Frequency Modulation in the Transmittivity of Wave Guides in Elastic-Wave Band-Gap Materials

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It is shown, for the first time, that the transmittivity of wave guides created as rectilinear defects in periodic elastic band-gap materials oscillates as a function of frequency. The results are obtained using the finite difference time domain method for elastic waves propagating in two-dimensional inhomogeneous media. The oscillations of the transmittivity are due to the richness of modes in the elastic systems and, mainly, due to the periodicity of the potential in the direction of the wave propagation. Results are presented for a periodic array of Pb and Ag cylinders inserted in an epoxy host, as well as for Hg cylinders in an Al host.

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The study of acoustic and elastic-wave propagation in periodic band-gap materials, known as phononic crystals, has received increasing amounts of interest in recent years [1–7]. Besides the possible application of these materials, e.g., in filter and transducer technology, the rich physics of the elastic waves (ELW) is one of the main reasons for the attention to their propagation. This rich physics stems mainly from their full vector character, the mixing of the longitudinal and the transverse component in the presence of inhomogeneities, and the variety of parameters (densities and velocities) that control their propagation in a complex system.

All the above may have as a result the appearance of novel characteristics in the ELW propagation, characteristics that do not appear in the scalar or electromagnetic (EM) wave case, where there are only longitudinal or transverse waves, respectively. In addition, ELW in ultrasonic frequencies have wavelengths of the order of millimeters and this permits the design of a variety of millimeter size structures, easily obtainable. An example is recent experimental work [4–6] where systems have been constructed by drilling millimeter diameter size cylinders in a metallic plate of Al and then filling the cylinders with Hg, oil, or air. These systems gave the chance for the study [4–6] of band gaps, linear defect whispering galleries, surface states, point defects, etc.

The main directions in the phononic crystal research have been inspired by corresponding studies in the photonic band-gap materials. Early work on photonic band-gap materials [8] has shown theoretically as well as experimentally the existence of full band gaps and all kinds of states, as, for example, surface states, defect states, etc. Recently, the guiding of the EM waves through defect modes created in a periodic crystal also has been studied [9,10]. It has been shown that these defect modes can act as wave guides (WG) which lead to the total transmission of an incident wave with frequency in the regime of the gap. The transmittivity of these guides has shown a similar frequency dependence with the transmittivity of the conventional EM guides [11]. It has to be mentioned, however, that only one mode has been allocated in the propagation through these guides, losing thus the phenomenology arising as a consequence of diffraction of several modes. Also, the problem which has been solved was the transmittance of the waves once they are in the WG, omitting thus all the information about the reflectance at the entrance of the guide.

In this work we study the guiding (directed propagation) through linear defects for the case of the ELW. More specifically, we study the ELW propagation through rectilinear defect modes created by removing rows of cylinders [see Fig. 1(a)] from two-dimensional (2D) periodic elastic systems, i.e., systems of cylinders embedded in a host material, exhibiting full band gaps. The first evidence of guiding through such defect modes has already been obtained experimentally for a system consisting of Hg cylinders in Al [5]. Here we intend to do a systematic examination of this guiding, mainly through transmission coefficient calculations. The main result is that *there is a lot of structure*

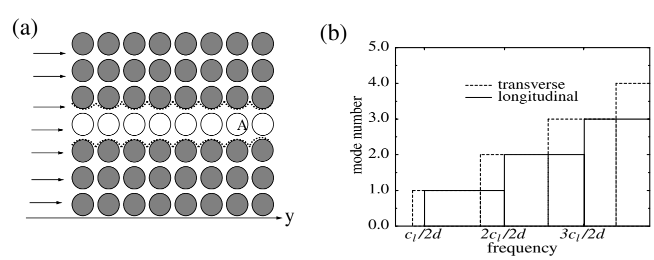

FIG. 1. (a) A cross section of the computational system. The transmitted flux is calculated at the region A. (b) Number of modes (n) that can propagate in a guide of width d as a function of frequency. The mode number is calculated through the simple relation $n = d/(\lambda/2)$, where $\lambda = c/\nu$ (ν: frequency; c: propagation velocity). The solid line shows the number for pure longitudinal modes ($c = c_l$); the dashed line shows the number for pure transverse modes ($c = c_s$).

in the frequency dependence of the transmission coefficient, something that has not been observed so far in EM or scalar wave band-gap materials. Thus one can have the impressive phenomenon of a succession of transmitted and not transmitted frequency regimes (inside the gap). As we discuss later, we attribute this phenomenon to the complexity of the elastic guiding modes (mixed longitudinal and transverse), the interference of these complex modes, and, mainly, the periodicity of the guide "boundaries." Because of the last factor we expect that the effect should be also observable in any wave propagation in guides with periodic boundaries, in particular, in electromagnetic and scalar waves.

Before the presentation and the discussion of results concerning specific systems we will try, generally and briefly, to discuss and demonstrate the particularity and the complexity of the problem of the ELW guiding through WG in phononic crystals.

In Fig. 1(a) we show a schematic representation of a WG formed in a 2D periodic elastic composite. If a wave with frequency inside the band gap impinges on the system, part of it is reflected and the rest is transmitted through the guide. It is important to say that *all the reflection effects take place close to the homogeneous-periodic interface and the entrance of the guide*. Thus, it is important to take into account the effects at the entrance of the guide. This can be done only if one calculates the transmittivity (transmission coefficient) by normalizing the transmitted energy flux by the energy flux of the incident wave and not of the wave that enters in the guide, which propagates through it almost without any losses.

From the theory of the conventional EM guides [11] it is known that there is a cutoff frequency before the wave has the ability to propagate in a guide of a certain width, d. This cutoff frequency is determined (through the corresponding wavelength, λ_o) as $\lambda_o = 2d$. The number of the propagating modes is given by the ratio λ/λ_o. The functional forms of these modes are simple sines and cosines. Thus the transmittivity in the conventional EM guides as a function of frequency, above a cutoff frequency, either is constant (when only one mode propagates) or has a step function behavior in analogy with the number of propagated modes. For the ELW case the situation is more complicated. Here, for a given frequency there are two wavelengths, one for the longitudinal component of the wave and one for the transverse. Thus, even in the artificial case of uncoupled longitudinal and transverse modes, the mode number function [Fig. 1(b)] and thus the transmittivity as a function of frequency encounters more complications than in the EM guides' case. The reality is even more complex as in ELW guides uncoupled longitudinal and transverse modes can not exist. The boundary conditions at the boundaries of the guide result to the mixing of the longitudinal with the transverse component of the wave and thus the formation of mixed and more complex modes (not single sines or cosines). Thus, in the presence of more than one mode, interference phenomena can exist that can lead to unexpected forms of the guide transmittivity. Another complication in the guides formed in periodic crystals arises from the *periodic form* of the guide boundaries [see the dotted, wavelike line in Fig. 1(a)]. This periodic form imposes a periodic potential in the propagation of the guided waves. Taking into account that the system is quasi-one-dimensional, this periodic potential can have a very significant influence in the wave propagation, e.g., the opening of gaps in otherwise allowed frequency regimes. The interplay of the above mentioned variety of phenomena makes the problem of the transmittivity in the elastic wave guides formed in phononic crystals not a trivial one.

In the following, we study the transmittivity of guides formed in realistic elastic-wave band-gap materials. This is done, mainly, through calculations of the transmission coefficient. To calculate the transmission coefficient we use the finite difference time domain method (FDTD). Within the FDTD the wave equation in 2D inhomogeneous media is discretized in both the space and the time domains. Through this discretization one obtains the components of the elastic field as functions of time at any point of a sample. The frequency dependence of the field at a particular point can be obtained by fast Fourier transforming the time results. The transmission coefficient is calculated by dividing the transmitted energy flux by the energy flux of the incident wave. In the calculations presented here we consider systems consisting of parallel infinitely long cylinders and wave propagation in a plane perpendicular to the axes of the cylinders. A brief presentation of the FDTD method as it is applied in this kind of problem is given in Ref. [6] while a more extended presentation can be found in Ref. [12].

To examine in detail the transmittivity in an elastic WG as a function of frequency we choose a periodic system consisting of Pb cylinders embedded in epoxy within a square arrangement and Pb volume fraction $f = 0.283$. This system has been found [7] to exhibit a wide full band gap. We form a wave guide by removing one row of Pb cylinders [see Fig. 1(a)] from a 7×8 unit cell slab of the periodic composite. (The slab is placed in a reservoir of epoxy.) The transmission through the guide formed is shown in Fig. 2(b) (solid line). To obtain the transmission coefficient we consider as an incident wave a longitudinal pulse with a Gaussian envelop in space, centered at $\omega a/c_{lo} \approx 2$ (a is the lattice constant and c_{lo} is the longitudinal wave velocity in epoxy). We obtain the transmission coefficient by calculating the energy flux vector at points close to the exit of the guide [region A in Fig. 1(a)]. The *flux vector is calculated by averaging the flux at different points on a line normal to the wave guide direction.* This averaging is very important as the transmission is not constant along a cross section of the WG.

As one can see in Fig. 2(b), the transmission coefficient (T) for the WG case is close to unity for most of the frequencies inside the gap. By considering monochromatic

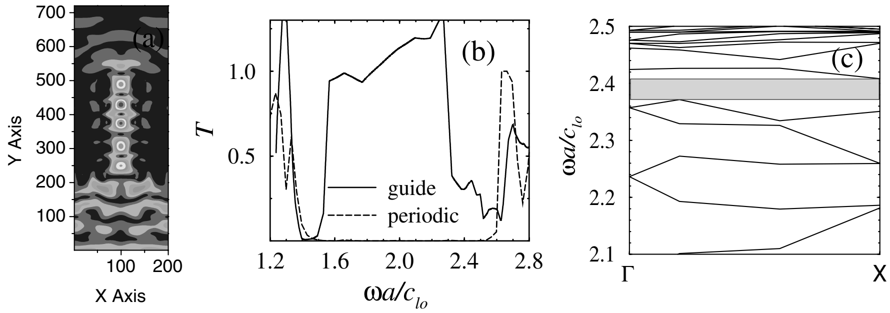

FIG. 2. (a) The square of the field amplitude over a sample consisting of Pb cylinders in epoxy (in a square periodic array with cylinder radius over lattice constant $r_c/a = 0.3$) with one missing row of cylinders (in the [10] direction). The incident wave is a longitudinal monochromatic plane wave with frequency $\omega a/c_{lo} \approx 2$. c_{lo} is the longitudinal sound velocity in epoxy. The axes are in grid units and $a = 30$ grid units. (b) Solid line: Transmission coefficient (T) as a function of frequency for elastic waves propagating in the guide described in connection with Fig. 2(a). Dashed line: The transmission coefficient for the Pb in epoxy periodic host. The incident wave is longitudinal, propagating in the [10] direction. (c) Supercell calculation of the dispersion relation for the system described in (a). The supercell contains 5×5 unit cells. The shadow region shows the gap formed. The results in panels (a) and (b) are calculated with the FDTD method and the results in panel (c) are calculated with the plane wave method.

incident plane waves with frequencies such that $T \approx 1$ one can see an excellent guiding of the waves through the linear defect produced by the removed rows. This guiding is demonstrated in Fig. 2(a) where we show the field over the sample at a given time for a WG formed in a 5×8 unit cells sample.

By examining the transmission coefficient [Fig. 2(b)] one can see, however, that before the end of the gap of the periodic system the transmission coefficient of the guide exhibits a pronounced dip. Sending monochromatic waves with frequencies at the regime of the dip one can see that *the waves are not allowed to enter into the guide*. This result can be understood if one takes into account (i) the complexity of the elastic mixed longitudinal and transverse modes and (ii) the periodic potential that the boundaries of the guide impose. Both can result in the *appearance of gaps in the WG otherwise allowed frequencies*.

Indeed, from the band structure for the system of the guide, one can see a small gap that coincides in position with the above mentioned dip in the transmission coefficient. The band structure for the system of the guide is shown in Fig. 2(c). It has been calculated by using a supercell method [7] based on the well known plane wave method (PW) [1] and considering only the field components in the plane of propagation. (It has to be noted that the FDTD method is also able to calculate dispersion relation [13], although through a more complicated procedure than that of the PW method.)

In order to gain a deeper understanding of the above results, instead of completely removing one row of cylinders from periodic systems, we started to reduce the radii (r_d) of these cylinders gradually, from $r_d = r_c$ (full periodic system) to $r_d = 0$ (system of the guide). For the case of Pb in epoxy we saw that the bands of localized states, which lead to the guiding of the waves, start to emerge mainly from the lower edge of the gap. As the r_d is decreased further, the bands move to the upper edge, trying to fill the gap. The above behavior, however, is not the only one that can be seen in the ELW case. Examining other material combinations we found that, by reducing the radii of one row of cylinders, the bands of localized states emerge *from the upper edge of the gap or from the lower or from both*. This, as far as we know, has not been observed either in the scalar wave case or in the EM one.

In order to demonstrate better the above complexity in the appearance of the localized states we also present results for a system of Ag cylinders embedded in epoxy. We form a guide again by removing one row of cylinders from a periodic slab (a square arrangement of cylinders with $r_c/a = 0.3$). The transmission coefficient for the system of the guide, as well as the corresponding band structure results, is shown in Fig. 3. Here, as one can see in Fig. 3(a), there is a dip in the transmission coefficient relatively close to the midgap of the periodic host [the transmission for the periodic host is shown with the dashed

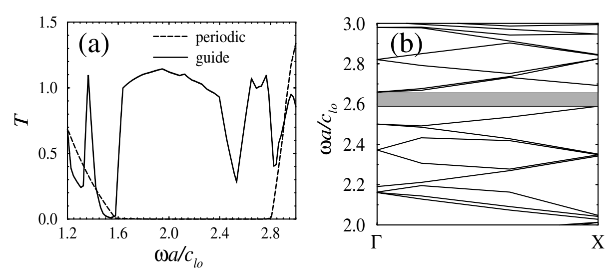

FIG. 3. (a) Solid line: Transmission coefficient (T) for elastic waves propagating in a guide formed in a periodic system of Ag cylinders in epoxy (with $r_c/a = 0.3$: square arrangement of the cylinders). The incident wave is longitudinal, propagating in the [10] direction. Dashed line: Transmission coefficient for the Ag-epoxy periodic host (before the formation of the guide). ω is the angular frequency, a is the lattice constant, r_c is the cylinder radius, and c_{lo} is the longitudinal sound velocity in epoxy. (b) Supercell calculation of the dispersion relation for the system of the guide described in connection with panel (a). We consider a 5×5 unit cell supercell.

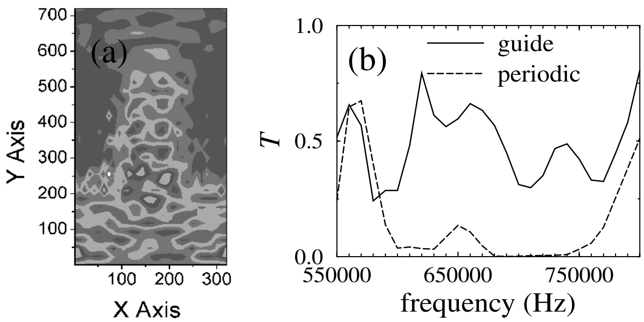

FIG. 4. (a) The square of the field amplitude (at a given time point) over a sample consisted of Hg cylinders in Al (in a square periodic array with cylinder radius over lattice constant $r_c/a = 0.357$) and two missing rows of cylinders (in the [10] direction). The incident wave is a longitudinal monochromatic plane wave with frequency $\nu = 7.5$ MHz. The axes are in grid units and $a = 30$ grid units. (b) Solid line: Transmission coefficient (T) as a function of frequency for elastic waves propagating in the guide described in panel (a). Dashed line: The transmission coefficient for the Hg in an Al periodic host (with $r_c/a = 0.357$). The incident wave is longitudinal, propagating in the [10] direction.

line in Fig. 3(a)]. Thus one can see the impressive result of a frequency regime in which the guide reflects back the incident wave while in the neighboring frequency regimes an excellent guiding of the wave takes place.

In all the above it has to be added also that although ELW present a richness of modes much larger than that of EM or scalar waves, we believe that gaps in the guiding wave propagation can exist also in guides formed in EM or scalar wave band-gap materials as the dominant parameter for this effect must be the periodicity in the direction of propagation rather than the richness of modes. This belief is supported by preliminary results which compare the transmittivity of guides formed in ELW band-gap materials with the transmittivity of the conventional elastic wave guides, and also by the fact that in "almost" one-dimensional systems the periodicity in the direction of propagation by itself is able to lead to gap formation.

The last system that we present here is one considered in a recent experimental study [5]: Hg cylinders in an Al host. In Fig. 4(a) we present the field over the sample at a given time for a guide formed by removing two rows of cylinders from a periodic slab. The result is in good agreement with the corresponding experimental one [5], proving the accuracy and the suitability of the FDTD method in this kind of problem [14]. In Fig. 4(b) we show the transmission as a function of frequency for the system of the guide (solid line) and for the host periodic system (dashed line). Here, again, the transmission coefficient in the regime of the gap exhibits the oscillating or rocking behavior discussed above.

In conclusion, we have shown that the propagation of the elastic waves in wave guides formed in periodic band-gap materials presents a very large richness of effects. This richness is due to the existence of mixed longitudinal and transverse modes and, mainly, due to the periodic modulation of the elastic constants at the WG boundaries. The transmittivity (T) in the WG as a function of frequency has an oscillating behavior with regions of $T \approx 1$ and regions of $T \approx 0$. Therefore, wave guides can be obtained with different transmittivities by changing the frequency within the gap range of the periodic host. All the above are demonstrated by presenting the transmittivity and the band structure in wave guide systems formed in periodic composites of Pb or Ag in epoxy, and Hg in Al.

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