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Surface polaritons of a left-handed medium

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Abstract

The dispersion relations of the surface polaritons of a semi-infinite dispersive medium, which is left-handed (having negative permittivity and permeability) over a frequency range, are obtained. The possibility of experimentally observing the surface polaritons by attenuated total reflection is demonstrated. © 2000 Elsevier Science B.V. All rights reserved.

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The peculiar properties of a medium, in which both the permittivity and the permeability assume negative values, have been demonstrated theoretically by Veselago [1]. Such a medium is called “left-handed” because for an electromagnetic plane wave propagating in it, $\vec{E} \times \vec{H}$ lies in the direction opposite to that of the wavevector. Recently, an artificial material, which is left-handed over a band of frequencies in the microwave region, has been built, using two dimensional arrays of split-ring resonators and wires [2]. Further development of this technique will probably lead to the production of isotropic left-handed materials. Our aim here is to investigate the surface polariton modes that exist near the boundary of such a left-handed medium, and to discuss a method by which they can be detected.

We consider an interface between two media, one of which is left-handed, and search for surface polaritons. These are polaritons that propagate along the interface, with their electric and magnetic fields localized near the interface. We assume that the left-handed medium occupies the $x > 0$ half-space, and that the medium at $x < 0$ has a dielectric constant ε_1 and a permeability μ_1, both of which are frequency independent. A left-handed material can only be realized when there exists frequency dispersion. For the medium at $x > 0$ we will employ dispersive forms of ε and μ similar to those that have been achieved with artificial structures. A dielectric constant of the form

$$\varepsilon(\omega) = 1 - \frac{\omega_p^2}{\omega^2} \tag{1}$$

with the plasma frequency ω_p in the GHz range, can be realized by using a network of thin wires [3]. A magnetic permeability of the form

$$\mu(\omega) = 1 - \frac{F\omega^2}{\omega^2 - \omega_0^2} \tag{2}$$

with the resonance frequency in the GHz range, can be achieved by using a periodic arrangement of split ring resonators [4]. A combination of the two structures yields a left-handed medium [2].

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Consider first the case of p polarization. The surface polariton fields are of the form

$$\vec{H}^{(1)} = \vec{a}_y A e^{k_0 x} e^{ihz}, \tag{3}$$

$$\vec{E}^{(1)} = \frac{c}{\omega\varepsilon_1} A\big(h\vec{a}_x + ik_0\vec{a}_z\big) e^{k_0 x} e^{ihz} \tag{4}$$

in the region $x < 0$, and

$$\vec{H}^{(2)} = \vec{a}_y B e^{-kx} e^{ihz}, \tag{5}$$

$$\vec{E}^{(2)} = \frac{c}{\omega\varepsilon(\omega)} B\big(h\vec{a}_x - ik\vec{a}_z\big) e^{-kx} e^{ihz} \tag{6}$$

in the region $x > 0$. Here $\vec{a}_x, \vec{a}_y, \vec{a}_z$ are unit vectors in the x, y and z directions, respectively. The wavevector components appearing in (3)–(6) are related by

$$k_0 = \left(h^2 - \varepsilon_1\mu_1\frac{\omega^2}{c^2}\right)^{1/2}, \tag{7}$$

$$k = \left(h^2 - \varepsilon(\omega)\mu(\omega)\frac{\omega^2}{c^2}\right)^{1/2}. \tag{8}$$

The boundary conditions at $x = 0$ require the tangential components of $\vec{E}$ and $\vec{H}$ to be continuous. This yields the surface mode dispersion relation

$$\frac{k}{\varepsilon(\omega)} + \frac{k_0}{\varepsilon_1} = 0. \tag{9}$$

In the s polarization case the fields have the form

$$\vec{E}^{(1)} = \vec{a}_y A e^{k_0 x} e^{ihz}, \tag{10}$$

$$\vec{H}^{(1)} = -\frac{c}{\omega\mu_1} A\big(h\vec{a}_x + ik_0\vec{a}_z\big) e^{k_0 x} e^{ihz} \tag{11}$$

for $x < 0$, and

$$\vec{E}^{(2)} = \vec{a}_y B e^{-kx} e^{ihz}, \tag{12}$$

$$\vec{H}^{(2)} = -\frac{c}{\omega\mu(\omega)} B\big(h\vec{a}_x + ik\vec{a}_z\big) e^{-kx} e^{ihz} \tag{13}$$

for $x > 0$. The continuity conditions at $x = 0$ yield the surface polariton dispersion relation

$$\frac{k}{\mu(\omega)} + \frac{k_0}{\mu_1} = 0. \tag{14}$$

For surface polaritons the field amplitudes decay in an exponential fashion as one moves away from the interface into either medium. Therefore both k_0 and k have to be real, and from (7) and (8) it follows that the surface polaritons are restricted to regions in which both

$$h^2 > \varepsilon_1\mu_1(\omega/c)^2 \tag{15}$$

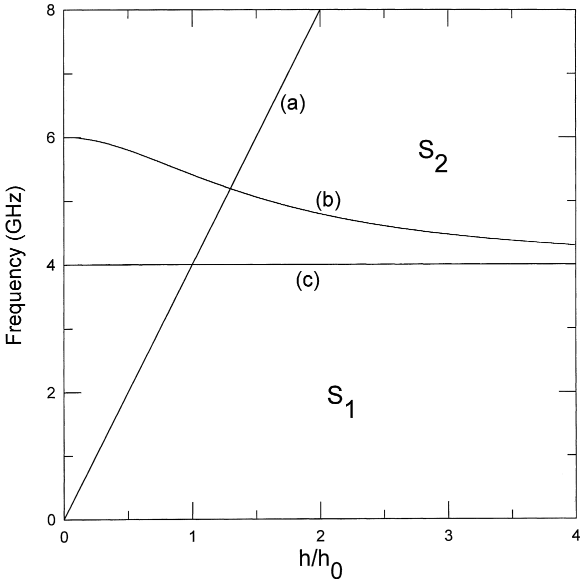

Fig. 1. Regions in the (h, ω) plane. h, the wavevector component parallel to the interface, is expressed in units of $h_0 = \omega_0/c$. The following curves are drawn: (a) $h = \sqrt{\varepsilon_1\mu_1}\,\omega/c$; (b) $h = \sqrt{\varepsilon(\omega)\mu(\omega)}\,\omega/c$; (c) $\omega = \omega_0$. Surface polaritons can exist only in regions S_1 and S_2.

and

$$h^2 > \varepsilon(\omega)\mu(\omega)(\omega/c)^2 \tag{16}$$

hold. These regions in the (h, ω) plane are shown in Fig. 1.

Condition (15) limits us to the right hand side of line (a). Condition (16) is satisfied in the following two regions: S_1, below the $\omega = \omega_0$ line, and S_2, above curve (b). We have solved Eqs. (9) and (14), using the following values for the parameters appearing in (1) and (2): $\omega_p = 10$ GHz, $\omega_0 = 4$ GHz, $F = 0.56$. In this case, the frequency range in which both $\varepsilon(\omega)$ and $\mu(\omega)$ are negative extends from ω_0 up to 6 GHz. For the medium at $x < 0$ we assume $\varepsilon_1 = 1$ and $\mu_1 = 1$. The calculated surface polariton dispersion curves are shown in Fig. 2. There exist two p polarized branches, i.e., solutions of Eq. (9), one in region S_1, branch (a), and one in region S_2, branch (c). Eq. (14), for the s polarized surface polaritons, yields one solution, branch (b), in region S_2. Only branch (b) lies completely within the frequency range in which the medium is left-handed. The upper branch, (c), is only partly inside this range, while the lowest branch, (a),

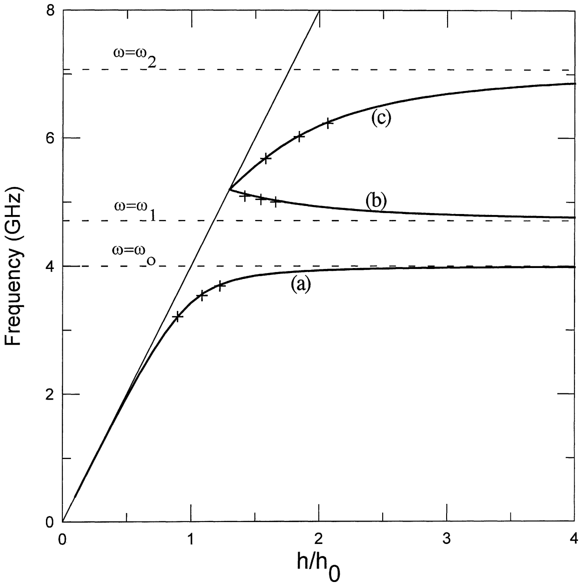

Fig. 2. Surface polariton dispersion curves. Curves (a) and (c) are for p polarization and curve (b) is for s polarization.

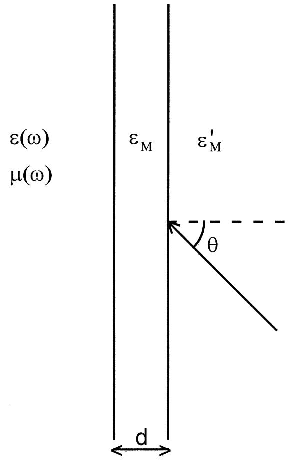

Fig. 3. Geometry of ATR experiment.

lies outside this range. The three dashed lines in Fig. 2 denote the frequencies which the three surface mode branches approach asymptotically at large h values: ω_0 is the resonance frequency of the magnetic permeability, ω_1 is the root of the equation $\mu(\omega) = -\mu_1$, and ω_2 is the root of the equation $\varepsilon(\omega) = -\varepsilon_1$.

Due to their evanescent nature, the surface polaritons will not interact directly with an incoming plane

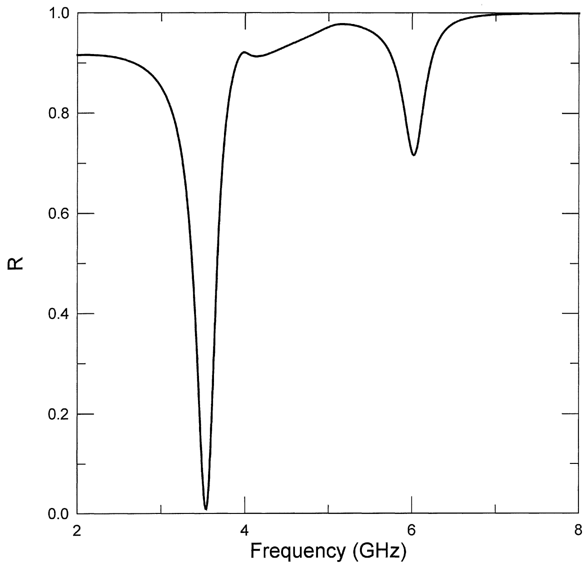

Fig. 4. Calculated reflectivity for the case of p polarization. The parameters used in this calculation are $\varepsilon_M = 1$, $\varepsilon'_M = 3$, $d = 3$ cm, $\theta = 45°$.

wave. However, they can be studied by the method of attenuated total reflection (ATR). This technique has previously been invoked for the investigation of various types of surface polaritons, e.g., plasmon-polaritons in metals [5,6], phonon-polaritons in ionic crystals [7,8], exciton-polaritons in semiconductors [9,10] and magnon-polaritons in magnetic materials [11,12]. We consider the ATR geometry shown in Fig. 3. The radiation is incident from the right hand side, at an angle of incidence θ, in a medium (usually a prism) having a dielectric constant ε'_M. A film of thickness d and dielectric constant ε_M acts as a spacer between the prism and the left-handed medium. If the angle of incidence is such that $\sqrt{\varepsilon'_M}\sin\theta > \sqrt{\varepsilon_M}$, the electromagnetic field will penetrate the gap as an evanescent wave, which can interact with the evanescent surface modes.

We have calculated the reflectivity of the left-handed medium in the configuration of Fig. 3, using classical electromagnetic theory. For this calculation, a damping term, with $\gamma = 0.03$, has been added to both $\varepsilon(\omega)$ and $\mu(\omega)$. A calculated ATR spectrum for the case of p polarization is shown in Fig. 4. There appear two sharp absorption peaks, due to the low and high frequency surface modes, i.e., branches (a) and (c) of

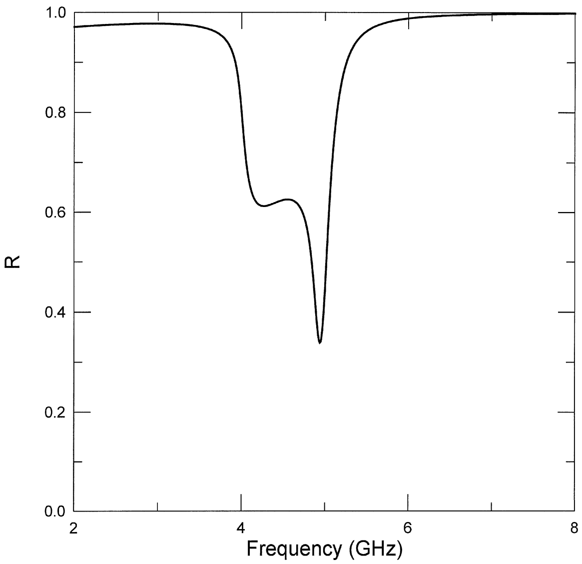

Fig. 5. Calculated reflectivity for the case of s polarization. The parameters used in this calculation are $\varepsilon_M = 1$, $\varepsilon'_M = 3$, $d = 1$ cm, $\theta = 60°$.

Fig. 2, respectively. In the s polarization case, Fig. 5, only one surface mode peak appears, due to branch (b) of Fig. 2. The weaker broader band, just above ω_0, is due to the excitation of bulk polaritons.

It should be noted that the presence of the prism material, having a dielectric constant ε'_M, perturbs the surface polaritons. Thus, the frequencies of the ATR peaks may deviate somewhat from the corresponding surface polariton frequencies, as calculated from (9) or (14). In the ATR experiment, the wavevector component parallel to the surface is given by $h = \sqrt{\varepsilon'_M}(\omega/c)\sin\theta$, and in Fig. 2 the positions of the points in the (h, ω) plane corresponding to the calculated ATR peaks are denoted by crosses. These were calculated for a spacing of $d = 3$ cm, and for the three angles of incidence $\theta = 40°$, $45°$, $50°$. It can be seen that the ATR peaks follow the surface mode dispersion curves rather closely.

In conclusion, we have obtained the dispersion curves of the surface polaritons of a semi-infinite left-handed medium, and demonstrated how they can be observed, using the ATR technique. In a right-handed material surface polaritons exist only in frequency regions of stop-bands, in which no propagating modes are possible. These are regions in which either the dielectric constant is negative (for p polarized modes), or the permeability is negative (for s polarized modes). However, in a bounded left-handed medium, surface polaritons exist also in the pass-band, in which both the dielectric constant and the permeability are negative.

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