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Extinction properties of a sphere with negative permittivity and permeability

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Abstract

Extinction spectra of spheres made of materials having dispersive permittivity and permeability are calculated. A dispersion in the microwave region, similar to that which has recently been achieved in artificial composite media is assumed. When either the permittivity or the permeability (but not both) is negative, there occurs a band of strong surface polariton absorption. However, when both quantities are negative, a band of highly suppressed extinction appears in the spectrum.

Keywords: D. Optical properties; E. Elastic light scattering

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A medium for which both the dielectric constant ϵ and the magnetic permeability μ assume negative values will have peculiar properties. This has been demonstrated theoretically by Veselago [1]. He has used the term "left-handed" for such a material, because for a plane wave propagating in such a medium, $\vec{E} \times \vec{H}$ lies in the direction opposite to that of the wavevector. Other unusual electrodynamic properties of left-handed materials are reverse Doppler shift, inverse Snell effect and reverse Cerenkov radiation [1]. Recently, a material which is left-handed for frequencies in the microwave range, has been built by combining two dimensional arrays of split-ring resonators and wires [2]. It is expected that these techniques will be further developed, so that in the near future isotropic left-handed materials will also become available. Here we investigate the electromagnetic scattering properties of a sphere made of a left-handed material. Simultaneous negative values of ϵ and μ can be realized only when there exists a frequency dispersion [1]. In our calculations we therefore employ dispersive forms of ϵ and μ, similar to those that can be attained artificially [2].

We consider a sphere of radius a, having relative permittivity $\epsilon_1(\omega)$ and relative permeability $\mu_1(\omega)$, placed in a medium having real, frequency-independent permittivity ϵ_2 and permeability μ_2. The sphere is irradiated by a plane wave, and its scattering and absorption properties can be specified in terms of the Mie coefficients. For these we employ the most general form, which includes magnetic effects. In Stratton's notation [3], they are given by

$$a_n = -\frac{\mu_1 j_n(k_1 a)[k_2 a j_n(k_2 a)]' - \mu_2 j_n(k_2 a)[k_1 a j_n(k_1 a)]'}{\mu_1 j_n(k_1 a)[k_2 a h_n(k_2 a)]' - \mu_2 h_n(k_2 a)[k_1 a j_n(k_1 a)]'} \quad (1)$$

$$b_n = -\frac{\epsilon_1 j_n(k_1 a)[k_2 a j_n(k_2 a)]' - \epsilon_2 j_n(k_2 a)[k_1 a j_n(k_1 a)]'}{\epsilon_1 j_n(k_1 a)[k_2 a h_n(k_2 a)]' - \epsilon_2 h_n(k_2 a)[k_1 a j_n(k_1 a)]'} \quad (2)$$

Here j_n and h_n are spherical Bessel and Hankel functions, respectively, and the primes denote differentiation with respect to their arguments. The propagation constants of the sphere and the surrounding medium are $k_1 = \sqrt{\epsilon_1(\omega)\mu_1(\omega)}\omega/c$ and $k_2 = \sqrt{\epsilon_2 \mu_2}\omega/c$, respectively. The extinction cross-section of the sphere, in units of the geometric cross-section, is given by

$$Q = -\frac{2}{(k_2 a)^2} \sum_{n=1}^{\infty} (2n+1)\,\mathrm{Re}(a_n + b_n) \quad (3)$$

For the sphere dielectric constant we use the plasma-like

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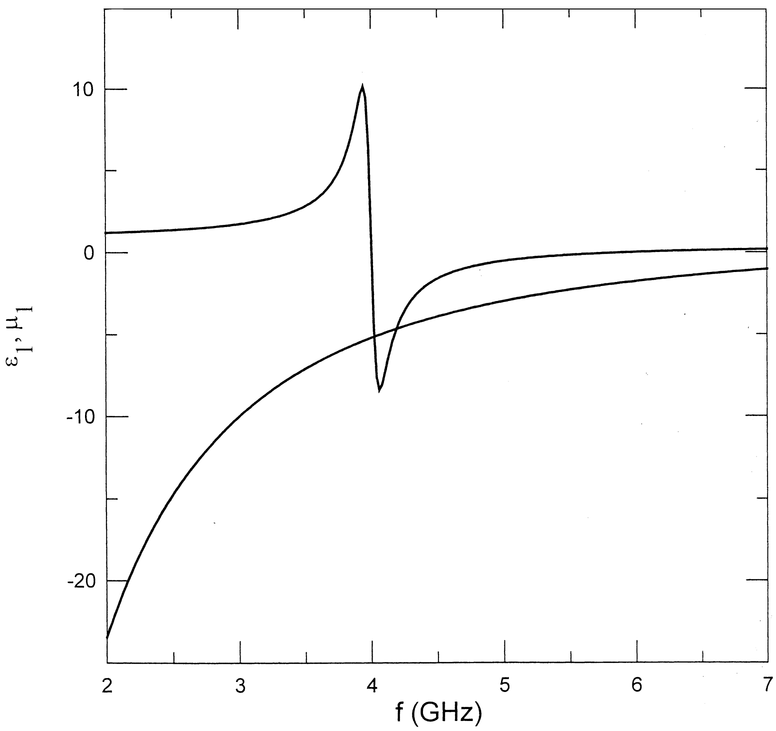

Fig. 1. Real part of the permittivity (lower curve) and the permeability (upper curve) used in the model calculations. The material is left-handed between 4 and 6 GHz.

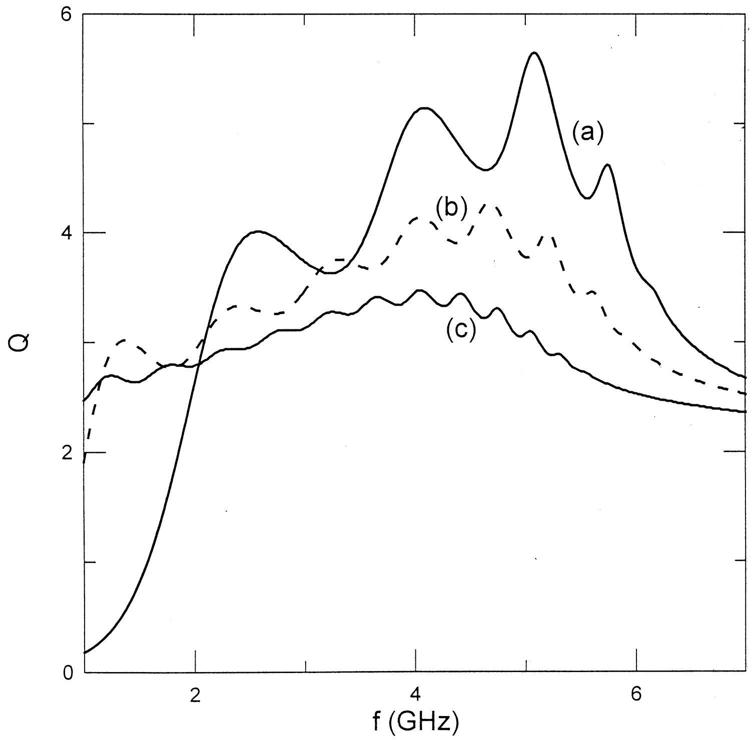

Fig. 2. Extinction cross-section of a sphere having a dielectric constant of the form (4), and $\mu_1 = 1$. The sphere radius is: (a) 2 cm; (b) 4 cm; (c) 8 cm.

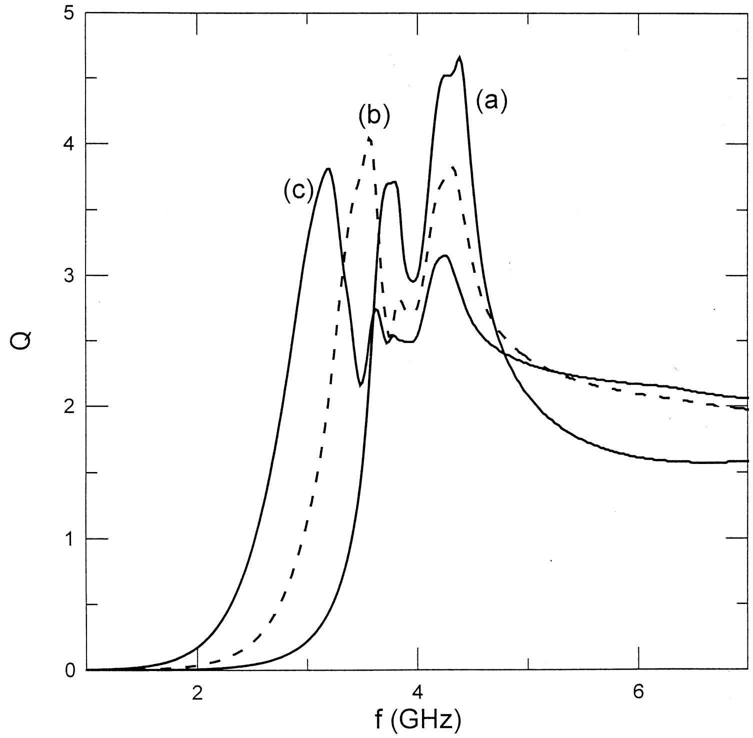

Fig. 3. Extinction cross-section of a sphere having a permeability of the form (5), and $\epsilon_1 = 1$. The sphere radius is: (a) 2 cm; (b) 4 cm; (c) 8 cm.

form

$$\epsilon_1(\omega) = 1 - \frac{\omega_p^2}{\omega(\omega + i\gamma)} \tag{4}$$

Such a dielectric constant, with the plasma frequency in the GHz range, can be realized by using a network of thin wires [4].

For the sphere permeability we employ the following form, which can be achieved by using a periodic arrangement of split ring resonators [5]:

$$\mu_1(\omega) = 1 - \frac{F\omega^2}{\omega^2 - \omega_0^2 + i\omega\Gamma} \tag{5}$$

In the numerical calculations we use the value $\omega_p = 10$ GHz for the plasma frequency, and $\omega_0 = 4$ GHz for the magnetic resonance frequency. For the damping parameters the values $\gamma = 0.03\omega_p$ and $\Gamma = 0.03\omega_0$ are assumed. For the parameter F the value of 0.56 has been chosen, so that the band of negative permeability, which begins at ω_0, will extend up to 6 GHz. The frequency dependence of the real part of ϵ_1 and μ_1 is shown in Fig. 1. In all calculations the values $\epsilon_2 = 1$ and $\mu_2 = 1$ are used for the surrounding medium.

First, we consider the independent contributions of the dielectric dispersion and the magnetic dispersion to the extinction. These we obtain by performing extinction calculations, once with $\mu_1(\omega) = 1$ and $\epsilon_1(\omega)$ from Eq. (4), and once with $\epsilon_1(\omega) = 1$ and $\mu_1(\omega)$ from Eq. (5).

Fig. 2 shows the extinction spectrum of spheres having a dielectric constant of the form (4), and no magnetic contribution. The real part of the dielectric constant is negative over the whole frequency region, and the spectrum consists of peaks which are due to surface plasmon-polaritons, as has been discussed previously in the case of small metallic spheres [6,7], where analogous peaks appear in the visible or ultraviolet range. The peaks are due to resonances of the Mie coefficients b_l ($l = 1, 2, \ldots$), and they are classified as surface mode peaks because the corresponding field amplitudes are highest at the surface of the sphere, and they decay towards its center. In the very small sphere limit (not shown here) only one peak, due to the $l = 1$ mode, appears. With increasing sphere size, more and more modes, corresponding to higher l values, contribute to the extinction, and the spectrum becomes broader and flatter.

Fig. 3 shows the extinction spectrum of spheres having a permeability of the form (5), and no dielectric contribution. The peaks, due to magnetic polaritons, are of two types. Those above ω_0 ($= 4$ GHz) in the region where $\mathrm{Re}(\mu_1) < 0$, are due to magnetic surface polaritons, resulting from resonances of the Mie coefficients a_l. The peaks below ω_0, where $\mathrm{Re}(\mu_1) > 0$, are due to magnetic bulk polaritons. It can be seen that for small spheres, the surface mode

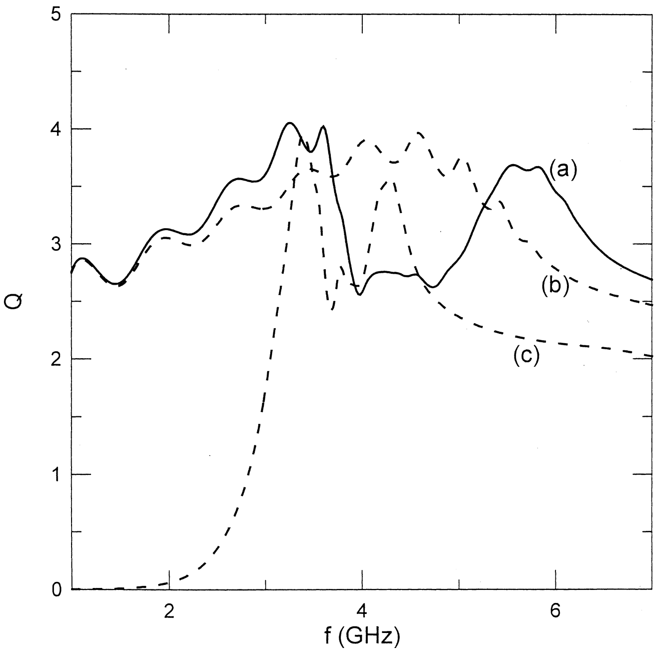

Fig. 4. Extinction cross-section of a sphere of radius 5 cm: (a) with permittivity and permeability given by Eqs. (4) and (5), respectively; (b) with permittivity given by Eq. (4) and $\mu_1 = 1$; (c) with permeability given by Eq. (5) and $\epsilon_1 = 1$.

absorption dominates, and with increasing sphere size the ratio of bulk to surface mode absorption increases.

Next, we proceed to the case in which both electric and magnetic contributions are present, i.e. the sphere material is characterized by $\epsilon_1(\omega)$ and $\mu_1(\omega)$ given by Eqs. (4) and (5), respectively. The extinction spectrum of a sphere of radius 5 cm is shown in Fig. 4a. The pure plasmon extinction, calculated with $\mu_1 = 1$ (Fig. 4b), and the pure magnetic

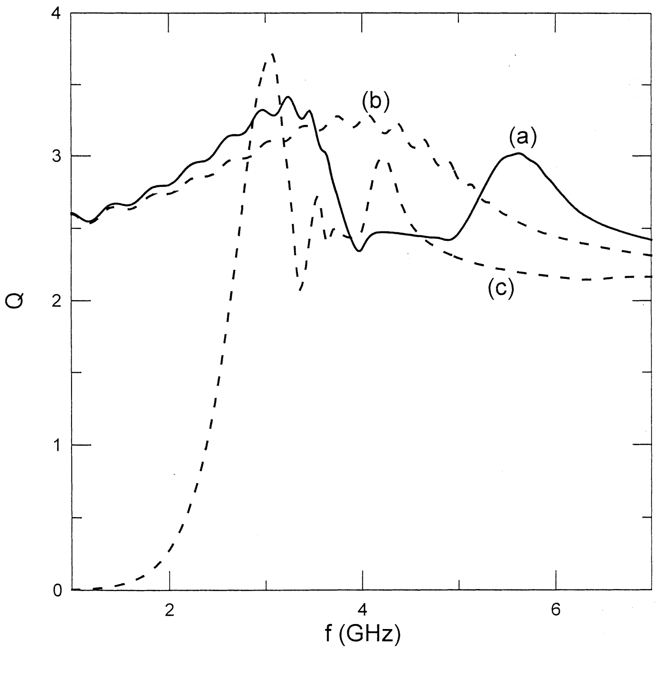

Fig. 5. Same as Fig. 4, but for a radius of 10 cm.

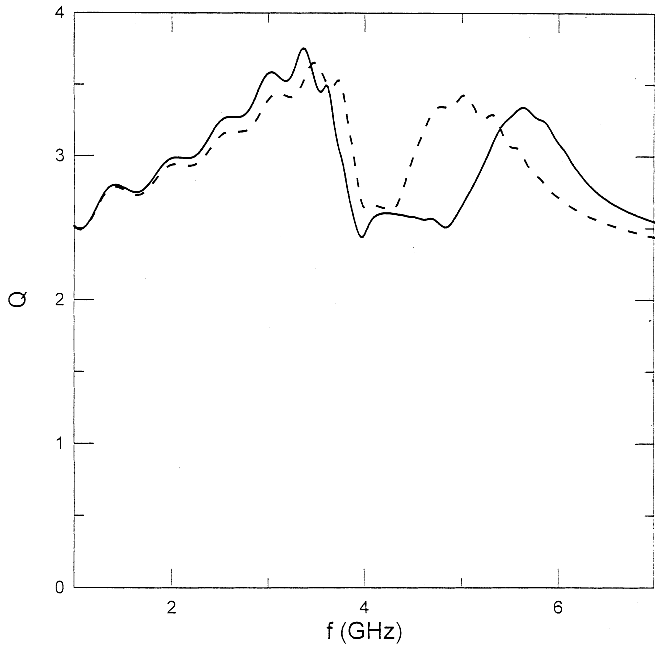

Fig. 6. Extinction cross-section of a sphere of radius 7 cm, having permittivity and permeability given by Eqs. (4) and (5), respectively. For the parameter F, the values of 0.56 (full curve) and 0.21 (dashed curve) have been used.

extinction, calculated with $\epsilon_1 = 1$ (Fig. 4c), are also shown. Since the plasmon-like and the magnetic excitations are roughly independent, we could expect the two extinction mechanisms to reinforce each other. This is indeed the case in the low-frequency region below ω_0. However, in the region just above ω_0, in which the real parts of both ϵ_1 and μ_1 are negative, this does not hold, and instead the two mechanisms are strongly suppressed. Thus, above about 4 GHz a strong, broad depression appears in the extinction spectrum. This feature persists when the sphere size is increased, as demonstrated by Fig. 5, which was calculated for a sphere of radius 10 cm. The width of the band of reduced extinction varies with the width of the range of negative permittivity and permeability. In Fig. 6 the extinction spectrum of a sphere of radius 7 cm is shown for two choices of F. The full curve was calculated with $F = 0.56$, as before. The dashed curve was obtained by using the reduced value of $F = 0.21$. In the latter case the range of negative permittivity and permeability extends only up to 4.5 GHz. This leads to a corresponding reduction of the width of the region of reduced extinction.

Finally, we present the physical interpretation of this extinction suppression effect. In frequency regions where either $\mathrm{Re}[\epsilon_1(\omega)] < 0$ or $\mathrm{Re}[\mu_1(\omega)] < 0$, but not both, the electromagnetic radiation cannot propagate inside the sphere material. However, it can resonantly excite surface polaritons, for which the amplitude decays inside the sphere. When both $\mathrm{Re}[\epsilon_1(\omega)] < 0$ and $\mathrm{Re}[\mu_1(\omega)] < 0$, the medium becomes transparent to the radiation (except for the small intrinsic absorption), so that no surface modes exist.

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