distance between the inner and outer rings of the SRRs suppresses the transmittance of the LH peak since losses are enhanced in the region between the rings.

Figure 5: Measured transmission spectra for off-plane CMMs composed of SRRs and continuous wires with a width of 1 mm (blue line), 1.2 mm (red line) and 1.5 mm (green line). In all three cases there are 10 u.c. in the propagation direction. It can be seen that increasing the width of the wires leads to a shift of $\omega_p$ to higher frequencies and thus to a more pronounced LH peak since $\omega_m$ is almost constant, but simultaneously the transmittance of the LH peak decreases due to losses induced by the wires.
Figure 6: Measured transmission spectra for off-plane CMMs composed of SRRs and continuous wires with (a) 3 u.c. (red curve), 10 u.c. (magenta curve), and 25 u.c. (blue curve) in the propagation direction. The width of the wires is 1mm; (b) 6 u.c. (green curve), and 10 u.c. (magenta curve) in the propagation direction. In this case the wires’ width is 1.2 mm. We can observe that decreasing the number of u.c. in the propagation direction leads to an increase of the LH peak’s transmittance.

Figure 7: Measured transmission spectra for off-plane CMMs composed of SRRs and continuous wires with 10 u.c. in the propagation direction. The green curve shows the spectrum for SRRs with a distance between the rings of 0.33 mm while the red curve for 0.2 mm. It can be seen that decreasing the distance between the rings suppresses the transmittance of the LH peak.

In conclusion, we have shown experimentally for the first time that the effective plasma frequency, $\omega_p'$, of the CMM composed of SRRs and continuous wires is lower than the wires-only plasma frequency, $\omega_p$. We have also demonstrated how to obtain experimentally an accurate value for $\omega_p'$. Furthermore, we showed that the in-plane wires, as opposed to the off-plane configuration, push the magnetic resonance frequency, $\omega_m'$, to a slightly higher value, $\omega_m'$. Finally, we tried to improve the transmittance of the observed LH peak by varying the wires’ width, the number of u.c.
in the propagation direction, and the distance between the rings. Increasing the wires’
width leads to a decrease in the transmittance due losses induced by the wires.
Decreasing the distance between the rings causes a drop in the transmission due to the
enhanced losses in the area between the inner and outer rings of the SRRs. On the
contrary, one can significantly enhance the transmittance of a LH peak by reducing
the number of u.c. in the propagation direction.

a.2) CMMs consisting of circular SRRs and wires with a LH passband at ~ 4
GHz.

The circular SRR (Fig. 8(a)) and CSRR (Fig. 8(b)) units are fabricated on circuit
boards (FR4) with a copper deposited layer of thickness 30 µm by etching. The
geometrical parameters of the SRR are \( d = t = 0.2 \) mm, \( w = 0.9 \) mm and \( r = 1.6 \) mm as
shown in Fig. 8(a). The circuit board has a thickness 1.6 mm and dielectric constant of
\( \varepsilon = 4.4 \). SRR units are arranged periodically with 5, 15, and 18 number of unit cells in
\( x \), \( y \) and \( z \) directions, respectively. The dimensions of the unit cell containing a single
SRR are \( a_x = a_y = 8.8 \) mm, and \( a_z = 6.5 \) mm. Transmission measurements are
performed in free space where, unlike a waveguide measurement, no restriction on the
size of the structures is imposed. Experimental set-up consists of a HP 8510C network
analyzer, and a set of microwave horn antennas. The incident field propagates along
the \( x \) direction with \( E \), and \( H \), along \( y \) and \( z \) directions, respectively.

Figure 8: Schematics: (a) split ring resonator (SRR) (b) a ring resonator with splits closed
(CSRR) (c) Periodic CMM composed of SRRs on one side, wires on the other side of dielectric board.

Figure 9: Measured transmission spectra of a periodic SRR medium (solid line) and periodic CSRR medium (dashed line) between 3-14 GHz.

Figure 9 shows the measured transmission spectra of periodic SRRs (solid line) and
CSRRs (dashed line) between 3-14 GHz. The first bandgap (3.55-4.05 GHz) of the
SRR medium is not present in the CSRR medium, indicating \( \mu < 0 \). The second
bandgap (8.1-11.9 GHz) is present for both the SRR medium and CSRR medium.
This measurement clearly shows that the stop bands of an SRR medium cannot be
automatically assumed as “negative \( \mu \)” behaviour. Some of the observed gaps could
also originate from the electrical response of the SRRs or from Bragg gaps due to
periodicity.
It was recently found (see first annual DALHM report) that the SRRs, in addition to their resonant magnetic response at $\omega_m$, exhibit a resonant electric response at $\omega_o$. The behavior is similar to that of a periodic cut-wire medium (wires of finite length), which exhibits a stop band with a well-defined lower edge due to the discontinuous wire geometry. As a result, the SRRs contribute to the effective permittivity of the CMM, causing a downward shift on the plasma frequency determined solely from wire structures. To demonstrate this effect, a CMM consisting of periodic alternating layers of CSRRs and wires is used (Fig. 8(c)).

Figure 8(c): Thickness, length and the width of the wires are 30 µm, 13.5 cm and 0.9 mm respectively. Figure 10 displays the measured transmission spectra of wire only medium and CMM consisting of CSRR and wire layers. The $\omega_p$ of the wire-only structure around 8 GHz, is reduced down to 5.3 GHz within the closed CMM structure. As seen in Fig. 10, $\omega_p$ of the CMM is lower than that of the wire-only medium alone. It is crucial to determine whether the shift in plasma cut-off frequency covers the magnetic resonance gap, which would render the CMM as a right-handed medium.

Figure 10: Measured transmission spectra of wires (dashed line) and closed CMM (solid line) composed by arranging closed SRRs and wires periodically.

Figure 11: Transmission spectra of SRRs (red dashed line), wires (blue dotted line) and CMM (solid line).

We have designed the present SRRs such that the first bandgap of SRR structure between 3.55 - 4.05 GHz is not obscured by this shift. The CMM structure is made of $N_x = 5$, $N_y = 15$, and $N_z = 24$ unit cells. Each unit cell has a single SRR, and a copper wire from stacked SRR and wire layers, with lattice spacings $a_x = a_y = 8.8$ mm, $a_z = 6.5$ mm. The transmission spectra for SRR only (solid line), wire only (dashed line) and CMM (bold solid line) periodic structures are displayed in Fig. 11.

The CMM structure allows propagation of EM waves between 3.6 and 4.1 GHz, where both $\varepsilon$ and $\mu$ are negative. The CMM pass band exactly coincides with the stop band of SRR. The transmission peak at 3.9 GHz is -1.2 dB, which is a significantly high value for a material made of metals. We stress that a similar transmission band is not present for a CMM composed of CSRRs and wires (Fig. 10). The electric response contribution of SRRs is also evident here: If the $\omega_p$ of the wire-only structure (dashed line in Fig. 11(a)) were used to identify the $\varepsilon < 0$ regime for the CMM, the transmission between 5.3 – 8 GHz would have occurred in a regime with $\varepsilon < 0$ and $\mu$
> 0, which is not possible. However, as shown in Fig. 10, the \( \varepsilon > 0 \) regime of the combined electric response of SRRs and wires starts at 5.3 GHz. These results are reported in Ref. 2.

Next, we investigate the effect of misalignment of SRR structures on their transmission spectrum. The misalignment can be introduced along the propagation direction, by shifting the SRR layers arbitrarily as shown in Fig. 12(c), or in the lateral direction, by changing the distance between successive layers arbitrarily, as shown in Fig. 14(b).

![Figure 12](image1.png)  
**Figure 12**: (a) Measured transmission spectra of aligned (black line) and slightly misaligned (red line) SRR structures. (b) Top view of aligned SRRs. (c) Top view of slightly misaligned SRRs.

![Figure 13](image2.png)  
**Figure 13**: (a) Measured transmission spectra of aligned (black line) and hardly misaligned (red line) SRR structures. (b) Top view of hardly misaligned SRRs.

![Figure 14](image3.png)  
**Figure 14**: (a) Measured transmission spectra of periodic (black line) and non-periodic (red line) SRR structures. (b) Front view of non-periodic SRRs.

![Figure 15](image4.png)  
**Figure 15**: Measured transmission spectra of empty PCB boards.

Fig. 12(a) shows the measured transmission spectra of aligned (Fig. 12(b)), and longitudinally misaligned (Fig. 12(c)) SRR boards. No significant change occurs as a result of this misalignment. Even in the exaggerated case depicted in Fig. 13(b), where the misalignment is larger than the period and the interfaces are completely randomized to the extend comparable to the wavelength, the SRR resonance persists as can be seen in Fig. 13(a). Similarly, the misalignment along the lateral direction, which, in fact, by destroying the lateral periodicity generates a non-periodic SRR medium, does not have a profound effect on the SRR resonance (Fig. 14(a)). Overall,
the misalignment does not impose tight restrictions on the stacking of SRR structures, provided that it remains smaller than the wavelength of the EM field propagating in the medium.

Finally, the transmission of all-dielectric (no metal) FR4 boards is measured in order to investigate the absorption in the dielectric substrate. A bulk dielectric medium is obtained by combining 20 boards, with a thickness of 3 cm. in the propagation direction. The transmission spectrum shown in Fig. 15 has an average around -2.5 dB between 3-7 GHz (~60% transmission). The reflection is obtained by the formula \( R = \frac{(n-1)^2}{(n+1)^2} \) around ~15% \((n = 2.15\) for FR4 boards). Therefore, ~25% of the signal is absorbed by the PCB boards. We note that the actual CMM structure has a FR4/air ratio 1/4, therefore absorption is around 5%, in agreement with our CMM transmission measurements.

b) Detailed study of 1D Split ring resonators.

b.1) Coupling of the incident electric field to the magnetic resonance of the SRRs.

For this study, a CMM consisting of only SRRs was fabricated. The geometrical parameters of the SRR are \( w = d = t = 0.33 \) mm and \( l = 3 \) mm. The CMM was then constructed by stacking together the SRR structures in a periodic arrangement. The u.c. contains one SRR and has the dimensions 5 mm (parallel to the cut sides), 3.63 mm (parallel to the continuous sides), and 5.6 mm (perpendicular to the SRR plane).

The transmission measurements were performed in free space on a CMM block consisting of 25×25×25 u.c. We considered the four nontrivial orientations of the SRR, which are shown in Fig. 16.

![Figure 16: Left-hand side: SRR geometry studied. Right-hand side: The four studied orientations of the SRR with respect to the triad \( k, E, H \) of the incident EM field.](image)

Figure 16: Left-hand side: SRR geometry studied. Right-hand side: The four studied orientations of the SRR with respect to the triad \( k, E, H \) of the incident EM field.

Figure 17 presents the measured transmission spectra, \( T \), of the CMM. The black line corresponds to the conventional case shown in Fig. 16(a), with \( H \) perpendicular to the SRR plane and \( E \) parallel to the symmetry axis of the SRR. Notice that \( T \) exhibits a stop band at ~8.5-10 GHz, due to the magnetic resonance. The red line shows \( T \) for the orientation of Fig. 16(b); here \( E \) is no longer parallel to the symmetry axis of the SRR and thus there is no longer mirror symmetry of the combined system of SRR plus EM field. Notice that now \( T \) exhibits a much wider stop band (8 to 10.5 GHz), starting at lower frequency. Very interesting results are obtained by comparing \( T \) for
the two cases shown in Figs. 16(c) and 16(d), for which there is no coupling to the external magnetic field since \( \mathbf{H} \) is parallel to the SRR plane. For the case of Fig. 16(c), where \( \mathbf{E} \) is parallel to the symmetry axis, no structure is observed around the magnetic resonance frequency line (green line in Fig. 17), as expected. However, for the case of Fig. 16(d), where the SRR plus EM field exhibit no mirror symmetry, a strong stop band in \( T \) around \( \omega_m \) is observed (blue line), similar to that of the conventional case, that of Fig. 16(a). This strongly suggests that the magnetic resonance can be excited by the electric field provided that there is no mirror symmetry. The effect is explained in more detail in Sect. 1.1.3, Fig. 1.

![Measured transmission spectra of a lattice of SRRs for the four different orientations shown in Fig. 16.](image)

In summary, it was found that the incident electric field couples to the magnetic resonance of the SRRs lattice, provided it is kept parallel to the wires of the SRRs with the gaps. There is an excellent agreement between theory and experiments (see WP1, theory). This unexpected electric coupling to the magnetic resonance of the SRRs is of fundamental importance for designing LHMs in higher dimensions.

b.2) **Free space transmission measurements on single ring SRRs.**

We have studied both experimentally and theoretically (see WP1-theory, Sect. 1.1.6) the propagation of EM waves through single split ring resonators. They also show a dip in the transmission spectrum, which is due to negative \( \mu \). \( \omega_m' \) is, for the same geometry of the single ring with the outer ring of a conventional SSR, shifted slightly to higher frequencies. One can also notice that for one-ring structures the transmission dip is somewhat shallower than for the conventional two-ring structures.
c) Free-space measurements of negative refraction and phase shift for 2D CMMs.

The 2D CMM in this work is a square lattice of FR4 boards with SRR and wire structures on the respective sides, as shown in Fig. 19(a) (we refer the reader to Sect. a.2 for the geometrical parameters of the SRR and wire elements). The shaded part is the unit cell consisting of two SRRs and two wires. The 2D CMM structure is made of $N_x = 5$, $N_y = 20$, and $N_z = 40$ unit cells, with lattice spacings $a_x = a_y = a_z = 9.3$ mm, respectively. The transmission measurement setup is the same as described in Section a.2, above.
Experimental setup used for refraction experiment. 2D CMM medium (bold solid line) between 3-7 GHz.

Figure 20 shows the measured transmission spectra of periodic SRRs, wires and 2D CMM between 3-7 GHz. The 2D CMM structure allows propagation of EM waves between 3.73 and 4.05 GHz, where both \( \varepsilon \) and \( \mu \) are negative. The CMM pass band exactly coincides with the stop band of SRR. The transmission peak at 3.92 GHz is -10.2 dB, which is a significantly high value for a material made of metals. The losses are mainly rising from the circuit boards that are perpendicular to the propagation direction, since for 1D CMM transmission peak is measured to be -1.3 dB (see Sec. a.2).

For the negative refraction measurements, a prism shaped 2D CMM of angle 26° (Fig. 19 (b)) is used. The minimum and maximum number of unit cells at the propagation direction are 3 and 19, respectively. The average thickness of the wedge is 10.5 cm. The refraction spectrum is measured by the setup depicted in Fig. 19 (c). The antenna mounted on the rotating arm detects the angular distribution of the transmitted EM wave through the wedge.

Figure 22: Unwrapped phase data obtained from different number of CMM layers varying from 5 to 9 in the propagation direction.

Figure 23: Average phase difference between consecutive number of layers of CMM a) between 3.73- 4.05 GHz, where left-handed transmission peak occurs. b) between 5.4-7 GHZ, where right-handed transmission peak takes place.
Figure 21(a) displays the transmission spectra as a function of frequency and refraction angle. The angle is scanned by $\Delta \theta = 2.5^\circ$ steps, while the frequency is swept from 3.73 GHz to 4.05 GHz in 400 steps. It is evident that the transmitted beam refractions to the negative side of the normal of the oblique interface. We stress that the frequency range corresponds to the left-handed pass band of the CMM. The angular cross section at $f = 3.92$ GHz is plotted in Fig. 21(b). The incident field has a Gaussian beam profile centered at $x = 0$ (not shown on figures). By applying a Gaussian profile fit to the lateral intensity distribution and employing Snell’s Law, an effective refractive index may be defined for the CMM. By the formula, $n_{\text{CMM}} \cdot \sin \theta_i = n_{\text{air}} \cdot \sin \theta_r$, where $\theta_i$ is the angle of incidence (equal to the wedge angle), and $\theta_r$ is the angle of refraction. At $f = 3.92$ GHz, we determine $\theta_r = 55^\circ$, hence, $n_{\text{eff}} = -1.86$.

Theoretical studies indicate that the phase velocity of an EM wave incident on a $\varepsilon < 0$, $\mu < 0$ medium reverses its sign at the interface [C.G. Parazzoli et al., PRL 90, 107401 (2003)]. It is well known that the phase delay of the EM wave propagating within a natural medium ($n > 1$) increases proportional to the thickness (i.e., length of the medium along the propagation direction). In contrast, this dependence should be reversed in a CMM exhibiting left-handed behaviour, that is, phase delay should decrease with increasing thickness. The CMMs are periodic media, therefore we can modify the thickness in a discrete manner, by the number of layers. Hence, we devise the following experiment to detect the presence of negative phase velocity: We measure the phase delay spectrum of a CMM within the left-handed transmission band, as a function of the number of layers along the propagation direction. The phase data is then unwrapped to obtain the dispersion curve for each CMM of different thickness. Figure 22 displays the phase dispersion for CMMs having number of layers successively from 5 to 9, respectively (total thickness from 38.8 mm to 74 mm). It is evident from the ordering of the curves that the phase decreases with increasing CMM thickness. Figure 23(a) shows the negative average phase shift between consecutive number of layers of CMM. In contrast, the average phase shift in the frequency range, where the same CMM has a right-handed transmission band is positive (Fig. 23(b)).

Using the definition $v_{ph} = c/n = \omega/k$ for the phase velocity, and $k = \Delta \Phi / \Delta L$ in terms of the phase difference and thickness difference between two CMMs with consecutive number of layers, we obtain the index of refraction as:

$$n = \frac{\Delta \phi}{\Delta L} \cdot \frac{c}{\omega} \cdot \frac{1}{\varepsilon} \cdot \frac{1}{\mu} \cdot \frac{n_{\text{air}}}{n_{\text{CMM}}} \cdot \sin \theta_i \cdot \sin \theta_r \cdot \frac{1}{\sin \theta_i} \cdot \frac{1}{\sin \theta_r} \cdot \frac{1}{n_{\text{CMM}}}.$$

(Eq. 1)

At $f = 3.92$ GHz, the measured parameters $\Delta \phi = -0.45 \pi$, and $\Delta L = 8.8$ mm, gives $n_{\text{eff}} = -1.95$, which is close to the value -1.86 obtained from the refraction data of the wedge experiment.

d) Negative refraction and focusing in a 2D photonic crystal.

d.1) Spectral analysis of negative refraction through a 2D dielectric photonic crystal.
In this study, following the work of the article PRL 90, 107402 (2003), by S. Foteinopoulou et. al., we have utilized a TE polarized upper band of a 2D dielectric photonic crystal (PC) (see Sect. 1.1.13) to obtain a medium $n_{\text{eff}} < 0$. The negative refraction of a Gaussian beam incident on a rectangular slab of PC at various incidence angles, and the focusing of a point source through the slab are successfully demonstrated experimentally and by FDTD simulations. The PC in this study is a hexagonal lattice of alumina rods in air. Lattice period is $a = 4.79$ mm. The rods have dielectric constant $\varepsilon = 9.61$, diameter $2r = 3.15$ mm, and length $l = 15$ cm. The schematic on the right describes the experimental setup: Assuming a 2D x-z coordinate system, the PC is located along the x-axis, centered at $x = 0$. The EM waves are emitted from an antenna, located on the $x<0$ side, incident on the crystal interface at $x = 0$. A monopole antenna scans the refracted field along the other interface. A HP8510C network analyzer is used to measure the transmission. Figure 24 summarizes the results for the negative refraction measurements:

![Figure 24](image.png)

**Figure 24:** (a) Spectral plot of the negative refraction at various incidence angles. Measured (b) and simulated (c) lateral field profiles at 41.7 GHz for the respective incidence angles. Solid curves denote the fitting Gaussian profiles. Incident beam has a Gaussian profile centered at $x = 0$ (not shown on figures). Measured data is a horizontal slice of the spectral plots given on the left, at 41.7 GHz.

Let us first comment on the spectral refraction maps. The relevant TE band of the PC extends from 40.65 to 46.27 GHz. Our antennas can measure up to 44 GHz. It is evident that the transmitted beam appears on the negative $x$ part of the interface, indicating negative refraction. The shift of the beam to the left with increasing
incidence angle is anticipated from Snell’s Law. In general, $n_{\text{eff}}$ is a function of frequency and the wavevector, i.e., $n(k, f)$. Here, the spatial broadening of the refraction pattern reflects the frequency dependent anisotropy. Nevertheless, in contrast to the large frequency band (~3 GHz) for negative refraction, the broadening is small. On the other hand, changing the incidence angle (i.e., $k$-parallel) at a particular frequency has profound effects (Fig. 24 (b), (c)). The shift of the beam is accompanied by a decrease in amplitude and broadening which are induced by the $k$-dependent anisotropy. In particular, the coupling to the photonic band decreases with increasing angle, and the loss and spreading of the beam increases with increasing path traversed within the crystal.

In order to verify that a single beam propagates within the crystal, we performed a time domain simulation of the structure for $30^\circ$ incidence angle at $f = 41.7$ GHz. As can be seen in Fig. 25, the incident beam gets higher order (mark 3) reflections at the interface. This is anticipated, since the condition $fa/c < a/2a_{\text{interface}}$ is not fulfilled (here, $a$ and $a_{\text{interface}}$ are the lattice period and interface period, respectively). The refracted beam, however, appears to be single, which is also suggested by the single transmitted beam on the other side of the PC. It has been shown theoretically that single beam refraction at higher bands is possible. Therefore, we conclude that most of the propagating power is coupled to the $0^{\text{th}}$ order diffracted wave, and employ Snell’s law; $n(f, k_i) \sin \theta_i = n_{\text{air}} \sin \theta_i$, where $\theta_i$ is the angle of incidence and $\theta_f$ is the angle of refraction inside PC. Using lateral field profiles in Fig.23, for $\theta_i = 15^\circ$, $30^\circ$, and $45^\circ$ we obtain $n_{\text{eff}} = -0.52$, -0.66, and -0.86, respectively. The simulation results for the same incidence angles give $n_{\text{eff}} = -0.66$, -0.72, and -0.80, respectively.

d.2) Focusing properties of the 2D photonic crystal.

The presence of negative refraction for large range of incidence angles brings the possibility that a PC slab structure may act like a lens for an omni-directional source. For the present PC, we first performed FDTD simulations for a TE polarized point source at $f = 42.07$ GHz, located at $d_{\text{src}} = 2\lambda$ away from air-PC interface. We stress that, in previous studies, the source is always located in the vicinity of the PC interface ($d_{\text{src}} < \lambda$). Figure 26(b) shows the resulting spatial intensity distribution in

**Figure 25:** Simulated negative refraction of a plane wave at $f = 41.7$ GHz incident at $\theta = 30^\circ$ to the PC interface (mark 1). Zero order (mark 2) and higher order (mark 3) reflections occur. The refracted (mark 4) the transmitted (mark 5) components appear to propagate as single beams.
the image plane. The PC-air interface is located at \( z = 0 \). The peak indicates focusing behaviour unambiguously (we also refer the reader to Fig. 27(a) for the simulated 2D field map, where the convergence of the wave fronts along the optical axis is clearly visible). We would like to emphasize that the focusing occurs away from the PC-air interface, with peak intensity of \( \sim -5 \text{ dB} \), observed at \( z \approx 8\lambda \). This is a very high value reported so far concerning focusing through PC structures. In this respect, channeling induced focusing effects are excluded, which occur close to the interface.

![Figure 26: Point focusing: (a) Lateral intensity profiles measured at six different positions along the propagation direction: \( z/\lambda = 1.78, 3.56, 5.34, 7.12, 8.90, \) and 10.68 for a point source located at \( d=2\lambda \) away from the interface. PC interface is at \( z=0 \). (b) Corresponding simulated 2D intensity in the image plane.](image)

In the experiment, a waveguide aperture and a monopole antenna is used as source-detector pair. For \( d_{\text{src}} = 2\lambda \), lateral intensity profiles at several \( z \) around the peak position are measured. In Fig. 27 (a), the focusing of the beam both in lateral and longitudinal directions is evident, with the maximum intensity at \( d_{\text{focus}}/\lambda \approx 8 \). We note the similarity of measured and simulated field patterns. The focusing has a longitudinal extend as well, which is induced by the anisotropy of \( n_{\text{eff}} \), and that \( n_{\text{eff}} \) deviates from the “perfect value” of negative unity.

When the source is shifted laterally or longitudinally, the field pattern mimics the shift, which is a clear indication of flat lens behavior. In Fig. 27 (a), we show the effect of longitudinal shift. When the source is moved from \( d_{\text{src}} = 2\lambda \) to \( d_{\text{src}} = 4\lambda \) away, the focus pattern on the other side shifts towards the PC-air interface. In Fig. 27 (b), the measured lateral profiles at focus points for various \( d_{\text{src}} \) are plotted. Interestingly, we observe that \( (d_{\text{src}} + d_{\text{focus}}) \) remains roughly constant. In the absence of the PC, the free space propagation at the focal distance has no features indicating the source location, which contrasts the drastic enhancement by focusing.
Figure 27(a): Simulated 2D field distribution $H_y(x,z)$ for $d_{sc} = 2\lambda$ (top) and $d_{sc} = 4\lambda$ (bottom). Source is located on the left.

Figure 27(b): The measured lateral intensity profiles at respective focal distances for different source distances. The intensity axis is normalized by the $d_{sc} = 2.0\lambda$ profile peak. The dashed line denotes the intensity profile in the absence of PC for $d_{sc} = 2\lambda$.

Finally we present results for the effect of PC thickness on the focusing pattern. For a fixed source distance of $d_{sc} = 2\lambda$, three PC slabs with number of layers 7, 9, and 11, respectively, are simulated and the field profile in the focusing plane is obtained. In Fig. 28, we observe that with increasing PC thickness, the focal pattern shifts away from the PC. To the extent of the isotropy of $n_{eff}$, this is completely expected behavior. For thick PC, the beam propagates within the PC to a wider extend until reaching the second interface. Upon leaving this interface, it then requires a longer path to focus. Having said this, we refrain to commit a geometric optic analysis using optic rays. The reason is the apparent anisotropy of the effective refractive index, and the fact that the EM wave propagates through the PC via Bloch modes and not as plane waves. Thus, a crossover (focal) point in the PC cannot be identified in these field maps. Again, what we see here is that the EM wave emerging at the second interface acquires the proper phase delays so that the resulting wave fronts converges along the optical path.

Figure 28: Field patterns for a point source at $d_{sc} = 2\lambda$ through PC slabs of thickness 7, 9, and 11 layers, respectively.
e) Transmission measurements of 100 GHz composite metamaterials.

We use the 100 GHz setup described in 1st year report, which is the same as in previous measurements. First, the pure SRR and pure wire samples are assembled in order to investigate the respective magnetic and dielectric responses in the transmission spectra. We note that the transmission CMM composed of CSSR and wire structures also have to be measured in order to determine the total dielectric response of the CMM. The fabrication of CSSR structures is currently under progress.

Figure 29: Measured transmission spectra of a pure-SRR sample. The dip around ~92.6 GHz is due to the magnetic
When stacked, the resulting pure SRR sample has $30 \times 30 \times 10$ unit cells along the lateral and propagation directions, respectively (the same applies for the pure-wire sample). In Fig. 29, the transmission spectrum of the pure SRR sample is shown. A strong resonance dip around $\sim 92.6$ GHz is observable. We note that this value matches the predictions derived from scaling arguments.

Currently we are working on the wire and CMM structures to demonstrate the left-handed transmission peak at 100 GHz. The measurements will be extended to other structures with larger number of layers along the propagation direction. In addition we are planning to include waveguide-to-waveguide measurements to increase the signal to noise ratio, which is crucial at this frequency regime.

f) Free space transmission measurements of 6 THz Split Ring Resonator structures.

During the second year of the project we fabricated (see WP2) and tested single SRR structures in the THz frequency region. The SRR size is $5 \times 5 \mu m^2$ while the u.c. dimensions are $7 \times 7 \times 5 \mu m^3$ (see Sect.1.1.7 for the detailed structure parameters). The transmission measurements were taken using a Bruker IFS 66v/S FT-IR spectrometer and a polarizer at the frequency range of 100-350 cm$^{-1}$. The results obtained for propagation perpendicular to the SRR plane are presented in Fig. 30 for both polarizations (see orientations (c) and (d) in Fig. 16). One can readily observe that for the polarization with the electric field parallel to the gap bearing sides of the SRRs (orientation (d) in Fig. 16), the transmission spectrum shows a gap at $\sim 5.5-8$ THz (Fig. 30, red curve), which is due to the electric coupling to the magnetic resonance of the SRRs as described in Section b.1. Thus, we demonstrate the occurrence of negative $\mu$ for the particular SRR design also in the THz region. One the other hand
for the polarization with $\mathbf{E}$ parallel to the SRR-sides with no gaps (orientation (c) in Fig. 16), we can notice $\omega_0$ at ~8.5 THz (Fig. 30, blue curve). There is in both cases a good agreement with the theoretical results.

**Figure 30:** Measured transmission spectra for SRRs with 5×5-μm$^2$ size. In both cases the propagation of EM waves is perpendicular to the SRR plane. The red curve shows $T$ for a polarization with $\mathbf{E}$ parallel to the sides of the SRRs with gaps, while the blue curve shows $T$ for $\mathbf{E}$ parallel to the sides with no gaps.
g) **Hexagonal and square Swiss Roll structures.**

A particular benefit of proving LHMs in the radio frequency (RF) regime is that we operate in the extreme near-field limit: the wavelength of the RF field is ~10 meters, whereas the scale of the metamaterials (i.e. the unit cell spacing) is ~10 mm. In this regime, the electric and magnetic field components of the RF radiation are effectively decoupled, so to manipulate the magnetic field, $H$, we need only control the permeability, $\mu$, of our materials. Working in this limit therefore simplifies both the construction of the material and the interpretation of its effects: we use the magnetic metamaterial in the RF as an exemplar for future applications of LHMs.

Accordingly, the emphasis of our effort in this workpackage has been to develop and characterise magnetic metamaterials for RF operation. Last year, we built an RF “lens” (or, more accurately, a “faceplate”, see Fig. 31), which was used to demonstrate geometry-preserving flux ducting in a 0.5 tesla MRI machine operating at 21.3 MHz.

![Figure 31](image)

**Figure 31:** (a) A typical roll used at 21.3 MHz, (b) Real (red line and axis) and imaginary (blue line and axis) parts of the permeability of a Swiss Roll as a function of frequency, and (c) the assembled hexagonal prism sample

This year, we fully characterised the material in two forms, the hexagonal prism from last year and also configured as a square prism, by measuring the RF field distribution obtained from a small dipole source, across the frequency range where the material has a magnetic response. These data have been interpreted using the effective medium model in three ways. Initially we correlated the observed resonances to specific permeability values; secondly we have developed an analytical theory of the square prism based on a waveguide model, and finally we have carried out numerical simulations using the Microwave Studio software package.

**The Hexagonal Prism**

As described in the previous report, the hexagonal sample was characterised using equipment at Oxford, where all three transmitted field components were measured. Initially, we concentrated on just the axial field $H_z$, and interpreted the results on resonance and at the magnetic plasma frequency. These were presented last year. We have now extended our analysis to the regime between these frequencies, where the permeability is negative. We have used both the axial and in-plane field ($H_i = \sqrt{(H_x)^2 + (H_y)^2}$) measurements to determine the nature of the field behaviour, and hence to interpret it using the continuum theory that we developed previously.
The magnetic plasma frequency, $\mu \to 0$

When $\mu_z = 0$, the dispersion relation

$$k_z^2 = \mu_z k_0^2 - \frac{\mu_z}{\mu_\|} k_{\|}^2$$  \hspace{1cm} (1)

demands that $k_z \to 0$, so that only the uniform Fourier component of an incident field can be propagated through the material. For a slab of infinite lateral extent, we expect that a point source would give rise to a plane wave. For a finite slab, however, there are boundary conditions at the edge of the slab, which impose $H_z = 0$ at the corners. Thus the field pattern is expected to have a broad plateau of $H_z$ in the centre, bounded by $H_z = 0$ at the periphery, whereas $H_r$ will be zero at the centre, but grow to a maximum close to the boundaries of the prism. This is indeed the field pattern that we observe in Fig. 32, and by analysing the field directions, we find that the overall field vector rotates through $\frac{1}{2}$ turn across the face of the prism.

**Negative permeability regime, $\mu < 0$**

With reducing frequency from the $\mu = 0$ value, we find the first resonant pattern lies at 24.75 MHz (see Fig. 33). On examining the two field components shown in Fig. 33, we see the clear evidence of a standing wave pattern. The overall field vector is purely axial at the centre, as defined by the excitation, and purely radial at the boundaries of the prism, as defined by the boundary conditions. Between these limits, the vector rotates, so that it is also purely radial 50 mm from the centre and purely axial again 80 mm from the centre, thus going through $1\frac{1}{2}$ turns from edge to edge.
The next resonance occurs at 23.25 MHz (see Fig. 34). Again, by examining the total field pattern, we see that it is a standing wave with the field vector rotating through 2½ turns from edge to edge: there are two minima in both the axial and radial components. Finally, we consider the next resonance at 22.45 MHz (see Fig. 35). Both the axial and radial patterns show three minima, so the total field vector here rotates through 3½ turns.

It is clear that this sequence of patterns represent a set of standing waves (½, 1½, 2½, 3½,… turns) on the surface of the prism. We interpret these measurements by first...
recognising that the characteristic patterns are the resonant modes of a hexagonal prism. We now appeal to the dispersion relation (1) and the frequency dependence of the permeability to determine the precise frequencies at which such modes would be expected. For simplicity, we approximate the hexagonal slab with a cylindrical prism, and confine our attention to the x-z plane. The z-component of the magnetic field within a cylindrical prism may be written as

\[ H_z(x, z) \sim J_0(k, x) \cos(k_z z) \]  

(2)

where \( J_0 \) is the zeroth order Bessel function of the first kind. At the input face, we have

\[ H_z(x, 0) \sim J_0(k, x) \]  

(3)

and, imposing \( H_z(R, 0) = 0 \) at the walls of the cylinder as observed experimentally, we can obtain the wavevectors \( k_x \) of the surface modes from the roots of \( J_0(k, R) \). Here, \( R \) is the radius of the equivalent cylinder to the hexagon, taken in this case to be \( R = 210 \text{ mm} \) since we are concerned with the x-z plane that contains the hexagonal diameter.

Now, in this frequency range, the free space wavelength is in excess of 10 m, so we can neglect \( k_0 \) in the dispersion relation (1) and write

\[ k_z = \pm \frac{k_x}{\sqrt{\mu_z}} \]  

(4)

Inserting this in (2), we have

\[ H_z(x, z) = J_0(k, x) \cos \left( \frac{k_x z}{\sqrt{\mu_z}} \right) \]  

(5)

and we obtain standing waves in the cavity (with antinodes at the surfaces) when

\[ \frac{k_x}{\sqrt{\mu_z}} = n\pi \]  

(6)

Thus, given \( k_x \) from the surface mode analysis, we can determine the values of \( \mu \) required to meet the resonance conditions. These are summarised in Table 1:

<table>
<thead>
<tr>
<th>Cylindrical Harmonic</th>
<th>Frequency (MHz)</th>
<th>( k_x ) (m(^{-1}))</th>
<th>( \mu' )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>29.45</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>24.75</td>
<td>53</td>
<td>-0.92</td>
</tr>
<tr>
<td>2</td>
<td>23.25</td>
<td>82</td>
<td>-2.26</td>
</tr>
<tr>
<td>3</td>
<td>22.45</td>
<td>112</td>
<td>-4.19</td>
</tr>
</tbody>
</table>
These values are plotted in Fig. 6 along with the curve for \( \mu' \) from the permeability data, in which the filling factor \( F \) had been determined as if the Swiss Rolls filled space. We retain the same values for the other parameters but treat \( F \) as adjustable, and determine the best fitting value from the data in Table 1. The resulting curve for \( \mu \) is also shown in Fig. 36. We find \( F_{opt} = 0.84F \), somewhat reduced from the ideal 0.91\( F \) for a hexagonal close packed cylinder, but still a realistic value, representing an increase in effective packing diameter of the rolls of < 0.5 mm.

**Square Prism**

The hexagonal prism, although providing good experimental data, is not amenable to an analytical study of the transmission characteristics of negative media. A simpler geometry is the square prism, so the sample was reassembled to make a square box, with the rolls packed on a square grid with spacers to define the lattice. A photo of the sample is shown in Fig. 37.

Measurements were made using both the Oxford table and our new in-house rig, recording all three-field components. The data are very similar, but the Oxford data was taken with higher spatial resolution (3 mm compared to 5 mm steps). Here we concentrate on these plots. Once again, we can identify the resonant frequency at 21.50 MHz, where the slab acts as a magnetic faceplate. The magnetic plasma frequency lies at 28.55 MHz, slightly lower than for the hexagonal prism, reflecting the reduced filling factor for the square lattice. Between these two frequencies, there is a sequence of resonant modes, whose patterns are shown in Figure 38.

We have analysed these patterns using the same approach as for the hexagon: each mode defines a wavevector in the surface, \( k_{//} \), whereas each resonance defines a longitudinal wavevector \( k_z \). Combining these gives the value for \( \mu \) at that frequency, as shown in the lower right panel of Fig. 38. The only parameter in the model is the filling factor, and the best fitting value of 0.79\( F_0 \) is almost identical to the value of 0.785\( F_0 \) for square packing. So it appears that the effective medium approach gives a good description of the propagation characteristics of the medium.

![Figure 36: Squares: values of \( \mu' \) deduced from the prism modes; blue line: plot of \( \mu' \) from Fig. 31, in which the rolls are assumed to fill space; red line: plot of \( \mu' \)](image)

![Figure 37: The square prism consisting of 289 rolls on a square lattice. The sides are](image)
Figure 38: The measured field patterns transmitted by the square prism, and the analysis of the corresponding modes. Squares: values of $\mu'$ deduced from the prism modes; Blue line: plot of $\mu'$ from Fig. 31, when the rolls are assumed to fill space; red line: plot of $\mu'$ with filling factor adjusted to fit the measured
WP2: Technologies for LH building blocks

The issues addressed in the second project year concerning LH building blocks development are:

- Fabrication of 1D double-side composite metamaterials (CMMs) operating around 100 GHz, using ultraviolet photolithography and thermal metal deposition.
- Process development of double-sided 30 and 100 GHz 2D CMM structures (optimization of the etching procedure).
- Fabrication of Split Ring Resonator (SRR) structures operating around 6 THz.

2.1 Fabrication of 1D double-side composite metamaterials (CMMs) operating around 100 GHz.

Having established the presence of a true left-handed transmission band at ~4 GHz (see the WP1 report), our next aim was to scale the present structure to 100 GHz regime. We employed the same fabrication technique, consisting of ultraviolet photolithography and thermal metal deposition, described in the 1st year report. For the sake of completeness, we briefly review the fabrication process here.

Three photomasks were designed for split ring-resonator (SRR) array, closed ring resonator array (CSRR) and continuous wire (CW) array patterns, respectively.

Commercial microscope cover slips (Corning glass; area: 22 mm x 22 mm; thickness: 250µm) were used as the substrate. The refractive index of the glass substrate was determined using phase delay measurements, and found to be ~2.4 for the 40 GHz regime. The dimensions of the SRR metamaterial parameters are depicted in Fig. 1. The pattern on each photomask consisted of 30, 20, 15, 10, and 5 layers of the relevant structure (i.e., CW, SRR, or CSRR) as shown in Fig. 2. A sample micrograph of a CMM unit layer consisting of a SRR layer on top of a wire layer is shown in Fig. 3.
The actual fabrication consisted of the following consecutive steps:

1. Cleaning the substrate (glass).
2. Photoresist (AZ5214E) coating by spinning and soft baking.
3. Patterning (CW, SRR, or CSRR) by UV exposure and post-exposure baking.
4. Flood exposure for removal of the photoresist residues.
5. Chlorobenzene bath for profiling photoresist sidewalls for lift-off.
7. Lift-off using acetone.

Parameters:

\[
\begin{align*}
d &= 4.44 \, \mu m & a_x &= 195.55 \, \mu m \\
r &= 35.5 \, \mu m & a_y &= 195.55 \, \mu m \\
w &= 20.0 \, \mu m & a_z &= 250 \, \mu m \\
t &= 4.44 \, \mu m
\end{align*}
\]

Figure 1: Schematic of the SRR unit and the dimensions, including the periodicity in a single SRR layer.

Figure 2: Photomask design for SRRs having 30, 20, 15, 10, and 5 layers along the propagation direction, respectively. Same applies for the photomask for wires (not shown).

Figure 3: Photomicrograph of CMM consisting of SRR layer on top of continuous wire layer.

### 2.2 Process development of double-sided 30 and 100 GHz CMM structures.

Microelectronics Research Group in FORTH/IESL developed in the second project year the technological background for fabrication of 2D LH structures for 30-50 GHz frequency range. Typical substrate for the test structures in this frequency range is a semi-insulated GaAs wafer, and the fabrication of 2D LH structures has two main steps:

- deposition and patterning of the metallic elements,
- cutting of the LH sample in building blocks for 2D structure.
The fabrication procedure for deposition and patterning of the LH structure is identical with the procedure used for the 1D LH structure and in the last DALHM year we developed a deep etching procedure able to etch 300-600 µm thick substrates.

**Wet etching procedure for etching 600 µm thick GaAs substrate**

Wet etching is one solution for etching GaAs substrate, but the etching profile depends on the orientation of the etching mask compared with the crystallographic planes of GaAs substrate for the majority of etching solutions. Several etching solutions were investigated and the dept of etch was measured. The results are included in following table:

<table>
<thead>
<tr>
<th>Etching solution</th>
<th>Composition</th>
<th>Etch time (min)</th>
<th>Etch depth (µm)</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>H₂O₂ : H₂SO₄ : H₂O</td>
<td>8 : 1 : 1</td>
<td>6</td>
<td>~60</td>
<td>Etching profile depends on mask orientation</td>
</tr>
<tr>
<td></td>
<td></td>
<td>10</td>
<td>~92</td>
<td></td>
</tr>
<tr>
<td>H₃PO₄ : HNO₃</td>
<td>1 : 1</td>
<td>-</td>
<td>-</td>
<td>The mask was destroyed for photoresist and polyamide masks</td>
</tr>
<tr>
<td>HCl : H₂O₂ : H₂O</td>
<td>40 : 4 : 1</td>
<td>10</td>
<td>~32</td>
<td>Lowest undercut</td>
</tr>
<tr>
<td>HF : H₂SO₄ : H₂O₂</td>
<td>1 : 1 : 1</td>
<td>10</td>
<td>~68</td>
<td></td>
</tr>
<tr>
<td>HF : HNO₃ : H₃PO₄ : H₂O</td>
<td>1 : 1 : 1 : 1</td>
<td>15</td>
<td>~37</td>
<td></td>
</tr>
<tr>
<td>H₂PO₄ : H₂O₂ : H₂O</td>
<td>1 : 1 : 1</td>
<td>15</td>
<td>~50</td>
<td></td>
</tr>
<tr>
<td>H₂SO₄ : H₃PO₄ : H₂O : H₂O₂</td>
<td>1 : 1 : 1 : 1</td>
<td>15</td>
<td>~50</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>30</td>
<td>~83</td>
<td></td>
</tr>
<tr>
<td>HF : HNO₃ : H₃PO₄</td>
<td>1 : 2 : 1</td>
<td>10</td>
<td>~250-300</td>
<td>Highest etching rate</td>
</tr>
</tbody>
</table>

For all solutions we investigated not only the etching depth but also the undercut of the etching and the profile of etched groove. Among etching solutions the first solution having a high etching rate was the solution H₂O₂ : H₂SO₄ : H₂O. Because of rather slow etch rate for this type of mixture, it would be good to be able to do at least two etching steps. The profile of the etched groove is presented in Fig. 4 and even if the etching rate is high and quite uniform, the profile of etched grooves is different depending on mask orientation. The worst case is presented in Fig. 4, were we can see that the slope of the profile is negative at the top of sample, which creates problems if a second etching is needed. Also the undercut can be seen in Fig. 4, where we can see the original photoresist mask on the top of the sample. The undercut is quite big at the top of the samples, around 60 µm, comparable with the depth of etching.
Several other etching solutions were investigated to find a solution that produces grooves with positive slope walls and reduced undercut. One choice is to use a HCl: H₂O₂: H₂O mixture even if the etching rate is quite small. The etching profile and the undercut are presented in Fig. 5.

In this case the slope of etched grooves is positive but the undercut is not uniform, even if is quite small. In Fig. 5(b) we can see the original photoresist mask and the edges of the groove after etching. Because of the small etching rate several steps of etching have to be used so we searched for solutions with higher etching rates. A solution based on H₃PO₄: H₂SO₄: H₂O₂: H₂O was used for etching and etching profile is presented in Fig. 6. In this case the etching profile is quite good, with positive...
slopes and quite uniform. Reduced undercut can be also seen in Fig. 6, since the photoresist mask is still on the sample. For this case after 30 min etching the depth obtained is ~83 \( \mu \)m with only ~57 \( \mu \)m undercut.

Even this last solution is etching quite slow the GaAs substrate so it can be used for 300 \( \mu \)m thick substrates, but is difficult to use for 600 \( \mu \)m thick substrates.

Figure 6: Etching profile and undercut for a \( \text{H}_3\text{PO}_4: \text{H}_2\text{SO}_4: \text{H}_2\text{O}_2: \text{H}_2\text{O} \) etching mixture.

For this case we have investigated a very fast etching solution based on HF: HNO\(_3\): \( \text{H}_3\text{PO}_4 \). The etching profile for this mixture and a general view of the etched groove are presented in Fig. 7.
Figure 7: Etching profile, (a), and general view of etched groove, (b), for a HF: HNO$_3$: H$_3$PO$_4$ etching mixture.

In this case we have a ~250-300 $\mu$m etched depth and ~150-200 $\mu$m undercut. In this case, because of the high etching rate, the etched groove is not very uniform as it can be seen from Fig. 7(b), but is very good for the size of our samples. Even in this case to ensure a good cutting of the sample we have to do two etchings, one on each side of the sample. For this etching mixture we have tried to use metallic or polyimide mask because the photoresist mask cannot be used if the etching time is longer than 5 minutes. After this time of exposure to the etching solution some holes are appearing in the photoresist layer. The metallic masks used were not good because one or other components of the solution are etching the majority of metals.

We have tried also a polyimide-based mask, which can be immersed into this etching mixture for more than 10 minutes, which is what we need for these samples. The polyimide used is PI2525 that gives an approx. 5 $\mu$m thick layers (depending on the spinning time and speed).

The process flow for fabrication of these samples is:

- Sample cleaning to remove any substance contaminating the sample
- Coating of the sample with polyamide PI2525
- Soft-baking of the polyamide at 125 °C
- Sample coating with photoresist by spinning. The photoresist used is AZ 5214
- Soft-baking of the photoresist
- UV exposure of the photoresist using a Karl Suss MA6/BA6 mask aligner
- Developing of the photoresist and polyamide pattern using AZ 400K photoresist developer
- Removal of photoresist in acetone
- Curing of polyimide at 250 °C, to remove all traces of solvent and achieve best chemical stability the mask
- Sample bonding on a silicon wafer to protect the front side area with LHM structure during etching
- Sample etching for 10 min. in H$_3$PO$_4$: H$_2$SO$_4$: H$_2$O$_2$: H$_2$O etching mixture
- Removal of the sample from silicon wafer and cleaning of remaining bonding wax
- Sample bonding on a silicon wafer to protect the back side area during etching
- Sample coating with photoresist by spinning
- Soft-baking of the photoresist
- UV exposure of the photoresist using a Karl Suss MA6/BA6 mask aligner
- Developing of the etching pattern using AZ 400K photoresist developer
- Sample etching for 5 min. in H₃PO₄: H₂SO₄: H₂O₂: H₂O etching mixture
- Removal of photoresist in acetone
- Removal of the sample from silicon wafer and cleaning of remaining bonding wax
- Sample cleaving

In Figure 8 is presented the edge of the sample after two-side etching and cleaving for the samples we have used for developing the procedure.

![Sample view after final cleaving.](image)

Figure 8: Sample view after final cleaving.

This etching procedure can be used for fabrication of 2D LH structures and depending of the thickness of the substrate used can be simplified for example to use only photoresist etching masks.

### 2.3 Fabrication of 6 THz Split Ring Resonator structures.

During the second year of the project we fabricated and tested (see WP1) single-ring SRR structures operating in the THz frequency region. The structure consists of single-ring SRRs (see a photo of the mask in Fig. 9 and a photo of the fabricated structure in Fig. 10) in polyimide PI2525 substrate. The fabrication of the structure was done using photolithography and e-beam evaporation (for the metal deposition). The SRR size is 5×5µm² while the unit cell dimensions are 7×7×5µm³.
Figure 9: View of the 6 THz mask.
The rings are from Ag and have width, depth and gap size all of 1 µm, and the spacing between rings is 2 µm. The mask permits fabrication of a multi-stacked structure with a polyimide layer between successive layers of SRRs. Also the mask has an area covered with SRRs of 25mm×25mm, designed to permit easy handling of the sample and good interaction between probing beam and the sample.

Figure 10: View of the 6 THz SRR metamaterials fabricated.
WP3: LH based demonstrators

Within the second project year we worked towards the development of the following demonstrators:
- Photonic crystal based high gain antennas.
- Components for the manipulation of the RF signal in MRI machines.

Below we give a brief presentation of this work.

3.1 Photonic crystal based high gain antennas.

3.1.1 A high gain antenna system operating at the band edge frequency of a two-dimensional photonic crystal.

Peculiar properties of photonic crystal band edges provide various ways to control the propagation of electromagnetic waves. Among the properties of the photonic band edge, enhanced local density of states and an index of refraction that is close to 0 near the upper band edge can be used to improve the gain and directivity of antennas.

As a starting point to the PC based high gain antennas, we studied the angular distribution of power emitted from a monopole source embedded inside a 2D PC. The far field radiation patterns near the lower and upper band edge frequencies are measured. Radiation patterns near the upper band edge shown in Fig. 1 indicate that high directivity is obtained only at the band edge of the PC. We checked the dependence of radiation patterns on the size of the photonic crystal. Radiation patterns at the upper photonic band edge for various crystal sizes are shown in Fig. 2. Evidently there is an optimum crystal size for which the directivity can be maximized. For the PC in this study, the optimum crystal size is 24x16 layers with a corresponding half power beam-width of 6 degrees. This is a very high directivity obtained from PC embedded sources.

Figure 1: Measured far field radiation patterns for 32x16 crystal size for the frequencies near the band edge.
3.1.2 Improvement of directivity in three dimensions by using a layer-by-layer photonic crystal.

The dimensionality of the PCs inherently defines the angular confinement: 2D PCs can achieve only a planar angular confinement for the emitted radiation, due to lack of confinement in the remaining spatial direction. In addition, the leakage of radiation out-of-plane direction within the PC also reduces the antenna efficiency. For antenna applications full 3D confinement into a solid angle is highly desirable. To overcome these shortcomings, we proposed a 3D PC for embedding the source.

The PC is a layer-by-layer structure having face tetragonal symmetry. Lattice constant is 1.28 cm in the stacking direction, and 1.1 cm in the plane perpendicular to the stacking direction. Dielectric material is alumina with refractive index $n = 3.1$. The embedded monopole antenna radiates near the band edge frequency. The measured far field radiation patterns in E-plane and in H-plane are shown in Fig. 3. Evidently, the emitted power is confined to a narrow angular region in both planes. Half power beam-width is $16^\circ$ corresponding to a directivity of 245.

**Figure 2:** Measured radiation patterns for various crystal sizes at the band edge frequency. Minimum half power beam-width is obtained for 24x16 layers.

**Figure 3:** Measured far field radiation patterns at the band edge frequency in the E-plane and H-plane.
3.1.3 Directive radiation by photonic crystal surface modes.

Several studies have revealed that finite PCs can sustain surface propagating modes if the surface of the PC is corrugated. Surface modes supported by the PC have decaying wave profiles both inside the air region and inside the PC. In addition, surface modes appear inside the photonic band gap and are below the light line.

One possible way to alter the PC surface morphology to create surface propagating waves in two dimensions is replacing the rods at the final layer with rods whose radius differ from the original rods. In this work we have followed this procedure to create surface modes. The crystal we used is a square array of alumina rods whose radius is 1.55 mm. We have replaced rods at the last layer with ones whose radiuses are 0.765 mm. Surface modes can be calculated by using a supercell. The supercell we used in our calculation contains 11 layers of alumina rods and an air layer of the same size. The corresponding band diagram is shown in Fig. 4. Measured field profile near the surface of the PC corresponding to a surface mode is shown in Fig. 5. The surface modes are excited by a monopole source placed inside the PC.

![Figure 4: Red curves represent the dispersion relation for the bulk modes. The part of blue curve inside the bandgap represents the dispersion diagram of the surface modes.](image1)

![Figure 5: Measured electric field intensity of the surface modes near the photonic crystal - air interface. Vertical direction is parallel to the crystal surface.](image2)

It is possible to couple the surface modes to the radiating modes by adding an extra layer of rods in front of the surface layer. Furthermore, coherent interference of the radiating modes may lead to directive radiation. To couple the surface modes to the radiating modes we placed another layer of alumina rods in front of the surface modes. The extra layer has a lattice constant equal to the twice of the bulk PC. The measured far field radiation patterns for 12.8 GHz and 12.9 GHz are shown in Fig. 6 (a) and (b). The emitted power is confined to a narrow angular region. Half power beam-widths are 8 degrees and 10 degrees for 12.8 GHz and 12.9 GHz, respectively. In our previous works, the directive radiation was obtained only at the band edges. Hence the bandwidth was very narrow. In this work, we have also improved the bandwidth of the antenna.
3.2 Development of components for use in the MRI machines.

Magnetic Resonance Imaging (MRI) offers an ideal proving ground for demonstrating the unique properties of left-handed metamaterials (LHM). In an MRI system, the main magnetic field (typically 0.5 – 3 tesla) needs to be homogeneous to a few parts per million, thus ruling out the introduction of any conventional magnetic material. Nevertheless, it would be very useful to have access to magnetic materials with which to manipulate the radiofrequency (RF) signals (in the range 20 – 120 MHz). Functions such as guiding, focussing and screening could substantially enhance the performance of MRI systems. Metamaterials can achieve this because they offer a means of obtaining magnetic properties at RF (for example large positive or negative permeability) without affecting the other magnetic fields in the system.

In this workpackage, therefore, we have been investigating potential metamaterial components for use at RF. One target is the development of an RF yoke, which will assist in the delivery or detection of RF signals. We have built and assessed a prototype yoke that has served to highlight which aspects are most important for achieving useful performance.

An important part of any future device will be a flux compressor, that can expand or concentrate flux from a single roll yoke to multiple roll pole-pieces. Last year we demonstrated a solenoid-based device, and also discussed the potential of capacitatively loaded loop resonators, the RF equivalent of the SRR used at microwave frequency. This year, we have investigated a compressor based on the loop resonator, which should have greater design flexibility.

3.2.1 RF Yoke

As a natural extension to this work we plan to study the surface modes in three dimensions. Currently, directive radiation is confined to one plane only. To extend the angular confinement to three dimensions we plan to use three dimensional photonic crystal surface modes.
We have investigated the concept of a yoke made of Swiss Roll metamaterial to provide a low reluctance pathway that could potentially assist in signal reception in magnetic resonance imaging and spectroscopy applications. For example, a yoke could enable a remote source to generate a field pattern between the pole-pieces of the yoke (see Fig. 7, using the magenta coil as the source loop), or conversely enable a remote detector (the magenta loop in Fig. 7) to receive signal from a source between the poles. Alternatively, we could achieve enhanced detection of a small source between the poles using a coplanar receiver coil (the blue loop in Fig. 7).

For this work, we used Swiss rolls were constructed from Pryalux flexible circuit board material, wrapped on mandrels 8 mm in diameter. These rolls were designed to resonate at 21.5 MHz and they had a Q of ~ 30. Preliminary tests were made on single rolls, by injecting a signal through a coupling loop at one end, and recording the detected signal through a second loop that could be moved along the roll. The detected signal was independent of the position of the receiver, but did depend on the length of the roll, being smaller for longer rolls. Thus the rolls act as good magnetic flux conductors, and the signal is determined by the reluctance (i.e. the length) of the flux return path.

We then considered joining individual rolls together. Each junction introduces extra loss, and 90° joints produce roughly double the loss of a straight joint between the same rolls. Because of these losses, a yoke constructed from butt-coupled single rolls would not be viable. To reduce the corner losses, we assembled bundles of seven rolls of different lengths, so that the corners were mitred at 45°, as shown in Fig. 7. This arrangement was much less sensitive to alignment, and significantly reduced the corner losses, so that a full yoke became viable.
The bundles were first assembled into a linear array with a 15 mm gap between the pole pieces (Fig. 8). An 11 mm diameter untuned loop was used as a source and a co-planar 33 mm untuned loop was used for reception. A reference level (0 dB) was taken from the source and receiver loops alone. These were then inserted between the pole pieces, and increasing amount of metamaterial components were added as the linear array was built up. Addition of the pole pieces introduced a parasitic tuned element and increased the output to +7.0 dB. As more components were added the signal level fell, presumably because of increasing losses and the longer return path. In the full collinear arrangement the received signal had fallen to +5.6 dB. When the elements were reconfigured to form the yoke, the signal rose to +6.1 dB.

The performance of the yoke was also investigated by using a remote receiver loop to detect flux circulating through the metamaterial bundles from a source between the pole pieces. Once again, the reference level was defined from the two loops in a co-planar configuration without the metamaterial. We then measured the signal being guided around the yoke (Fig. 9). At first sight, this result shows perfect coupling, but it must be recalled that this is a resonant system, so we expect the signal on resonance...
to be much higher than the reference (we found +7 dB for the pole pieces alone, see above). So although this result is encouraging, it also shows that the device must be much improved to be truly useful. Nevertheless, it does provide a proof of principle that metamaterial yokes could enhance signal reception by introducing a low reluctance flux return path.

We have also examined our higher performance materials in which the permeability and Q are at least a factor of 2 higher than in the Pyralux material. In these, the signal down a 200 mm roll was increased from 4.2 dB to 10.3 dB, showing much improved flux ducting. However, the losses at joints, while reduced compared to those in the Pyralux system, are still unacceptable (see Table 1). We have therefore investigated the effect of an additional coupler in the form of two connected loops, that links the end of one roll with the next. This significantly improves the flux linkage, as shown in Table 1, but further work is necessary to optimise this approach.

<table>
<thead>
<tr>
<th>Signal (dB)</th>
<th>Pyralux</th>
<th>Espanex</th>
<th>Espanex + coupler</th>
</tr>
</thead>
<tbody>
<tr>
<td>Straight joint</td>
<td>-5.9</td>
<td>-2.7</td>
<td>-1.1</td>
</tr>
<tr>
<td>90° joint</td>
<td>-11.3</td>
<td>-6.8</td>
<td>-0.9</td>
</tr>
</tbody>
</table>

Another issue that affects the yoke performance is how uniform a field can be generated between the pole pieces or equivalently what resolution can be detected from a source in the gap. In our study last year of the resolution of the face-plate, we found that the transmitted flux tends to be confined to the individual rolls. In that work, we applied a numerical filter to the data and showed that the faceplate resolution was accounted for by the loss in the material. However, if we consider rather shorter pole-pieces, the resolution will be dominated by the size of the individual elements, producing a field distribution with uniform areas that are the same size as the rolls. This effect was also seen in our MRI experiments on the faceplate, and is clear in the data from the square prism sample. This suggests that the yoke needs to be constructed of rolls with the smallest possible diameter.

However, in very recent experiments, we have found evidence, yet to be confirmed, to suggest that some details of the flux patterns may be transmitted through a single roll. Small, 3 mm diameter, loops were used as both source and detector, with the source being placed on axis at the entrance of a very large, 37 mm diameter, hollow roll, resonant near 6 MHz. The characteristic minima in the axial field of a small loop were also seen at the output of the roll, suggesting that some mapping of the input flux distribution to the output face does occur. More detailed scans of the output field distribution must be carried out to determine whether this is a real effect or an artefact.

### 3.2.2 Flux Compressor
Last year, we reported our initial feasibility study last year of the solenoid flux compressor. The advantage of this type of concentrator is that, being tightly wound, the flux cannot escape, and hence it is easier to realize a large concentration ratio. Its main disadvantage is that a separate solenoid must be built for each experiment. We have investigated a more flexible approach using resonant loop elements. We reported last year that arrays of such loops could transfer flux: could an array of capacitively loaded loops whose diameters are progressively reduced in some predetermined manner be used to concentrate flux?

Here we consider a compressor built from a sequence of resonant loops, all tuned to the same frequency, and wound on different diameter formers. The coils were then assembled on a common insulating mandrel (Fig. 10). The number of turns in each coil was allowed to vary to make it possible to tune each coil of the set to 21.3 MHz (for compatibility with a 0.5T MRI system) using readily available capacitors. In the simplest case the loops are equidistant from each other but this construction allowed the distance between the elements to be a free parameter so we could investigate the impact on concentration ratio of altering the element spacing.

Numerical calculations were performed to investigate the performance of the device, in particular whether improved performance could be achieved by modifying the inter-element spacing. The calculations suggested that the concentration efficiency was unaffected, but the pass-band was reduced.

The device was tested in two configurations: first with all the elements pushed close together, leading to a spacing of 2.2 mm, and second with the inter-element spacing increased to 5.2 mm. The comparisons between the measured and calculated spectra are shown in Fig. 11. Although the bandwidths of the devices and the details of the

![Figure 10: The prototype flux compressor consists of 13 segments, each of width 2.2 mm, whose diameter reduces by ~ 1 mm per segment from 20.5 mm to 7 mm. The number of turns, diameter and capacitance of each segment were selected to give resonance](image-url)
resonant peaks change, their overall transmission levels are little affected, so that the design of these devices can be seen to be quite robust.