Transmission studies of left-handed materials

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Left-handed materials are studied numerically using an improved version of the transfer-matrix method. The transmission, reflection, phase of reflection, and absorption are calculated and compared with experiments for both single split-ring resonators with negative permeability and left-handed materials, which have both the permittivity \( \epsilon \) and permeability \( \mu \) negative. Our results suggest ways of positively identifying materials that have both \( \epsilon \) and \( \mu \) negative, from materials that have either \( \mu \) or \( \epsilon \) negative.

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The TMM has been used to simulate the reflection and transmission from an array of square SRRs. Figure 1(a) shows a diagram of a single square SRR of the type used for
In our case we have $c = 0.33$ mm and the size of SRR $w = 3$ mm. Figure 1(b) shows a three-dimensional realization of the actual LHM that we have simulated.

Figure 2 presents the results of the transmission versus frequency for the split ring resonators alone, and of the LHM, which consists of the SRRs with metallic wires placed uniformly between the SRRs. The square array of metal wires alone behaves as a high pass filter with a cutoff frequency $\nu_c = 19$ GHz. The cutoff frequency $\nu_c$ of the metallic wires is given by $\nu_c = c_{\text{light}}/2a\sqrt{\varepsilon_0}$, where $a$ is the distance between the wires, $c_{\text{light}}$ is the velocity of light in the air, and $\varepsilon_0$ is the dielectric constant of the background. In our case $a = 5$ mm and $\varepsilon_0 = 1$. The cutoff frequency is independent of either value of $\text{Re} \varepsilon_m$ or $\text{Im} \varepsilon_m$ of the metal, provided that either $|\text{Re} \varepsilon_m| > 1000$ or $|\text{Im} \varepsilon_m| > 1000$. The dot-dashed curve is that of the SRR array with $a = 5$ mm. By adding wires uniformly between the SRRs, a pass band occurs where both $\mu_{\text{eff}}$ and $\varepsilon_{\text{eff}}$ are negative (solid line). The transmitted power of the LHM is very high (close to one), because no $\text{Im} \varepsilon_m$ is taken for the metal. We also studied more realistic cases with nonzero $\text{Im} \varepsilon_m$. We found that the resonance frequency $\nu_0$ increases as $|\varepsilon_m|$ increases and saturates to a value of $\nu_0 \approx 8.5$ as $|\varepsilon_m| \to \infty$. This is clearly shown in Fig. 3 where we plot $\nu_0$ versus the magnitude of $\varepsilon_m = \varepsilon_r + i\varepsilon_i$. The resonance frequency is also sensitive to the permittivity of the board that the SRR is lying and of the embedding medium. We have found that $\nu_0$ drops as the dielectric constant of the embed-
dielectric board.

Authors have not done a systematic effort to fit the experimental results of effective permittivity and permeability:

\[ \epsilon_{\text{eff}}(\nu) = 1 - \frac{\nu_p^2}{\nu^2 + i \nu \gamma} \]  

where \( \nu_p \) is the plasma frequency or cutoff frequency \( \nu_c \) of the wires, \( \nu_c = \nu_p \approx 19 \text{ GHz} \) in our case. A negative \( \mu_{\text{eff}} \) accounts for the deep in the transmission in SRRs and its form is given by

\[ \mu_{\text{eff}}(\nu) = 1 - \frac{F \nu^2}{\nu^2 - \nu_0^2 + i \nu \Gamma} \]

where \( F \) is the filling factor which is in our case close to 0.3 and \( \nu_0 = 7.9 \text{ GHz} \) is the resonance frequency of the SRR. We used that \( \gamma = \Gamma = 0.2 \text{ GHz} \). There is very good agreement between the solution of present model and numerical simulations of real structures.

Equations (4) and (5) enable us also to explain the difference between the absorption of SRR and LHM. The portion \( A \) of the field absorbed in the system is

\[ A = (1 - R)(1 - e^{-\alpha z}) \]

where \( R \) is the reflection, \( \alpha \) is the absorption coefficient, and \( z \) is the length of the system. \( \alpha \) is proportional to the imaginary part of the refraction index. Owing to the form of Eqs. (4) and (5), both \( \alpha \) and \( 1 - \exp(-\alpha z) \) possess a sharp maximum near the resonance frequency and are small far outside the resonance gap. This is true both for SRR and LHM. The main difference between the absorption of SRR and LHM is given by the first term in Eq. (6). In SRR, \( 1 - R \) is close to unity for all frequencies outside the resonance gap, where \( 1 - R \) is zero. The absorption \( A \), given by the product in Eq. (6), is therefore nonzero only on the borders of the resonance gap where both functions are nonzero. In contrast to SRR, \( 1 - R \) possess a maximum inside the resonance gap. Then, absorption \( A \) should have therefore one maximum as shown in Fig. 4(b).

In Fig. 5 we present the typical results of the phase of the reflection for both the LHM and the array of SRRs. Obtained data are in qualitative agreement with that obtained from the homogeneous model given by Eqs. (4) and (5). Notice that there is a substantial difference in their characteristics. The
phase of reflection increases with frequency in SRR. In a LHM, the phase as a function of frequency possesses two extrema on the two sides of the resonance gap and decreases with the frequency inside the gap. We derived the same behavior of phase from the standard textbook formulas for the reflection, if the frequency dependent permittivity and permeability follow relations (4) and (5).

We used an improved transfer-matrix method for numerical studies of complex meta-materials. Our numerical data of transmission confirms the presence of a resonant gap, in agreement with theoretical predictions and experiments. Our method enables us to analyze also the reflection and absorption of the light and also the change of the phase of the reflected field. We compare our numerical data that of a simple homogeneous model defined by effective permittivity and permeability [Eqs. (4), and (5)] and found very good qualitative agreement either with numerical simulations or with theoretical predictions. This supports our belief that for the frequencies in the resonance gap our structure possesses negative effective $\mu_{\text{eff}}$ and negative effective $\varepsilon_{\text{eff}}$. Although our method does not allow us to vary the size parameters of SRR continuously, we are able to predict, at least qualitatively, how the position of resonance gap depends on various parameters of the system. We believe that generalization of our numerical method to nonhomogeneous discretization of space will enable us to analyze more realistic structures.

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15J.B. Pendry and P.M. Bell, in Photonic Band Gap Materials, Ref. 1 p. 203.
18For the structures described in Fig. 1, the typical CPU time necessary to calculate transmission for a given frequency is approximately 55 sec/unit cell on an alpha workstation (533 MHz Alpha 21164 processor with 512 MB memory). Most of the CPU time is spent in the normalization of the actual numerical data. As our structure is highly nonhomogeneous (permittivity of systems varies in three or more orders from site to site), we have to normalize the numerical data after 3–4 steps. For studies of more homogeneous samples, the CPU time could be reduced by a factor of 2 to 5.
19For wider resonance gap, the right absorption peak in the SRR may disappear.