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תודה מיוחדת לשירי כפיר על הסבלנות, ההקשבה התמיכה והאהבה.
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I. ABSTRACT

The field of quantum information (QI) is attracting enormous interest, in view of its fundamental nature and its potentially revolutionary applications to cryptography, teleportation and computing. These applications rely on the ability to "engineer", transfer and maintain the *entanglement* of quantum logic units by their interaction or measurement. Thus entanglement is a key resource of QI.

Among the various quantum logic schemes, those based on photons have the advantage of using very robust and versatile carriers of QI. Yet the main impediment towards their operation at the few-photon level is the weakness of photon-photon interaction in conventional media. A promising avenue has been opened by studies of enhanced nonlinear coupling via electromagnetically induced transparency (EIT) in multilevel atomic vapors, allowing highly efficient cross-phase modulation (XPM) of two optical beams. We explore this new avenue, aiming towards the ultimate goal of achieving unprecedented degree of control over the entanglement of photons in these schemes.

Throughout the investigation of these QI processing schemes, we are mainly interested in getting new insights and deep understanding of quantized field-matter interactions and their control.

A more classical avenue that has emerged in the course of the research, is concerned with vector solitons formed by two optical beams via XPM within the EIT media.

Contents

II. INTRODUCTION

A. The role of entanglement in Quantum Information

An information theory that is based on quantum mechanics extends and completes classical information theory. Apart from quantum generalizations of classical notions such as sources, channels and codes, the Quantum Information Theory (QIT) includes two complementary kinds of information: classical information and quantum entanglement. Entanglement is a unique feature of quantum mechanics, whereby the quantum states of two or more particles (e.g., atom, photon or ion) are highly correlated and cannot be separated, that is, each particle's quantum state is described with reference to the other particle. If two particles are entangled, then by performing a measurement on one of the particles we can immediately know the state of the other particle, even if they are separated in space. While classical information can be copied but can only be transmitted forward in time, entanglement cannot be copied but can connect any two points in space-time [1]. Conventional processing operations destroy entanglement but quantum operations can create it and use it for example, to speed-up certain classical computations.

B. Quantum computing and teleportation

A new class of computers is possible using physical components that obey quantum mechanical laws. The register of a quantum computer is composed of many two-state systems (quantum bits or qubits). A qubit is typically a microscopic system, such as an atom, nuclear spin or photon, where the states 0 and 1 are represented by two distinguishable states of the microscopic system (e.g., vertical and horizontal photon polarizations: $|0\rangle \equiv \leftrightarrow$, $|1\rangle \equiv \updownarrow$). Unlike its classical analogue, a qubit can not only be in one of the basis states $|0\rangle$ or $|1\rangle$, but also in any superposition state of the form: $|\psi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle$, where $|\alpha|^2$ and $|\beta|^2$ represent the probability to be in states $|0\rangle$ and $|1\rangle$ respectively. A qubit can therefore represent a continuum of states in a 2D complex vector space.

A quantum computation is being processed by a succession of two-qubit quantum gates: coherent interactions between specific pairs of qubits, by analogy to classical digital computation as a succession of Boolean logic gates. The time evolution of an arbitrary quantum state is computationally more powerful than the evolution of a digital logic state. The quantum computation can be thought of as a coherent superposition of digital computations proceeding in parallel. Shor has shown [2] how this parallelism can be exploited to develop polynomial-time quantum algorithms for computational problems, such as factoring numbers into primes, which was considered an intractable problem before Shor's algorithm. The factorization of large composite numbers into primes is a problem which is the basis of the security of many classical-key cryptography systems. Quantum computers also provide exponential speed-up for simulating many-particle physical systems [3].

The mathematical definition of a two-qubit state is:

$$|\psi_{AB}\rangle = c_{00}\,|0_A 0_B\rangle + c_{01}\,|0_A 1_B\rangle + c_{10}\,|1_A 0_B\rangle + c_{11}\,|1_A 1_B\rangle \tag{1}$$

In general, this state is an entangled two qubit state, thats is, it cannot be factorized into a product state of the two individual qubits:$(\alpha\,|0\rangle + \beta\,|1\rangle)_A(\alpha'\,|0\rangle + \beta'\,|1\rangle)_B$. Only when the coefficients in (1) satisfy $c_{00} = \alpha\alpha'$, $c_{01} = \alpha\beta'$, $c_{10} = \beta\alpha'$, and $c_{11} = \beta\beta'$, the state can be factorized into a product of two states and is therefore non-entangled.
An important example of entangled two-qubit states are the Bell-states: $|B_{00}\rangle = \frac{1}{\sqrt{2}}(\,|00\rangle + |11\rangle)$, $|B_{01}\rangle = \frac{1}{\sqrt{2}}(\,|01\rangle + |10\rangle)$, $|B_{10}\rangle = \frac{1}{\sqrt{2}}(\,|00\rangle - |11\rangle)$, and $|B_{11}\rangle = \frac{1}{\sqrt{2}}(\,|01\rangle - |10\rangle)$. These maximally entangled states (often called Einstein-Podolsky-Rosen or EPR pair) are widely employed in quantum communication protocols, such as teleportation and quantum dense-coding, as well as in fundamental tests of the locality of quantum mechanics [4]. In the first stage in such protocols, a pair of particles in a Bell-state is shared between two parties. In the second stage, this shared entanglement is used to achieve transmission of a qubit via two classical bits or transmission of two classical bits via a single qubit.
Superdense coding and teleportation have recently received considerable experimental attention. The Innsbruck group [5] implemented a version of superdense coding in which three distinguishable states are created by manipulating one member of the EPR pair of polarization-entangled photons. The same group have also demonstrated teleportation using these photon states[6].

The ability to preserve and manipulate an entangled states is the distinguishing feature of quantum computers and teleportation schemes, responsible both for their advantages and for the difficulty involved in building them.

C. Quantum gates

It was shown [7] that any quantum computation can be expressed as a sequence of one- and two- qubit quantum gates, that is, unitary operations acting on one or two qubits at a time.

The most general one-qubit gates are described by a 2×2 unitary matrix $\begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix}$, mapping $|0\rangle$ to $\alpha\,|0\rangle + \beta\,|1\rangle$, and $|1\rangle$ to $\gamma\,|0\rangle + \delta\,|1\rangle$. One-qubit gates can be physically implemented quite easily, for example, by quarter- or half-wave plates acting on polarized photons.

The standard two-qubit gate is the controlled-NOT gate, which flips the 'target' (second) qubit if its 'control' (first) qubit is $|1\rangle$: it interchanges $|10\rangle$ with $|11\rangle$ and does not change the $|01\rangle$ and $|00\rangle$ states. Another example is the controlled phase gate which can be represented by the matrix $\begin{pmatrix} 1 & 0 \\ 0 & e^{i\phi} \end{pmatrix}$, mapping the state $|11\rangle$ to $e^{i\phi}\,|11\rangle$ while leaving the other states unchanged.

Two-qubit gates are much more difficult to realize because they require two separated QI carriers to be brought into strong and controlled interaction. Theoretical suggestions for optical implementations of the controlled-phase logic gate are the main theme of the thesis and will be discussed in detail in the first three chapters.

D. QI Schemes

The requirements for realizing a quantum computer are scalable quantum bits (qubits) that can (i) be well isolated from the environment, preventing decoherence and entanglement loss; (ii) be initialized, measured and undergo controllable interactions to implement a universal set of quantum logic gates.

NMR techniques have been used to implement the most advanced algorithms [8]. The limitations of such systems, where the information is encoded in a *mixed* state and the measurement is done on an ensemble, is the difficulty to scale it to many qubits with high fidelity.

In contrast, ion-trap systems appear to be more promising [9]. The basic idea is to

store a qubit in a state of an ultracold trapped ion. Ions strongly interact via mutual coulomb repulsion, allowing unitary manipulation and control of the qubits by lasers [10]. The drawback of quantum computations with ions is their strong interaction with the environment due to their charge. This leads to decoherence caused by technical noise sources. Nevertheless, beautiful experiments for QI processing with trapped ions have recently been demonstrated at NIST [11].
Neutral atoms trapped in an optical lattice were suggested in [12]. This technology appears to have good prospects in the long run [13]. Solid state systems including spin qubits in semiconductors [14], quantum dots [15, 16] and nitrogen vacancies in diamonds [17] can offer high-scalability. These are also good candidates for the future.
Recently, entanglement between two superconducting qubits was demonstrated [18]. The disadvantage here is that strong interaction in a condensed matter environment makes decoherence a difficult problem.
Single photon qubits [19–22] offer both the advantages of isolation from the environment, as well as the best level of control over their quantum states. They are therefore our chosen candidates for quantum information processing, as will be described in detail in this thesis.

E. QI with photons

As stated above, the main impediment towards the successful development of the QI field is *decoherence*, i.e, the loss of entanglement and QI by the effect of the environment [23]. We plan to overcome this impediment by using photon-based schemes in nonconventional media [24–26].
Photons are ideal carriers of QI, as they travel at the speed of light and are negligibly affected by decoherence. Therefore, photon-based schemes [19–22] have the advantage of using very robust and versatile carriers of QI.
However, the disadvantage of operating at the few-photon level is the weakness of their interactions (optical nonlinearities) in conventional media [27]. This weakness of photon-photon interactions has precluded so far the construction of *deterministic* quantum teleportation and cryptography schemes, which require the entanglement of few-photon fields and their sharing by distant partners [28]. The alternative chosen by many workers has been *proba-*

bilistic, based on linear operations followed by measurements and *postselection* of only the desired measurement outcomes [22, 29].

A promising avenue has been opened by studies of enhanced nonlinear coupling via electromagnetically induced transparency (EIT) in atomic vapors, which will be explained in detail in the following section.

F. Control of photon propagation and interactions for quantum computing

1. EIT and Dark State Polaritons

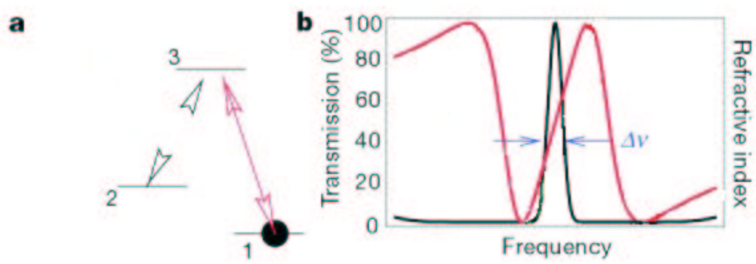

FIG. 1: EIT

Our main vehicle is based on EIT and the associated dark-state polaritons. These occur in three-level atoms with a pair of low-energy states (Fig. 1a), for example, sublevels of the ground electronic state of alkali atoms, having different magnetic moments. These atoms interact with two near-resonance laser fields: a probe field, coupling levels $|1\rangle$ and $|2\rangle$ and a control field, coupling levels $|2\rangle$ and $|3\rangle$.

Starting with all the atoms in the ground state $|1\rangle$, and applying first the control field, we stimulate the system into the so-called "dark" coherent superposition:

$$|Dark\rangle = \frac{\Omega_c\,|1\rangle - \Omega_p\,|2\rangle e^{[i(\mathbf{k}_p-\mathbf{k}_c)\cdot\mathbf{r}-i(\omega_p-\omega_c)t]}}{\sqrt{\Omega_c^2+\Omega_p^2}} \tag{2}$$

which is a stationary eigenstate of this three-level system. Note that with $\Omega_p = 0$ this superposition reduces simply to $|1\rangle$, our initial state.

If one switches on the probe field slowly enough (adiabatically), the system will remain in this eigenstate, which turns into a superposition of levels $|1\rangle$ and $|2\rangle$, eliminating absorbtion and preventing the population of the excited level $|3\rangle$. The perfect EIT condition is two-photon resonance, i.e, the energy difference between the two laser fields exactly matches the energy difference between the levels $|1\rangle$ and $|2\rangle$. This condition leads to a very narrow transparency window, which can be increased by applying stronger control field. As illustrated in Fig. 1b, within this transparency window, the refractive index experiences steep variation as a function of the frequency, resulting in very low group velocity of the probe pulse:

$$v_g = \frac{c}{n + \frac{dn}{dw}} \tag{3}$$

where n is the refractive index and c is the speed of light in vacuum. The group velocity is proportional to the intensity of the control field and inversely proportional to the atom density [30].

If we now switch off the control field, letting its intensity fall off to zero, the result is a complete halt of the probe field. (see references [31–33] about "*stopping the light*").

The dynamics of light propagation in EIT is usually discussed in terms of *polaritons*, which are a superposition of the electromagnetic field and the collective atomic component (atomic spin coherence). As the light field enters the medium, its energy is compressed by a factor v_g/c, i.e., the energy of the pulse is much smaller inside the medium, while the remaining energy is used to establish the coherence between the levels $|1\rangle$ and $|2\rangle$, flipping atomic dipoles. This combined excitation of photons and dipoles (atomic "pseudospins") spins is the dark-state polariton [30].

2. *Cross-phase modulation (XPM)*

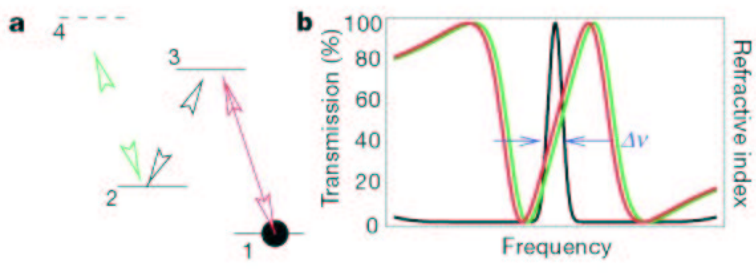

FIG. 2: Cross phase modulation

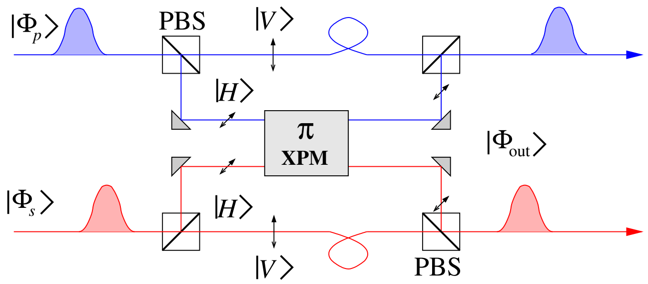

FIG. 3: Proposed implementation of the optical CPHASE logic gate between two single-photon qubits, using polarizing beam-splitters (PBS), and π cross-phase modulation (XPM) studied here.

Among the effects made possible by EIT, we find XPM to be most useful for our purposes. As described above, in the vicinity of two-photon resonance the refractive index dispersion is very steep. Therefore, small energy shifts result in large refractive-index change (giant Kerr effect), allowing highly efficient cross-phase modulation (XPM) of two optical beams, namely, two-field interaction resulting in their mutual nonlinear phase shift [24, 30, 34, 35]. We now consider a four-level system in the N-shaped configuration (Fig. 2a), where in addition to the EIT conditions we apply an off-resonant field $\mathcal{E}_s$, which couples the transition $|2\rangle$ to $|4\rangle$. This field induces an AC Stark shift and effectively changes the energy of level $|2\rangle$, resulting in a change of the resonant-frequency for the probe field (EIT condition). This change in the resonant frequency leads to a change in the index of refraction (Fig. 2b), corresponding to giant XPM [34, 36, 37]. For a recent experimental realization see [38].
We exploit this highly efficient XPM of two optical beams in each chapter of this thesis, either for the purpose of achieving the deterministic entanglement of two photons (Chapters III-V), or, for a more classical communication application, which is the unique new type of vector solitons in EIT media (Chapter VI).

3. *Photonic band gap materials (PBG)*

One way of maximizing XPM involves the use of PBG materials. Photonic crystals (PCs) are structures having spatially periodic dielectric constant. They are also known as photonic band gap (PBG) materials and are considered to be the photonic analogues of solid-state electronic crystals. They were originally proposed by John and Yablonovitch [39] in order to realize two new optical phenomena: localization (trapping) of light and complete inhibition of spontaneous emission [40]. The band structure of PCs refers to the spectral density of propagating photon modes. For the range of frequencies inside the gap, the density of modes (DOM) is identically zero, which means that the propagation of electromagnetic waves with frequency within the PBG is forbidden in all directions, for $3D$ PBG, or in the directions of periodicity, for $D < 3$ PBG. These 'forbidden' modes undergo total (Bragg) reflection in the periodic structure.

A $3D$ PBG reflects light coming from any direction, while a $2D$ ($1D$) PBG reflects only light coming from the plane (direction) of periodicity. A $1D$ PBG is simple enough to realize, by arranging an periodic array of dielectric layers, since for any difference in the index of refraction a band gap will appear. The width of this gap will grow with the index-contrast. This is not the case with $2D$ and $3D$ PBG, where the index-contrast should be large enough for a PBG to appear. In addition, the lattice constant must be of the order of the wavelength of the EM waves. One stronger condition on the density of dielectric scatterers is that the microscopic (Mie) scattering resonance (within a unit cell) and the macroscopic Bragg resonance should overlap (see [41] and references therein).

The first experimental PBG was created by Yablonovitch and co-workers [39] with a band gap in the microwave region. Unfortunately, the Yablonovitch fabrication method cannot be easily extended to the optical region. $3D$ PBGs have been developed by other methods [42–44] down to the near infra-red region [45].

We have considered a PBG to be a powerful means of trapping a photon, allowing it to repeatedly interact with another photon and thus maximize their XPM (Chapter III-IV). This requires the ability to dynamically shift or switch-on and off the PBG, so as to allow the photon to enter and exit the medium freely (Chapter IV).

G. Existing schemes of Photon-Photon Entanglement

Several studies have predicted the ability to achieve an appreciable nonlinear phase shift of extremely weak optical fields [46] or a two-photon switch [37], using the driven N-configuration of atomic levels. One of the main hindrances of such a scheme is the *mis-match* between the group velocities of the field that is subject to EIT and its nearly-free propagating partner, which severely limits their effective interaction length [36].

There are schemes [34, 35] that can remove this bottleneck, by basically modifying the nonlinear interaction of weak optical pulses. The scheme in Ref [35] consists in *symmetric photon-photon coupling by cold atoms with Zeeman-split sublevels* and it relies on simultaneous EIT for both fields interacting with magnetically (Zeeman-) split sublevels in the presence of two driving fields. It thereby renders their group velocities equal and allows their cross-coupling over long distances, bringing it to its ultimate limit of efficiency. Another scheme [34] for achieving large interaction lengths and highly efficient two-field coupling requires an equal mixture of two isotopic species, interacting with two driving fields and an appropriate magnetic field: Both schemes can either yield giant cross-phase modulation or ultrasensitive two-photon switching in cold vapor, without resorting to photonic crystals or cavities [47].

The drawback of these or any other schemes that involve co-propagating pulses lies in the fact that, since the phase shift of each pulsed field is proportional to the intensity of the other, it will be maximal at the pulse peak and vanish at the tails. This feature produces *frequency chirping of the pulses* and, therefore, their spectral broadening. Even if the resulting spectral width does not exceed the transparency window of the EIT resonance, the multimode character of the interaction between two cross-coupled few-photon pulses unduly complicates their entanglement.

We have suggested that this drawback may be remedied [48] via controlled modification of the photonic density of states in gaseous EIT media, by spatially modulating their refractive index with an off-resonant standing light wave. By dynamically varying the spectral width and edges of the resulting photonic band gap (PBG) in time, a propagating light pulse can be converted into a stationary photonic excitation inside the PBG, where its propagation is forbidden. The concept of optically-induced PBGs [49] together with a recent experimental progress in trapping and manipulating light pulses in dynamically controlled PBGs in atomic vapors [50], can be highly instrumental. The trapped photonic excitation can induce large nonlinear phase shifts at the *single-photon level* that can be chirp-free, for specific schemes [24, 25] exploiting its standing-wave character. These features open up the way for possible QI applications without the limitations associated with travelling wave configurations [36] and without invoking cavity QED techniques [47, 51].

H. Spatial Optical Solitons

In addition to the photon-photon entanglement, we have found that XPM has unique advantages in the field of optical solitons.
Optical beams have natural tendency to diffract upon propagation in an homogenous linear media. For example, a quasi-monochromatic Gaussian beam propagating in a linear medium, will have the narrowest width at some particular plane in space at which the wavefront of the beam is planar. However, as the beam propagates away from that plane, the wavefront becomes quadratic and the beam diffracts. In nonlinear medium, the refractive index is modified by the presence of light, and therefore the propagation of an optical beam can be different from its propagation in a linear material. In particular, under certain conditions it is possible to keep the beam wavefront planar, thus allowing a non-diffracting beam to propagate. This self-trapped beam is called an optical spatial soliton [52]. Such solitons will appear whenever the nonlinearity can compensate the diffraction, leading to a stable soliton propagation.
Spatial solitons may provide a powerful means of creating reconfigurable all-optical circuits where light is guided and controlled by light itself. One of the goals of modern nonlinear optics is the development of the ultimate fast, all-optical device in which light can be used to control light. The unique possibilities of reconfigurable circuits created in nonlinear bulk media without any fabricated optical waveguide can be achieved by employing the fundamental concept of light guiding light, based on the propagation of optical spatial solitons [52]. Spatial solitons are considered to be efficient information-carrying units. The process of classical all-optical switching can be associated with the evolution of different types of spatial optical solitons and interactions between them. Other applications center on some of the unique properties of soliton collisions, where one can exploit soliton collisions for implementing logic functions, or even classical computation. The use of solitons for long-distance communications is under active research from the viewpoint of fundamental physics and industrial applications. [53]

I. Scope of the work

In this thesis we have identified and explored in depth several promising novel approaches to photon-photon entanglement in gases and solids, as well as to optical solitons. Our findings suggest the feasibility of a controlled-phase quantum logic gate via XPM between two single photons,

Namely, we have proposed promising novel approaches to optically-induced (-controlled) polariton-polariton entanglement with suppressed decoherence in gases and solids, via XPM of two optical beams. These findings suggest the possibility to realize a controlled-phase logic gate between two single photons, and new soliton effects. Our novel approaches include: (a) giant nonlinearity and entanglement of single photons via electromagnetically induced transparency (EIT) within optically-induced photonic bandgaps (Chapter III) and (b) within solid-state photonic bandgap (PBG) structures (Chapter IV); (c) long-range interaction between EIT-polaritons via dipole-dipole forces (Chapter V), leading to entanglement of two single-photon pulses with moderated transverse confinement; (e) Classical vector solitons formed by two optical beams via XPM within EIT media (Chapter VI).

In Chapter III, we study a novel regime of giant Kerr-nonlinear interaction between two ultraweak optical fields in which the cross-phase modulation is not accompanied by spectral broadening of the interaction pulses. This regime is realizable in atomic vapors, when a weak probe pulse, upon propagating through the EIT medium, interacts with a signal pulse that is dynamically trapped in a PBG created by spatially-periodic modulation of its EIT resonance. We find that conditional phase shifts as large as π, resulting in photon-photon entanglement, can be obtained in this regime. The attainable π phase shift, accompanied by negligible absorption and quantum noise, is shown to allow a high-fidelity realization of the CPHASE (controlled-phase) universal logic gate between two single-photon pulses.
Notwithstanding its highly promising advantages, deterministic EIT-polariton entanglement faces other serious difficulties. Small group velocities that correspond to long interaction times, and thus large conditional phase shifts, in a medium of finite length (typically of a few centimeters) [34, 54] impose limitations on the photonic component of the signal polariton, whose magnitude determines the conditional phase shift. Copropagating pulses pose yet another difficulty: since the conditional phase-shift of each pulse is proportional to the local

intensity of the other pulse, different parts of the interacting pulses acquire different phase shifts, which causes their frequency chirp and spectral broadening.

As we show in Chapter III, the foregoing difficulties may be overcome via controlled modification of the photonic density of states in gaseous EIT media, by modulating their refractive index with an off-resonant standing light wave [48]. By properly tuning the resulting photonic band structure, a propagating signal pulse can be converted into a standing-wave polaritonic excitation inside the photonic band gap (PBG). The trapped signal polariton, having an appreciable photonic component, can impress a large phase shift that is *spatially-uniform* (across the pulse) upon the propagating probe, at the single-photon level.

We formulate the basic theory underlying our scheme and give an analytical solution of the equations of motion for the two interacting quantum fields. We study the cross-phase modulation between the fields. Using our solution, we calculate the output states for two multimode coherent fields, as well as for two single-photon multimode Fock-states, enabling the realization of a deterministic CPHASE logic gate between two single-photon pulses representing qubits.

In Chapter IV we put forward a mechanism that may produce strong photon-photon interactions along with suppressed quantum noise and give rise to their entanglement with high fidelity, by combining the advantages of their dispersion in PBG structures and of the strongly enhanced nonlinear optical coupling achievable via EIT in an appropriately doped medium. The main idea is that a single-photon signal pulse is adiabatically converted into a standing-wave polaritonic excitation inside the periodic structure. This trapped polariton, having an appreciable photonic component, can impress a large, spatially-uniform phase shift upon a slowly propagating probe polariton. This task can further be facilitated by employing 2D- or 3D-periodic structures with defects where the two pulses interact via tightly confined modes [55–58].
Giantly enhanced cross-phase modulation with suppressed spectral broadening is predicted between optically-induced dark-state polaritons whose propagation is strongly affected by photonic bandgaps of spatially periodic media with multilevel dopants. This mechanism is shown to be capable of fully entangling two single-photon pulses with high fidelity.

In Chapter V, we present our study of a novel scheme for the generation of conditional phase shifts between two colliding slow-light polaritons in atomic vapors. The nonlinear interaction is provided by strong, long-range dipole–dipole interactions between Rydberg states of the multi-level atoms in a ladder configuration. In contrast to the previous schemes, this mechanism allows for a homogeneous conditional phase shift of π even for moderate transverse confinement of single-excitation wavepackets, which is necessary for the realization of deterministic phase gate between single-photon pulses.

Instead of resorting to the rather weak short-range collisional interactions of atoms [59], here we follow a different approach that is based on the long-range dipole-dipole interactions between atoms in the internal Rydberg states, which are populated only in the presence of polaritons. In a static electric field, such states possess large permanent dipole moments [60]. Thus the strong nonlocal dipole-dipole interaction between the atoms can further enhance the effective interaction time between the polaritons. We derive and solve the effective one-dimensional equations of motion for the polariton operators. We show that under experimentally realizable conditions, the conditional phase shift in a collision of two single-quantum polaritons is spatially homogeneous and can be sufficiently large for the implementation of the quantum phase gate, even for moderate focusing or transverse confinement of interacting pulses.

The advantageous features of the present scheme pave the way for possible QI applications based on deterministic photon-photon entanglement, without the limitations associated with traveling wave configurations [36] and without invoking cavity QED techniques [20, 61, 62].

Despite the extensive discussion of the giant XPM in EIT media, and its recent experimental demonstration [38], its analysis has been mainly restricted to one dimensional (1D) propagation, without considering transverse (diffraction) effects of the cross-coupled beams. In Chapter VI we study unexplored aspects of the giantly-enhanced XPM between two beams subject to EIT: the formation of low-power spatial solitons that arise solely from the balance between diffraction and XPM, with no contribution from SPM.

We theoretically show that the giant Kerr nonlinearity in the regime of electromagnetically induced transparency (EIT) in vapor may cause the formation of hitherto unobserved 1D and 2D spatial Thirring-like vector solitons, wherein the nonlinear terms are solely due to the cross-phase modulation that couples two parallel light beams. This is, to our knowledge, the first physical system supporting spatial solitons with solely XPM interaction.

III. DETERMINISTIC QUATUM LOGIC WITH PHOTONS VIA OPTICALLY INDUCED PHOTONIC BAND GAPS

Deterministic quantum logic with photons via optically induced photonic band gaps

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We study the giant Kerr nonlinear interaction between two ultraweak optical fields in which the cross-phase-modulation is not accompanied by spectral broadening of the interacting pulses. This regime is realizable in atomic vapors, when a weak probe pulse, upon propagating through the electromagnetically induced transparency (EIT) medium, interacts with a signal pulse that is dynamically trapped in a photonic band gap created by spatially periodic modulation of its EIT resonance. We find that large conditional phase shifts and entanglement between the signal and probe fields can be obtained with this scheme. The attainable π phase shift, accompanied by negligible absorption and quantum noise, is shown to allow a high-fidelity realization of the controlled-phase universal logic gate between two single-photon pulses.

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I. INTRODUCTION

The field of quantum information (QI) is attracting enormous interest, in view of its fundamental nature and its potentially revolutionary applications in cryptography, teleportation, and computing [1]. QI processing schemes rely on the ability to "engineer" and maintain the entanglement of coupled systems. Among the various QI processing schemes of current interest [2–7], those based on photons [6,7] have the advantage of using very robust and versatile carriers of QI. Yet the main impediment toward their operation at the few-photon level is the weakness of photon-photon interaction (optical nonlinearities) in conventional media [8]. One way to circumvent these difficulties is to use linear optical elements, such as beam splitters and phase shifters, in conjunction with single-photon sources and detectors, to achieve *probabilistic* photon-photon entanglement, conditioned on the successful outcome of a measurement performed on auxiliary photons [7].

A promising avenue for *deterministically*, rather than probabilistically, entangling single photons has been opened up by studies of giantly enhanced nonlinear coupling in the regime of electromagnetically induced transparency (EIT) in atomic vapors [9,10]. EIT relies on the classical driving fields to induce coherence between atomic levels and transform the field into an atom-dressed polariton propagating in the medium with controllable, arbitrarily small group velocity [11,12]. These studies have predicted the ability to achieve an appreciable *conditional* phase shift, impressed by one weak field upon another [13], or a two-photon switch [14], using the driven N-shaped configuration of atomic levels. One drawback of these schemes has been the mismatch between the group velocities of the probe pulse moving as a slow EIT polariton and the nearly free propagating signal pulse, which severely limits their effective interaction length and the maximal conditional phase shift [15]. This drawback may be remedied by using an equal mixture of two isotopic species, interacting with two driving fields and an appropriate magnetic field, which would render the group velocities of the two weak pulses equal [16]. Alternative schemes to achieve the group velocity matching and strong nonlinear interaction between the pulses employ a single species of multilevel atoms that couple to both fields in a *symmetric* fashion [17,18].

Notwithstanding its highly promising advantages, deterministic EIT-polariton entanglement faces other serious difficulties. Small group velocities that correspond to long interaction times, and thus large conditional phase shifts, in a medium of finite length (typically of a few centimeters) [16–18] impose limitations on the photonic component of the signal polariton, whose magnitude determines the conditional phase shift. Copropagating pulses pose yet another difficulty: since the conditional phase shift of each pulse is proportional to the local intensity of the other pulse, different parts of the interacting pulses acquire different phase shifts, which causes their frequency chirp and spectral broadening.

As we show here, the foregoing difficulties may be overcome via controlled modification of the photonic density of states in gaseous EIT media, by modulating their refractive index with an off-resonant standing light wave [19]. By properly tuning the resulting photonic band structure, a propagating signal pulse can be converted into a standing-wave polaritonic excitation inside the photonic band gap (PBG). The trapped signal polariton, having an appreciable photonic component, can impress a large, *spatially uniform* phase shift upon the propagating probe, at the single-photon level. The advantageous features of the present scheme pave the way for possible QI applications based on deterministic photon-photon entanglement, without the limitations associated with traveling-wave configurations [15] and without invoking cavity QED techniques [20].

In Sec. II we formulate the basic theory underlying our scheme and give an analytical solution of the equations of motion for the two interacting quantum fields. In Sec. III we

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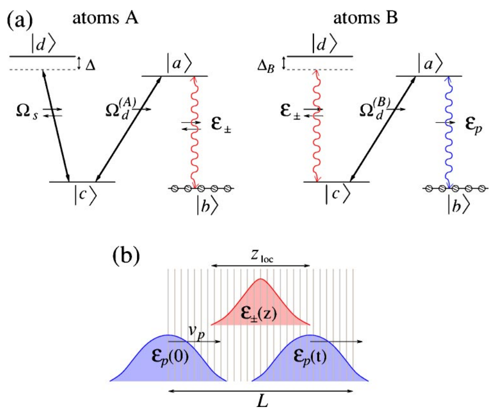

FIG. 1. (Color online) (a) Atomic level scheme involving two species of atoms A and B, aimed at trapping the signal field $\mathcal{E}_{\pm}$ in a PBG and its cross coupling with the slowly propagating probe field $\mathcal{E}_p$. (b) Probe pulse propagation and interaction with the trapped signal pulse.

study the cross-phase-modulation between the fields while in Sec. IV we discuss an explicit realization of a deterministic controlled-phase (CPHASE) logic gate between two single-photon pulses representing qubits. Our conclusions are summarized in Sec. V.

II. EQUATIONS OF MOTION

We consider a cold atomic medium containing two species of atoms A and B, with N-shaped level configurations [Fig. 1(a)]. Atoms A and B correspond to two isotopic species of trapped alkali-metal atoms subject to an appropriate magnetic field that shifts the Zeeman sublevels and tunes the relevant atomic transitions outlined below. All the atoms are assumed to be optically pumped to the ground states $|b\rangle_{A,B}$. Atoms A and B resonantly interact with two running-wave classical fields driving the atomic transitions $|c\rangle_{A,B}\to|a\rangle_{A,B}$ with the Rabi frequencies $\Omega_d^{(A,B)}$, respectively. In the absence of levels $|d\rangle_{A,B}$, this situation corresponds to the usual EIT for the weak (quantum) signal $\mathcal{E}$ and probe $\mathcal{E}_p$ fields which are acting on the transitions $|b\rangle_{A,B}\to|a\rangle_{A,B}$: In the vicinity of a frequency corresponding to the two-photon Raman resonances $|b\rangle_{A,B}\to|c\rangle_{A,B}$, the medium becomes transparent for both weak fields [9–11]. This transparency is accompanied by a steep variation of the refractive index. Atoms A, in addition, dispersively interact with a standing-wave classical field having the Rabi frequency $\Omega_s(z)=2\Omega_s\cos(k_s z)$ and detuning $\Delta\gg\Omega_s$ from the atomic transition $|c\rangle_A\to|d\rangle_A$. This field induces a spatially periodic ac Stark shift of level $|c\rangle_A$ that results in a spatial modulation of the index of refraction for the signal field according to [19]

$$\delta n=\frac{c}{v_s}\frac{4\Delta_s}{\omega_{ab}}\cos^2(k_s z),$$

where $\Delta_s=\Omega_s^2/\Delta$ is the amplitude of the Stark shift, c/v_s is the ratio of the speed of light in vacuum to the group velocity in the medium, $v_s\propto|\Omega_d^{(A)}|^2$, and $\omega_{\mu\nu}$ is the frequency of the atomic resonance $|\mu\rangle\leftrightarrow|\nu\rangle$. When the modulation depth is sufficiently large, the forward propagating signal field $\mathcal{E}_+$ with a carrier wave vector k near $k_s=w_s/c$ undergoes Bragg scattering into the backward propagating field $\mathcal{E}_-$ with the wave vector $-k$. This scattering of counterpropagating fields into each other forms a standing-wave pattern and modifies the photonic density of states such that a range of frequencies appears in which light propagation is forbidden—a PBG [19]. Both components $\mathcal{E}_{\pm}$ of the signal field dispersively interact with atoms B via the transition $|c\rangle_B\to|d\rangle_B$ with the detuning Δ_B. Thus atoms of species B simultaneously provide EIT for the slowly propagating probe field $\mathcal{E}_p$ and its cross coupling with the signal field $\mathcal{E}_{\pm}$ [13,15,16].

We assume that initially the signal pulse of duration T_s enters the EIT medium, where, in the absence of the standing-wave field $\Omega_s=0$, it is slowed down and spatially compressed, by a factor of $v_s^0/c\ll 1$, to the length $z_{\text{loc}}\simeq v_s^0 T_s$. Once the signal pulse has been fully accommodated in the medium of length L, which requires that $z_{\text{loc}}<L$, it is converted into a standing-wave polaritonic excitation according to the procedure described in [19]. To this end, the driving field $\Omega_{d,0}^{(A)}$, corresponding to the input group velocity $v_s^0\propto|\Omega_{d,0}^{(A)}|^2$, is adiabatically switched off and the pulse is halted in the medium. Next the standing-wave field Ω_s is switched on, thereby establishing the PBG, and finally the driving field is switched back on to a value $\Omega_d^{(A)}>\Omega_{d,0}^{(A)}$, releasing the signal pulse into the PBG. The amplitude of the photonic component of the signal pulse, which is responsible for the cross-phase modulation, is now larger than that at the input by a factor of $\sqrt{v_s/v_s^0}=\Omega_d^{(A)}/\Omega_{d,0}^{(A)}$ [12]. Then, upon propagating through the medium with the group velocity v_p, the probe pulse interacts with both forward and backward components of the signal over its localization length z_{loc} [Fig. 1(b)]. For a large enough product of the signal field intensity $|\mathcal{E}_{\pm}|^2$ and interaction time L/v_p, both pulses accumulate uniform conditional phase shifts which can exceed π. Finally, the signal pulse is released from the medium by reversing the sequence that resulted in its trapping.

Let us now consider the scheme more quantitatively. To describe the quantum properties of the medium, we use collective slowly varying atomic operators $\hat{\sigma}_{\mu\nu}^{(\iota)}(z,t)=(1/N_\iota^z)\sum_{j=1}^{N_\iota^z}|\mu_j\rangle_{\iota\iota}\langle\nu_j|e^{-i\omega_{\mu\nu}^{(\iota)}t}$, averaged over small but macroscopic volume containing many atoms of species $\iota=A,B$ around position z [11]: $N_{A,B}^z=(N_{A,B}/L)dz\gg 1$, where $N_{A,B}$ is the total number of the corresponding atoms. The quantum radiation is described by the traveling-wave (multimode) electric field operators $\hat{\mathcal{E}}_{\pm}(z,t)=\sum_q a_{\pm}^q(t)e^{\pm iqz}$ and $\hat{\mathcal{E}}_p(z,t)=\sum_q a_p^q(t)e^{iqz}$, where a_j^q is the annihilation operator for the field mode with the wave vector k_j+q, k_j being the carrier wave vector of the corresponding field. These single-mode operators possess the standard bosonic commutation relations $[a_i^q,a_j^{q'\dagger}]=\delta_{ij}\delta_{qq'}$, which yield $[\hat{\mathcal{E}}_i(z),\hat{\mathcal{E}}_j^\dagger(z')]=L\delta_{ij}\delta(z-z')$. In a frame rotating with the frequencies of the optical fields, the interaction Hamiltonian has the following form:

$$H = \frac{\hbar N_A}{L}\int dz\{\Delta\hat{\sigma}_{dd}^{(A)} - g_A[\hat{\mathcal{E}}_+ e^{ikz} + \hat{\mathcal{E}}_- e^{-ikz}]\hat{\sigma}_{ab}^{(A)} - \Omega_d^{(A)} e^{ik_d^{(A)}z}\hat{\sigma}_{ac}^{(A)} - 2\Omega_s \cos(k_s z)\hat{\sigma}_{dc}^{(A)}\} + \frac{\hbar N_B}{L}\int dz\{\Delta_B\hat{\sigma}_{dd}^{(B)} - g_p\hat{\mathcal{E}}_p e^{ik_p z}\hat{\sigma}_{ab}^{(B)} - \Omega_d^{(B)} e^{ik_d^{(B)}z}\hat{\sigma}_{ac}^{(B)} - g_B[\hat{\mathcal{E}}_+ e^{ikz} + \hat{\mathcal{E}}_- e^{-ikz}]\hat{\sigma}_{dc}^{(B)}\} + \text{H.c.}, \quad (1)$$

where $g_A = \wp_{ab}^{(A)}\sqrt{\omega_{ab}^{(A)}/(2\hbar\epsilon_0 SL)}$, $g_p = \wp_{ab}^{(B)}\sqrt{\omega_{ab}^{(B)}/(2\hbar\epsilon_0 SL)}$, and $g_B = \wp_{dc}^{(B)}\sqrt{\omega_{dc}^{(B)}/(2\hbar\epsilon_0 SL)}$ are the atom-field coupling constants, $\wp_{\mu\nu}^{(\iota)}$ being the corresponding atomic dipole matrix element and S the cross-sectional area of the quantum fields. To facilitate the analysis, we decompose the induced atomic coherences as

$$\hat{\sigma}_{ba}^{(A)} = \hat{\sigma}_{ba}^{+(A)} e^{ikz} + \hat{\sigma}_{ba}^{-(A)} e^{-ikz},$$

$$\hat{\sigma}_{bc}^{(A)} = \hat{\sigma}_{bc}^{+(A)} e^{i(k-k_d)z} + \hat{\sigma}_{bc}^{-(A)} e^{-i(k+k_d)z},$$

$$\hat{\sigma}_{cd}^{(B)} = \hat{\sigma}_{cd}^{+(B)} e^{ikz} + \hat{\sigma}_{cd}^{-(B)} e^{-ikz},$$

and make the transformations

$$\hat{\sigma}_{ba}^{(B)} \to \hat{\sigma}_{ba}^{(B)} e^{ik_p z}, \quad \hat{\sigma}_{bc}^{(B)} \to \hat{\sigma}_{bc}^{(B)} e^{i(k_p - k_d)z}.$$

Using the slowly varying envelope approximation, we have the following equations of motion for the weak quantum fields:

$$\left(\frac{\partial}{\partial t} + c\frac{\partial}{\partial z}\right)\hat{\mathcal{E}}_p(z,t) = i g_p N_B \hat{\sigma}_{ba}^{(B)}, \quad (2a)$$

$$\left(\frac{\partial}{\partial t} \pm c\frac{\partial}{\partial z}\right)\hat{\mathcal{E}}_\pm(z,t) = i g_A N_A \hat{\sigma}_{ba}^{\pm(A)} + i g_B N_B \hat{\sigma}_{cd}^{\pm(B)}. \quad (2b)$$

The evolution of the atomic operators is governed by the Heisenberg-Langevin equations [10], which are treated perturbatively in the small parameters $g\hat{\mathcal{E}}/\Omega_d$ and in the adiabatic approximation for all the fields [11],

$$\hat{\sigma}_{ba}^{\pm(A)} = -\frac{i}{\Omega_d^{(A)}}\left[\left(\frac{\partial}{\partial t} + \gamma_A - 2i\Delta_s\right)\hat{\sigma}_{bc}^{\pm(A)} - i\Delta_s\hat{\sigma}_{bc}^{\mp(A)} e^{\pm 2i\delta kz}\right] + \frac{i}{\Omega_d^{(A)}}\hat{F}_{bc}^{\pm(A)}, \quad (3a)$$

$$\hat{\sigma}_{bc}^{\pm(A)} = -\frac{g_A\hat{\mathcal{E}}_\pm}{\Omega_d^{(A)}} - i\frac{\hat{F}_{ba}^{\pm(A)}}{\Omega_d^{(A)}}, \quad (3b)$$

$$\hat{\sigma}_{ba}^{(B)} = -\frac{i}{\Omega_d^{(B)}}\left[\left(\frac{\partial}{\partial t} + \gamma_B\right)\hat{\sigma}_{bc}^{(B)} - i\frac{g_B^2(\hat{\mathcal{E}}_+^\dagger\hat{\mathcal{E}}_+ + \hat{\mathcal{E}}_-^\dagger\hat{\mathcal{E}}_-)}{\Delta_B}\hat{\sigma}_{bc}^{(B)}\right] + \frac{i}{\Omega_d^{(B)}}\hat{F}_{bc}^{(B)}, \quad (3c)$$

$$\hat{\sigma}_{bc}^{(B)} = -\frac{g_p\hat{\mathcal{E}}_p}{\Omega_d^{(B)}} + i\frac{\hat{F}_{ba}^{(B)}}{\Omega_d^{(B)}}, \quad (3d)$$

$$\hat{\sigma}_{cd}^{\pm(B)} = \frac{g_B\hat{\mathcal{E}}_\pm}{\Delta_B}\hat{\sigma}_{cc}^{(B)} = \frac{g_B}{\Delta_B}\frac{g_p^2(\hat{\mathcal{E}}_p^\dagger\hat{\mathcal{E}}_p)}{|\Omega_d^{(B)}|^2}\hat{\mathcal{E}}_\pm, \quad (3e)$$

where $\delta k = k_s - k$ is the phase mismatch, $\gamma_A = \gamma_{bc}^{(A)} + \gamma_d^{(A)}\Delta_s/\Delta$ and $\gamma_B = \gamma_{bc}^{(B)}$ are the Raman coherence relaxation rates, $\gamma_d^{(\iota)}$ is the spontaneous decay rate of state $|d\rangle_\iota$, and $F_{\mu\nu}^{(\iota)}$ are δ-correlated Langevin noise operators associated with the relaxation.

To solve the coupled set of Eqs. (2a), (2b), and (3a)–(3e), we introduce new quantum fields $\hat{\Psi}_p$ and $\hat{\Psi}_\pm$ (dark-state polaritons [11]) via the canonical transformations

$$\hat{\Psi}_p = \cos\theta_B\hat{\mathcal{E}}_p - \sin\theta_B\sqrt{N_B}\hat{\sigma}_{bc}^{(B)}, \quad (4a)$$

$$\hat{\Psi}_\pm = \cos\theta_A\hat{\mathcal{E}}_\pm - \sin\theta_A\sqrt{N_A}\hat{\sigma}_{bc}^{\pm(A)}, \quad (4b)$$

where the mixing angles $\theta_{A,B}$ are defined as $\tan\theta_{A,B} = g_{A,p}\sqrt{N_{A,B}}/\Omega_d^{(A,B)}$. It follows from Eqs. (3b) and (3d) that

$$\hat{\Psi}_p = \frac{\hat{\mathcal{E}}_p}{\cos\theta_B} = -\frac{\sqrt{N_B}\hat{\sigma}_{bc}^{(B)}}{\sin\theta_B},$$

$$\hat{\Psi}_\pm = \frac{\hat{\mathcal{E}}_\pm}{\cos\theta_A} = -\frac{\sqrt{N_A}\hat{\sigma}_{bc}^{\pm(A)}}{\sin\theta_A},$$

i.e., inside the medium the photonic component of each polariton is proportional to $\cos\theta_\iota$ while the atomic component to $\sin\theta_\iota$ of the corresponding mixing angle θ_ι. From Eqs. (2a), (2b), and (3a)–(3e), the equations of motion for polaritons are then obtained as

$$\left(\frac{\partial}{\partial t} + v_p\frac{\partial}{\partial z}\right)\hat{\Psi}_p = -\kappa_p\hat{\Psi}_p + i\eta\hat{I}_s\hat{\Psi}_p + \hat{\mathcal{F}}_p, \quad (5a)$$

$$\left(\frac{\partial}{\partial t} \pm v_s\frac{\partial}{\partial z}\right)\hat{\Psi}_\pm = -\kappa_s\hat{\Psi}_\pm + i\eta\hat{I}_p\hat{\Psi}_\pm + i\beta\hat{\Psi}_\mp + \hat{\mathcal{F}}_s, \quad (5b)$$

where $v_p = c\cos^2\theta_B$ and $v_s = c\cos^2\theta_A$ are the group velocities, $\hat{I}_p \equiv \hat{\Psi}_p^\dagger\hat{\Psi}_p$ and $\hat{I}_s \equiv \hat{\Psi}_+^\dagger\hat{\Psi}_+ + \hat{\Psi}_-^\dagger\hat{\Psi}_-$ the intensity (excitation-number) operators for the probe and signal polaritons, respectively, $\kappa_{s,p} = \gamma_{A,B}\sin^2\theta_{A,B}$ the absorption rates, $\hat{\mathcal{F}}_{s,p}$ the associated δ-correlated noise operators, $\eta = \cos^2\theta_A\sin^2\theta_B g_B^2/\Delta_B$ the cross-phase-modulation rate between the polaritons, and $\beta = \Delta_s\sin^2\theta_A$ the coupling rate of the forward and backward propagating components of the signal polariton. In Eq. (5b), the linear phase modulation has been absorbed in the signal polariton via the unitary transformation $\hat{\Psi}_\pm \to \hat{\Psi}_\pm e^{2i\beta t}$, and we have assumed that the effective phase matching condition $2\delta kz \ll 1$ remains satisfied for $0 \le z \le L$ [19]. We have also assumed that the cross absorption is negligible, which requires that $\Delta_B \gg \gamma_d^{(B)}$ [13,14]. Then the cross-phase-modulation η is purely real and is proportional to the intensity of the photonic component of the signal polariton, $\cos^2\theta_A = \hat{\mathcal{E}}_\pm^\dagger\hat{\mathcal{E}}_\pm/(\hat{\Psi}_\pm^\dagger\hat{\Psi}_\pm)$, multiplied by the intensity of the atomic component of the probe polariton, $\sin^2\theta_B = N_B\hat{\sigma}_{cc}^{(B)}/(\hat{\Psi}_p^\dagger\hat{\Psi}_p)$.

Equations (5a) and (5b) are similar to the corresponding equations derived for the case of cross-phase-modulation in a doped photonic crystal [21]. Their general analytical solution for arbitrary initial and boundary conditions of the traveling-wave quantized fields $\hat{\Psi}_{p,\pm}$ is not known. However, when the absorption is small enough to be neglected (see below), for a given time and space dependence of the signal-polariton intensity $\hat{I}_s(z,t)$, the solution for the probe is

$$\hat{\Psi}_p(z,t) = \hat{\Psi}_p(0,\tau)\exp\left[i\frac{\eta}{v_p}\int_0^z \hat{I}_s(z',\tau+z'/v_p)dz' \right], \quad (6)$$

where $\tau = t - z/v_p$ is the retarded time. An analytic solution for the two counterpropagating components of the signal polariton can be obtained only in the case when the spatial dependence of the probe-polariton intensity can be neglected on the scale of z_{loc}, $\hat{I}_p(z,t) \simeq \hat{I}_p(t)$. This requires that $v_p T_p > z_{\text{loc}}$, where T_p is the duration of the probe pulse. Alternatively, the spectral width of the probe $\delta\omega_p \sim T_p^{-1}$ should satisfy $\delta\omega_p < v_p/z_{\text{loc}}$. Then Eq. (5b) can be solved using the Fourier transform technique [19]. The solution for the polariton-mode operators $\hat{\psi}_\pm^q(t) = \int dz\, e^{\mp iqz}\hat{\Psi}_\pm(z,t)$ is given by

$$\hat{\psi}_+^q(t) = \hat{\psi}_+^q(0)e^{i\hat{\phi}_s(t)}\left[\cos(\chi t) - i\frac{q v_g}{\chi}\sin(\chi t) \right], \quad (7a)$$

$$\hat{\psi}_-^q(t) = i\hat{\psi}_+^{-q}(0)e^{i\hat{\phi}_s(t)}\frac{\beta}{\chi}\sin(\chi t), \quad (7b)$$

where $\chi = \sqrt{q^2 v_s^2 + \beta^2}$. Note that all the spatial modes $\hat{\psi}_\pm^q$ of the signal polariton acquire the same q-independent phase shift $\hat{\phi}_s(t) = \eta\int_0^t \hat{I}_p(t')dt'$, with $\hat{I}_p(t) = \hat{\Psi}_p^\dagger(0,\tau)\hat{\Psi}_p(0,\tau)$. It follows from Eqs. (7a) and (7b) that a signal pulse containing only the modes with $|q| \ll \beta/v_g$ will be strongly trapped inside the medium, its wave packet periodically cycling between the forward and backward components while interacting with the probe polariton. We then obtain

$$\hat{\Psi}_+(z,t) = \hat{\Psi}_+(z,0)e^{i\hat{\phi}_s(t)}\cos(\beta t), \quad (8a)$$

$$\hat{\Psi}_-(z,t) = i\hat{\Psi}_+(z,0)e^{i\hat{\phi}_s(t)}\sin(\beta t), \quad (8b)$$

$$\hat{\Psi}_p(z,t) = \hat{\Psi}_p(0,\tau)e^{i\hat{\phi}_p(z)}, \quad (8c)$$

where $\hat{\phi}_p(z) = (\eta/v_p)\int_0^z \hat{I}_s(z')dz'$, with $\hat{I}_s(z) = \hat{\Psi}_+^\dagger(z,0)\hat{\Psi}_+(z,0)$, is the probe phase-shift operator.

Equations (8a)–(8c) are our central result. Let us dwell upon the approximations involved in the derivation of this solution. During the conversion of the signal pulse into a standing-wave polaritonic excitation inside the PBG, the nonadiabatic corrections resulting in its dissipation are negligible provided the medium is optically thick [11],

$$\frac{g_A^2 N_A z_{\text{loc}}}{c\gamma_a^{(A)}} \simeq \varsigma_A \rho_A L \gg 1,$$

where $\varsigma_A = \wp_{ab}^{(A)2}\omega_{ab}^{(A)}/(2\hbar\epsilon_0 c\gamma_a^{(A)})$ is the resonant absorption cross section for the transition $|b\rangle_A \to |a\rangle_A$ and $\rho_A = N_A/(SL)$ is the density of atoms A. Due to nonzero values of q, the trapped signal pulse spreads and eventually leaks out of the medium at a rate

$$\kappa_l \simeq \frac{q^2 v_s^2}{\pi\beta}, \quad 0 \leqslant |q| < \beta/v_s.$$

We can estimate the bandwidth of the signal pulse from its spatial extent as $\delta q \sim v_s/(cL) < |\Omega_d^{(A)}|^2/(\gamma_a^{(A)}c)$, thus obtaining the upper limit for the leakage rate

$$\kappa_l \leqslant \frac{v_s^4}{\pi c^2 \beta L^2}. \quad (9)$$

Hence, the interaction time $t_{\text{int}} = L/v_p$ is limited by $t_{\text{int}} \times \max\{\kappa_s, \kappa_p, \kappa_l\} \ll 1$. The corresponding fidelity of the cross-phase modulation is given by

$$F = \exp[-(\kappa_s + \kappa_p + \kappa_l)L/v_p]. \quad (10)$$

Thus, to minimize the standing-wave field-induced absorption of the signal polariton, due to the enhanced relaxation of Raman coherence γ_A and index modulation exceeding the transparency window, the ac Stark shift should be limited by

$$\Delta_s < \frac{\gamma_{bc}^{(A)}\Delta}{\gamma_d^{(A)}}, \frac{|\Omega_d^{(A)}|^2}{\gamma_a^{(A)}}. \quad (11)$$

At the same time, the bandwidth of the probe is limited by the length of the medium [22],

$$\delta\omega_p < \frac{|\Omega_d^{(B)}|^2}{\sqrt{g_p^2 N_B \gamma_a^{(B)} L/c}} = \frac{|\Omega_d^{(B)}|^2 k_p}{\gamma_a^{(B)}\sqrt{(3\pi/2)\rho_B L}}, \quad (12)$$

where $\rho_B = N_B/(SL)$ is the density of atoms B.

Under these conditions, as can be deduced from Eqs. (5a), (5b), and (8a)–(8c), the time evolution of the system is described by the effective interaction Hamiltonian

$$H_{\text{eff}} = -\frac{\hbar}{L}\int dz[\eta\hat{\Psi}_p^\dagger\hat{\Psi}_p(\hat{\Psi}_+^\dagger\hat{\Psi}_+ + \hat{\Psi}_-^\dagger\hat{\Psi}_-) + \beta(\hat{\Psi}_+^\dagger\hat{\Psi}_- + \hat{\Psi}_-^\dagger\hat{\Psi}_+)]. \quad (13)$$

Its first term is responsible for the cross-phase-modulation between the probe and signal polaritons, while the second term describes the scattering between the forward and backward components of the signal polariton into each other. Since the probe polariton propagates with the group velocity v_p, the implicit time dependence of the effective Hamiltonian (13) is contained in the probe polariton operators as $\hat{\Psi}_p = \hat{\Psi}_p(z-\zeta)$, where $\zeta = v_p t$. Employing the plane-wave decomposition of the polariton operators

$$\hat{\Psi}_p(z) = \sum_q \hat{\psi}_p^q e^{iqz}, \quad (14a)$$

$$\hat{\Psi}_\pm(z) = \sum_q \hat{\psi}_\pm^q e^{\pm iqz}, \quad (14b)$$

where the mode operators $\hat{\psi}_i^q$ obey, to a good approximation [11], the bosonic commutation relations $[\hat{\psi}_i^q, \hat{\psi}_j^{q'\dagger}] \simeq \delta_{ij}\delta_{qq'}$,

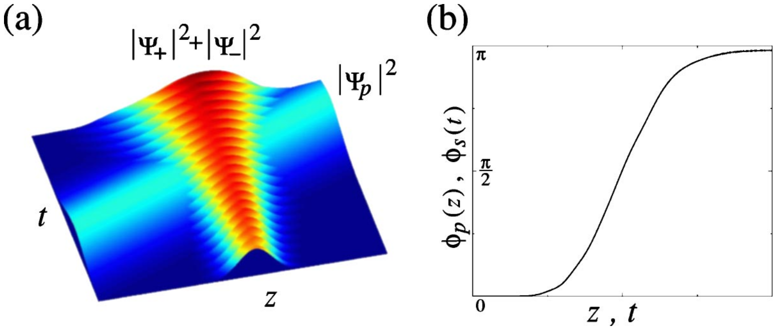

FIG. 2. (Color online) (a) Probe-polariton propagation and interaction with the trapped signal polariton. (b) Probe (signal) phase shift $\phi_p(z) \equiv \phi_p(z, t_{\max}+z/v_p)$ $[\phi_s(t) \equiv \phi_s(z,t)]$ as a function of $z[t]$.

we have $[\hat{\Psi}_i(z), \hat{\Psi}_j^\dagger(z')] \simeq L\delta_{ij}\delta(z-z')$. It is then easy to show that the first and second terms of the effective Hamiltonian (13) commute.

III. CROSS-PHASE-MODULATION

In this section we study the nonlinear interaction between the signal and probe polaritons by exploring the classical as well as fully quantum treatments of the system.

A. Classical fields

We begin with the classical limit of the theory, in which the operators $\hat{\Psi}_{p,\pm}$ and $\hat{I}_{p,s}$ are replaced by the corresponding c numbers. Let us consider two single-photon pulses, which, upon entering the medium, are converted into two polaritons, each containing a single excitation,

$$\frac{1}{L}\int I_s dz = \frac{v_p}{L}\int I_p dt \simeq 1. \quad (15)$$

Then the conditional phase shifts, accumulated by the probe and signal pulses during the interaction, are given by

$$\phi_p = \phi_s = \frac{\eta L}{v_p} = \frac{g_B^2 L \cos^2\theta_A \tan^2\theta_B}{c\Delta_B} \equiv \phi. \quad (16)$$

We note again that the phase shift is proportional to the intensity of the photonic component of the signal polariton, as attested by the presence of the $\cos^2\theta_A$ term in the numerator of Eq. (16).

Expressing the atom-field coupling constants g through the decay rate γ of the corresponding excited state as

$$g = \frac{3\pi c\gamma}{2k^2 SL},$$

and assuming that $\gamma_a^{(A)} \simeq \gamma_a^{(B)}$ and $g_A^2 N_A \gg |\Omega_d^{(A)}|^2$ $(v_s \ll c)$, from Eq. (16) we have

$$\phi \simeq \frac{3\pi\gamma_d |\Omega_d^{(A)}|^2 \rho_B}{2k_p^2 \Delta_B |\Omega_d^{(B)}|^2 S\rho_A}. \quad (17)$$

For realistic experimental parameters, relevant to a cold atomic gas ($T \lesssim 1$ mK) with $L \simeq 1$ cm, $\rho_{A,B} \simeq 10^{12}$ cm^{-3}, $S \simeq 10^{-8}$ cm^2, $\omega_{p,s} \simeq 3\times 10^{15}$ rad/s, $\Omega_d^{(A)} \simeq 5\times 10^8$ rad/s, $\Omega_d^{(B)} \simeq 2\times 10^7$ rad/s, $\Delta_B \simeq \Delta_s \simeq 10^8$ rad/s, $\gamma_{a,d} \simeq 10^7$ s^{-1}, and $\gamma_{bc} \simeq 10^3$ s^{-1}, we obtain $\phi \simeq \pi$ with the fidelity $\mathcal{F} \geqslant 0.98$, the main limiting factor being the collisional relaxation of Raman coherence. In Fig. 2 we show the results of our numerical simulations of the probe polariton propagation and interaction with the trapped signal polariton. One can see in Fig. 2(a) that the trapped signal polariton slowly spreads with the rate κ_l and simultaneously undergoes rapid spatiotemporal oscillations, whose period is determined by the reflection rate $T_{\text{osc}} = \pi/\beta$, while the spatial amplitude is given by the penetration depth $\pi v_s/(2\beta) = \pi c/(2\Delta_s \tan^2\theta_A)$ which is much smaller than its spatial extent z_{loc}. The resulting phase shifts of the probe and signal pulses are shown in Fig. 2(b).

B. Quantum fields: The evolution operator

We now turn to the fully quantum treatment of the system. Given an input state of the probe and signal polaritons $|\Phi_{\text{in}}\rangle$, the state of the system subject to the effective interaction Hamiltonian H_{eff} evolves according to

$$|\Phi(t)\rangle = U(t)|\Phi_{\text{in}}\rangle, \quad (18)$$

where the evolution operator $U(t)$ is defined via

$$U(t) = \exp\left(-\frac{i}{\hbar}\int_0^t H_{\text{eff}} dt'\right). \quad (19)$$

We are interested in the output state of the system at time $t_{\text{out}} > L/v_p$, when the probe pulse has left the active medium. Since the first term of Hamiltonian (13), which is responsible for the cross-phase-modulation between the probe and signal polaritons, commutes with the second term, which describes the scattering between the forward and backward components of the signal polariton, the evolution operator (19) can be factorized into the product of two commuting operators U_{I} and U_{II}, corresponding to the respective terms of the Hamiltonian,

$$U_{\text{out}} = U_{\text{I}} U_{\text{II}}. \quad (20)$$

Using the plane-wave decompositions for the polariton operators, Eqs. (14a) and (14b), and recalling that in Eq. (13) $\hat{\Psi}_p = \hat{\Psi}_p(z-\zeta)$ with $\zeta = v_p t$, we have

$$U_{\text{I}} = \prod_{q_p} \exp\left[i\phi \hat{\psi}_p^{q_p\dagger} \hat{\psi}_p^{q_p} \sum_q (\hat{\psi}_+^{q\dagger}\hat{\psi}_+^{q} + \hat{\psi}_-^{q\dagger}\hat{\psi}_-^{q}) \right], \quad (21a)$$

$$U_{\text{II}} = \prod_q \exp[i\beta t_{\text{out}}(\hat{\psi}_+^{q\dagger}\hat{\psi}_-^{-q} + \hat{\psi}_-^{-q\dagger}\hat{\psi}_+^{q})]. \quad (21b)$$

Note that U_{I} does not contain time explicitly, because for $t_{\text{out}} > L/v_p$ the cross-phase-modulation is over, as the probe pulse has already left the medium. However, the interaction time L/v_p is contained implicitly in $\phi=\eta(L/v_p)$. Below we will employ the evolution operator of Eqs. (20), (21a), and (21b) to calculate the output state of the system for the single-photon and coherent input states.

C. Single-photon states

Consider first the evolution of two single-photon input pulses, which in the medium correspond to the initial state

$$|\Phi_{\rm in}\rangle = |1_p\rangle \otimes |1_+\rangle \otimes |0_-\rangle, \tag{22}$$

consisting of two single-excitation polariton wave packets

$$|1_p\rangle = \sum_{q_p} \xi_p^{q_p}|1_p^{q_p}\rangle, \quad |1_+\rangle = \sum_q \xi_+^q|1_+^q\rangle,$$

where $|1_p^{q_p}\rangle=\hat{\psi}_p^{q_p\dagger}|0\rangle$ and $|1_+^q\rangle=\hat{\psi}_+^{q\dagger}|0\rangle$. The Fourier amplitudes $\xi_{p,+}^q$, normalized as $\Sigma_q|\xi_{p,+}^q|^2=1$, define the spatial envelopes $f_{p,+}(z)=\Sigma_q\xi_{p,+}^q e^{iqz}$ of the probe and forward signal pulses that initially (at $t=0$) are localized around $z=0$ and $z=L/2$, respectively [see Fig. 2(a)]. After the interaction, at time $t_{\rm out}>L/v_p$, the output state of the system is found to be

$$\begin{aligned}|\Phi_{\rm out}\rangle &= U_{\rm out}|\Phi_{\rm in}\rangle = e^{i\phi}|1_p\rangle \otimes [\cos(\beta t_{\rm out})|1_+\rangle \otimes |0_-\rangle \\ &\quad + i\sin(\beta t_{\rm out})|0_+\rangle \otimes |1_-\rangle],\end{aligned} \tag{23}$$

where $|1_-\rangle\equiv\Sigma_q\xi_+^q|1_-^{-q}\rangle$ with $|1_-^{-q}\rangle=\hat{\psi}_-^{-q\dagger}|0\rangle$. Thus, while the signal pulse periodically cycles between the forward and backward modes, the combined state of the system acquires an overall conditional phase shift $\phi=\eta L/v_p$. When $\phi=\pi$ and $t_{\rm out}$ is such that $\beta t_{\rm out}=2\pi n$ (n being any integer), the output state of the two photons is given by

$$|\Phi_{\rm out}\rangle = -|\Phi_{\rm in}\rangle, \tag{24}$$

which can be used to realize a deterministic controlled-phase (CPHASE) logic gate between the two photons representing qubits, as described in Sec. IV.

D. Multimode coherent states

Consider finally the evolution of input wave packets composed of the multimode coherent states

$$|\alpha_p\rangle = \prod_{q_p}|\alpha_p^{q_p}\rangle, \quad |\alpha_+\rangle = \prod_q|\alpha_+^q\rangle, \quad |0_-\rangle = \prod_q|0_-^q\rangle. \tag{25}$$

The states $|\alpha_p\rangle$ and $|\alpha_+\rangle$ are the eigenstates of the input operators $\hat{\Psi}_p(0,t)$ and $\hat{\Psi}_+(z,0)$ with the corresponding eigenvalues

$$\alpha_p(t) = \sum_{q_p}\alpha_p^{q_p}e^{-iq_pct}, \quad \alpha_+(z) = \sum_q \alpha_+^q e^{iqz}. \tag{26}$$

From the operator solutions (8), the expectation values for the polariton operators are then obtained as

$$\langle\hat{\Psi}_p(z,t)\rangle = \alpha_p(\tau)\exp\left[\frac{e^{i\phi}-1}{L}\int_0^z|\alpha_+(z')|^2dz'\right], \tag{27a}$$

$$\langle\hat{\Psi}_\pm(z,t)\rangle = \alpha_+(z)\exp\left[\frac{e^{i\phi v_p/c}-1}{L}c\int_0^t|\alpha_p(\tau')|^2dt'\right] \times \begin{bmatrix}\cos(\beta t)\\ i\sin(\beta t)\end{bmatrix}, \tag{27b}$$

where ϕ is given in Eq. (16). These equations are notably different from those obtained for single-mode [23] and multimode copropagating fields [16,18] because all parts of the probe pulse interact with the whole signal pulse (and the other way around), which is reflected in the space (time) integration. Similarly to the cases discussed in Refs. [16,18,23], only in the limit $\phi\ll 1$ do Eqs. (27a) and (27b) reproduce the classical result,

$$\phi_p = \frac{\phi}{L}\int|\alpha_+|^2dz' = \frac{\eta}{v_p}\int I_s(z')dz', \tag{28a}$$

$$\phi_s = \frac{\phi v_p}{L}\int|\alpha_p|^2dt' = \eta\int_0^t I_p(t')dt', \tag{28b}$$

whereby a phase shift of π can be obtained when

$$\frac{1}{v_p}\int|\alpha_+|^2dz' = \int|\alpha_p|^2dt' = \frac{\pi}{\eta}.$$

This restriction on the classical correspondence of the coherent states comes about since, for large enough cross-phase-modulation rates η, these states exhibit periodic collapses and revivals as ϕ and $\phi v_p/c$ change from 0 to 2π. This fact severely limits the usefulness of weak coherent states for QI applications based on the polarization degrees of freedom of optical fields.

Let us also calculate the time evolution of the input state

$$|\Phi_{\rm in}\rangle = |\alpha_p\rangle \otimes |\alpha_+\rangle \otimes |0_-\rangle. \tag{29}$$

Using Eqs. (20), (21a), and (21b), and the fact that

$$\exp[i\beta t(a^\dagger b + b^\dagger a)]|n_a\rangle|0_b\rangle = \sum_{k=0}^n \binom{n}{k}^{1/2}[\cos(\beta t)]^{n-k}[i\sin(\beta t)]^k|(n-k)_a\rangle|k_b\rangle, \tag{30}$$

where $a^\dagger,a$ and $b^\dagger,b$ are the bosonic creation and annihilation operators for the corresponding field modes [24], we obtain a rather cumbersome, but nevertheless useful result,

$$\begin{aligned}|\Phi_{\rm out}\rangle &= U_{\rm out}|\Phi_{\rm in}\rangle \\ &= \prod_{q_p}e^{-|\alpha_p^{q_p}|^2/2}\sum_{m_1,m_2,\ldots,m_l}\frac{(\alpha_p^{q_p^1})^{m_1}(\alpha_p^{q_p^2})^{m_2}\cdots(\alpha_p^{q_p^l})^{m_l}}{\sqrt{m_1!\,m_2!\,\cdots m_l!}} \\ &\quad\times|(m_1)_p^{q_p^1}\rangle|(m_2)_p^{q_p^2}\rangle\cdots|(m_l)_p^{q_p^l}\rangle\otimes\prod_q e^{-|\alpha_+^q|^2/2} \\ &\quad\times\sum_n[\alpha_+^q e^{i\phi(m_1+m_2+\cdots+m_l)}]^n \\ &\quad\times\sum_{k=0}^n\frac{[\cos(\beta t_{\rm out})]^{n-k}[i\sin(\beta t_{\rm out})]^k}{\sqrt{k!\,(n-k)!}}|(n-k)_+^q\rangle|k_-^{-q}\rangle.\end{aligned} \tag{31}$$

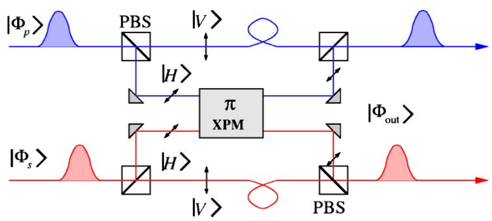

FIG. 3. (Color online) Proposed implementation of the optical CPHASE logic gate between two single-photon qubits, using polarizing beam splitters (PBS's), and π cross-phase-modulation (XPM) studied here.

For $\phi=\pi$, one has $e^{i\phi(m_1+m_2+\cdots+m_l)}=\pm 1$, the "+1" or "−1" corresponding to the sum $(m_1+m_2+\cdots+m_l)$ being, respectively, even or odd. Accordingly, Eq. (31) simplifies to

$$|\Phi_{\rm out}\rangle = \frac{1}{2}(|\alpha_p\rangle + |-\alpha_p\rangle) \otimes \prod_q e^{-|\alpha_+^q|^2/2} \sum_n (\alpha_+^q)^n \times \sum_{k=0}^{n} \frac{[\cos(\beta t_{\rm out})]^{n-k}[i\sin(\beta t_{\rm out})]^k}{\sqrt{k!\,(n-k)!}} |(n-k)_+^q\rangle |k_-^q\rangle + \frac{1}{2}(|\alpha_p\rangle - |-\alpha_p\rangle) \otimes \prod_q e^{-|\alpha_+^q|^2/2} \sum_n (-\alpha_+^q)^n \times \sum_{k=0}^{n} \frac{[\cos(\beta t_{\rm out})]^{n-k}[i\sin(\beta t_{\rm out})]^k}{\sqrt{k!\,(n-k)!}} |(n-k)_+^q\rangle |k_-^q\rangle, \tag{32}$$

where $|-\alpha_p\rangle=\Pi_{q_p}|-\alpha_p^{q_p}\rangle$. A particularly simple and important case is realized for $\beta t_{\rm out}=(\pi/2)n$ (n being any integer), when either $\sin(\beta t_{\rm out})$ or $\cos(\beta t_{\rm out})$ is zero and the signal polariton is found in one of the four possible states $|\alpha_+\rangle\otimes|0_-\rangle, |-\alpha_+\rangle\otimes|0_-\rangle, |0_+\rangle\otimes|i\alpha_-\rangle$, and $|0_+\rangle\otimes|-i\alpha_-\rangle$. As an example, when $\beta t_{\rm out}=2\pi n$, we have

$$|\Phi_{\rm out}\rangle = \frac{1}{2}(|\alpha_p\rangle + |-\alpha_p\rangle) \otimes |\alpha_+\rangle \otimes |0_-\rangle + \frac{1}{2}(|\alpha_p\rangle - |-\alpha_p\rangle) \otimes |-\alpha_+\rangle \otimes |0_-\rangle, \tag{33}$$

which is an entangled coherent superposition of macroscopically distinguishable states of two fields. Such entanglement of coherent Schrödinger-cat states [25] can find important applications in schemes of quantum-information processing and communication with continuous variables [26]. Thus, using our scheme, one could contemplate the feasibility of deterministic quantum computation with optical coherent states [27].

IV. DETERMINISTIC LOGIC GATE

Utilizing the scheme of Fig. 3 and the results of Sec. III C, one can realize a transformation corresponding to the CPHASE logic gate between two traveling single-photon pulses representing qubits. To this end, suppose that the qubit basis states $\{|0\rangle,|1\rangle\}$ are represented by the vertical $|V\rangle\equiv|0\rangle$ and horizontal $|H\rangle\equiv|1\rangle$ polarization states of the photon. After passing through a polarizing beam splitter (PBS), the vertically polarized component of each photon is not reflected, while the horizontally polarized component is directed into the active medium. Employing the procedure discussed above, whereby the $|H_s\rangle$ component of the signal pulse is first trapped in the medium, then interacts with the $|H_p\rangle$ component of the probe, and finally is released, the two-photon state $|\Phi_{\rm in}\rangle=|H_pH_s\rangle$ acquires the conditional phase shift π, as per Eq. (24). At the output, each photon is recombined with its vertically polarized component on another PBS, where the complete temporal overlap of the vertically and horizontally polarized components of each photon is achieved by delaying the $|V\rangle$ wave packet in a fiber loop or sending it though another EIT vapor cell. The resulting transformation corresponds to the truth table of the CPHASE gate,

$$|V_pV_s\rangle \to |V_pV_s\rangle,$$
$$|V_pH_s\rangle \to |V_pH_s\rangle,$$
$$|H_pV_s\rangle \to |H_pV_s\rangle,$$
$$|H_pH_s\rangle \to -|H_pH_s\rangle. \tag{34}$$

Together with the Faraday rotations of photon polarization (implementing arbitrary single-qubit rotations) and linear phase shift, the CPHASE gate is *universal* as it can realize any unitary transformation [1].

V. CONCLUSIONS

In this paper we have proposed a scheme for highly efficient Kerr nonlinear interaction between two weak optical fields. We have shown that large conditional phase shifts and entanglement can be obtained in atomic vapors, in which a weak (quantum) probe pulse, upon propagating through the medium, interacts with a weak signal pulse that is dynamically trapped in a photonic band gap created by spatially periodic modulation of the electromagnetically-induced-transparency resonance. The attainable π phase shift accompanied by negligible absorption and spectral broadening can be used for high-fidelity implementation of the CPHASE universal quantum logic gate between the two single-photon pulses. The proposed scheme may therefore pave the way to quantum-information applications, such as deterministic all-optical quantum computation, dense coding, and teleportation [1].

Before closing, we note that our central equations (5a), (5b), and (8a)–(8c) and consequently the main results Eqs. (23), (27a), and (27b) are similar to those obtained by us for the case of cross-phase-modulation in doped photonic crystals [21], whose practical realization represents a formidable experimental challenge. By contrast, the recent experimental progress in trapping and manipulating light pulses in dynamically controlled photonic band gaps in atomic vapors [28] puts the present scheme within easy experimental reach considering present day technology.

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IV. GIANT NONLINEARITY AND ENTANGLEMENT OF SINGLE PHOTONS IN PHOTONIC BAND GAP STRUCTURES

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Giant nonlinearity and entanglement of single photons in photonic bandgap structures

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Abstract. – Giantly enhanced cross-phase modulation with suppressed spectral broadening is predicted between optically induced dark-state polaritons whose propagation is strongly affected by photonic bandgaps of spatially periodic media with multilevel dopants. This mechanism is shown to be capable of fully entangling two single-photon pulses with high fidelity.

Introduction. – The main impediment towards the use of single photons in schemes for deterministic quantum logic and teleportation [1] as very robust and versatile carriers of quantum information [2] is the weakness of optical nonlinearities in conventional media. A major trend aimed at the enhancement of optical nonlinearities exploits one-dimensional (1D) periodic distributed Bragg reflectors and 2D- or 3D-periodic photonic crystals (PCs), where light can be slowed down or trapped via multiple reflections in the vicinity of photonic bandgaps (PBGs) [3]. Giantly enhanced nonlinearity has been predicted when dopants with transition frequencies within the PBG are implanted in the structure, so that light near such frequencies resonantly interacts with the dopants and is concurrently affected by the PBG dispersion [4]. The resulting soliton-like transmission of very weak pulses within the PBG while filtering out spurious noise is highly advantageous for classical communication [5]. However, this mechanism is incompatible with the goals of quantum logic and communications, particularly with photon-photon entanglement, because of the quantum noise associated with resonant field-atom interactions.

Another pathway to enhance the nonlinearities is based on electromagnetically induced transparency (EIT) in atomic media, which comes about when classical driving fields induce coherence between atomic levels and transform the weak fields into atom-dressed dark-state polaritons [6, 7]. The ultrahigh sensitivity of the EIT polaritonic dispersion to a small field-induced Stark shift of its atomic level can result in an appreciable nonlinear phase shift, impressed by one ultraweak field upon another [8]. Notwithstanding this promising sensitivity, EIT-polariton entanglement by a large conditional phase shift of one photon in the presence of another (also known as cross-phase modulation) faces serious challenges in spatially uniform

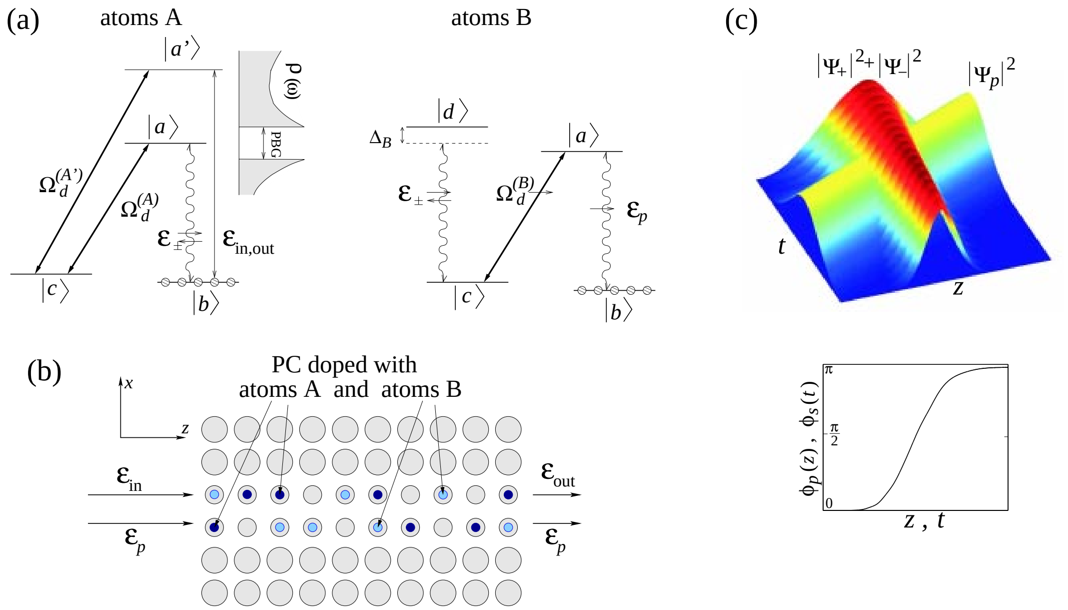

Fig. 1 – (a) Level scheme of atomic species A and B. Atoms A convert the input signal field $\mathcal{E}_{\rm in}$ at frequency outside the PBG to the trapped field $\mathcal{E}_{\pm}$ at frequency inside the PBG. Atoms B provide EIT for the probe field $\mathcal{E}_p$ and its cross-coupling with the signal field $\mathcal{E}_{\pm}$. (b) 2D-periodic photonic crystal (PC) having the density of modes $\rho(\omega)$, is doped with atomic species A and B. (c) Interaction and the resulting phase shift of the probe and signal pulses.

media. One drawback of these schemes has been the mismatch between the group velocities of the probe pulse moving as a slow EIT polariton and the nearly free propagating signal pulse, which severely limits their effective interaction length and the maximal conditional phase shift [9]. To enable long interaction times and thus large conditional phase shifts in a medium of finite length (up to a few centimeters), the group velocities of *both* interacting pulses should be small [10, 11]. This, however, imposes a limitation on the photonic component of the signal polariton whose magnitude determines the phase shift. Copropagating pulses pose yet another difficulty: since the phase shift of each pulse is proportional to the local intensity of the other pulse, different parts of the interacting pulses acquire different phase shifts, which results in their spectral broadening.

Here we put forward a mechanism that may produce strong photon-photon interactions along with suppressed quantum noise and give rise to their entanglement with high fidelity, by combining the advantages of their dispersion in PBG structures and of the strongly enhanced nonlinear optical coupling achievable via EIT in an appropriately doped medium. The main idea is that a single-photon signal pulse is adiabatically converted into a standing-wave polaritonic excitation inside the periodic structure. This trapped polariton, having an appreciable photonic component, can impress a large, spatially uniform phase shift upon a slowly propagating probe polariton. This task can further be facilitated by employing 2D- or 3D-periodic structures with defects where the two pulses interact via tightly confined modes [3, 12, 13].

Photon-photon interaction in PBG structures. – The proposed scheme is based upon a periodic structure containing uniformly distributed dopants —atoms A and B (see fig. 1). Atoms A, having double-Λ level configuration and interacting with classical driving fields on the transitions $|c\rangle_A \to |a\rangle_A$, $|a'\rangle_A$, with the Rabi frequencies $\Omega_d^{(A,A')}$, respectively, facilitate the trapping of the signal pulse inside the periodic structure, by converting its frequency from outside to inside of the PBG. On the other hand, atoms B, having N level configuration and interacting with the $\Omega_d^{(B)}$ driving field on the transition $|c\rangle_B \to |a\rangle_B$, serve to simultaneously slow down the probe pulse and cross-couple it with the signal.

The following procedure is foreseen to this end. Initially, all atoms A are in the ground state $|b\rangle_A$, the driving fields $\Omega_d^{(A)} = 0$ and $\Omega_d^{(A')} > T_{\text{in}}^{-1}$, where T_{in} is the temporal width of the input signal pulse $\mathcal{E}_{\text{in}}$. The carrier frequency of $\mathcal{E}_{\text{in}}$ is outside the PBG, close to the $|b\rangle_A \to |a'\rangle_A$ transition frequency, so that the usual EIT for the input signal due to the Λ configuration $|b\rangle_A \leftrightarrow |a'\rangle_A \leftrightarrow |c\rangle_A$ is realized. Upon entering the medium, the signal pulse is slowed down and spatially compressed, by a factor of $v_s'/c \ll 1$, to the length $z_{\text{loc}} \simeq T_{\text{in}} v_s'$, where $v_s' \propto |\Omega_d^{(A')}|^2$ is its group velocity inside the medium. Once the signal pulse has fully accommodated in the medium of length L, which requires that $z_{\text{loc}} < L$, the driving field $\Omega_d^{(A')}$ is adiabatically switched off. As a result, the signal is stopped, its photonic component being converted into the stationary atomic (Raman) coherence $\sigma_{bc}^{(A)}$ [6, 7]. Next, the driving field $\Omega_d^{(A)}$ is adiabatically switched on to a value $\Omega_d^{(A)} > \Omega_d^{(A')}$, converting the atomic coherence into the signal field $\mathcal{E}_\pm$, whose frequency is inside of the PBG and amplitude is larger than that of the input signal $\mathcal{E}_{\text{in}}$ by a factor of $\sqrt{v_s/v_s'} \simeq \Omega_d^{(A)}/\Omega_d^{(A')} > 1$ [7]. Due to the Bragg scattering of the forward and backward propagating components of the signal pulse with the wave vectors $\pm k$, it remains localized (trapped) within the medium [3]. Both components $\mathcal{E}_\pm$ of the signal field dispersively interact with atoms B via transition $|c\rangle_B \to |d\rangle_B$ with the detuning Δ_B. This off-resonant interaction causes an ac Stark shift of level $|c\rangle_B$, thereby strongly affecting the EIT dispersion for the probe field $\mathcal{E}_p$, which interacts with atoms B on the transition $|b\rangle_B \to |a\rangle_B$ [8]. For a large enough product of the signal-field intensity $|\mathcal{E}_\pm|^2$ and interaction time L/v_p (v_p being the probe group velocity), both pulses accumulate a *uniform* conditional phase shift which can reach π (see fig. 1(c)). Finally, reversing the sequence that resulted in trapping of the signal pulse, its frequency is converted back to the original frequency and the $\mathcal{E}_{\text{out}}$ pulse leaves the medium.

Let us now consider the scheme more quantitatively. To describe the quantum properties of the medium, we use collective slowly varying atomic operators $\hat{\sigma}_{\mu\nu}^{(\iota)}(z,t) = \frac{1}{N_\iota^z}\sum_{j=1}^{N_\iota^z} |\mu_j\rangle_{\iota\iota}\langle\nu_j| \times e^{-i\omega_{\mu\nu}^{(\iota)}t}$, averaged over a small but macroscopic volume containing many dopants of species $\iota = A, B$ around position z [6]: $N_{A,B}^z = (N_{A,B}/L)\,\mathrm{d}z \gg 1$, where $N_{A,B}$ is the total number of the corresponding dopants. The quantum radiation is described by the traveling-wave (multimode) electric-field operators $\hat{\mathcal{E}}_\pm(z,t) = \sum_q a_\pm^q(t) e^{\pm iqz}$ and $\hat{\mathcal{E}}_p(z,t) = \sum_q a_p^q(t) e^{iqz}$, where a_j^q are the annihilation operators for the field mode with the wave vector $k_j + q$, k_j being the carrier wave vector of the corresponding field. These single-mode operators possess the standard bosonic commutation relations $[a_i^q, a_j^{q'\dagger}] = \delta_{ij}\delta_{qq'}$, which yield $[\hat{\mathcal{E}}_i(z), \hat{\mathcal{E}}_j^\dagger(z')] = L\delta_{ij}\delta(z-z')$.

Using the standard technique [6], we perturbatively solve the Heisenberg equations for the atomic coherences $\hat{\sigma}_{\mu\nu}^{(\iota)}$ under EIT conditions [6–11] and substitute the solution into the propagation equations for the slowly varying field operators $\hat{\mathcal{E}}_j(z,t)$. These operators are related to the polariton operators $\hat{\Psi}_\pm = \hat{\mathcal{E}}_\pm/\cos\theta_A$ and $\hat{\Psi}_p = \hat{\mathcal{E}}_p/\cos\theta_B$, which represent the coupled excitation of the corresponding field and atomic Raman coherence $\hat{\sigma}_{bc}^{(\iota)}$. The mixing angles $\theta_{A,B}$ are defined via $\tan\theta_{A,B} = g_{A,B}\sqrt{N_{A,B}}/\Omega_d^{(A,B)}$, where $g_{A,B} = \wp_{ab}^{(A,B)}\sqrt{\omega_{ab}^{(A,B)}/(2\hbar\epsilon_0 SL)}$ are the atom-field coupling constants, $\wp_{\mu\nu}^{(\iota)}$ being the corresponding atomic dipole matrix element and S the cross-sectional area of the quantum fields. Note that the amplitude of the photonic component of each polariton is proportional to the $\cos\theta_\iota$ of the corresponding mixing angle θ_ι. Under the Bragg resonance condition $k \simeq \pi/p_s$, where p_s is the structure period, the equations of motion for polaritons are obtained as

$$\left(\frac{\partial}{\partial t} \pm v_s \frac{\partial}{\partial z}\right)\hat{\Psi}_\pm = -\kappa_s \hat{\Psi}_\pm + i\eta \hat{I}_p \hat{\Psi}_\pm + i\beta\hat{\Psi}_\mp + \hat{\mathcal{F}}_s\,, \tag{1a}$$

$$\left(\frac{\partial}{\partial t} + v_p \frac{\partial}{\partial z}\right)\hat{\Psi}_p = -\kappa_p \hat{\Psi}_p + i\eta \hat{I}_s \hat{\Psi}_p + \hat{\mathcal{F}}_p\,. \tag{1b}$$

Here $v_s = c\cos^2\theta_A$ and $v_p = c\cos^2\theta_B$ are the group velocities, $\hat{I}_s \equiv \hat{\Psi}_+^\dagger\hat{\Psi}_+ + \hat{\Psi}_-^\dagger\hat{\Psi}_-$ and $\hat{I}_p \equiv \hat{\Psi}_p^\dagger\hat{\Psi}_p$ are the intensities (excitation numbers) of the signal and probe polaritons, respectively; $\kappa_{s,p} = \gamma_{bc}^{(A,B)}\sin^2\theta_{A,B}$ are the absorption rates, $\gamma_{bc}^{(\iota)}$ being the Raman coherence decay rate of the corresponding atoms, $\hat{\mathcal{F}}_{s,p}$ are the δ-correlated noise operators associated with the relaxation; $\eta = [1 + i\gamma_d^{(B)}/(2\Delta_B)]\cos^2\theta_A\sin^2\theta_B g_B'^2/\Delta_B$ is the cross-coupling rate between the polaritons, $g_B' = \wp_{dc}^{(B)}\sqrt{\omega_{dc}^{(B)}/(2\hbar\epsilon_0 SL)}$ being the atom-field coupling on the transition $|c\rangle_B \to |d\rangle_B$ and $\gamma_d^{(B)}$ the decay rate of $|d\rangle_B$ (for $\Delta_B \gg \gamma_d^{(B)}$, the cross-absorption vanishes [8–10] and then η, being purely real, represents the cross-phase modulation rate). Finally, $\beta = \frac{1}{2}\delta\omega_{\mathrm{PBG}}\cos^2\theta_A$ is the Bragg reflection rate, $\delta\omega_{\mathrm{PBG}}$ being the PBG bandwidth.

Equations (1) constitute the starting point of our analysis. Their general solution, for arbitrary initial/boundary conditions of the traveling-wave quantized polaritons $\hat{\Psi}_{p,\pm}$, is not known. When absorption is negligible (see below), for a given time- and space-dependence of the signal-polariton intensity $\hat{I}_s(z,t)$, the solution for the probe is

$$\hat{\Psi}_p(z,t) = \hat{\Psi}_p(0,\tau)\exp\left[i\frac{\eta}{v_p}\int_0^z \hat{I}_s\big(z', \tau + z'/v_p\big)\,\mathrm{d}z'\right],$$

where $\tau = t - z/v_p$ is the retarded time. An analytic solution for the two counter-propagating components of the signal polariton can be obtained in the long probe limit, *i.e.*, when the spatial dependence of the probe-polariton intensity is negligible on the scale of z_{loc}: $\hat{I}_p(z,t) \simeq \hat{I}_p(t)$. This requires that $v_pT_p > z_{\mathrm{loc}}$, where T_p is the duration of the probe pulse (its spectral width being $\delta\omega_p \sim T_p^{-1} < v_p/z_{\mathrm{loc}}$). Then eq. (1a) is soluble by the Fourier transform $\hat{\Psi}_\pm(z,t) = \frac{1}{2\pi}\int \mathrm{d}q\, e^{\pm iqz}\hat{\psi}_\pm(q,t)$, with the result

$$\hat{\psi}_+(q,t) = \hat{\psi}_+(q,0)\,e^{i\hat{\phi}_s(t)}\left[\cos(\chi t) - i\frac{qv_s}{\chi}\sin(\chi t)\right], \tag{2a}$$

$$\hat{\psi}_-(q,t) = i\hat{\psi}_+(-q,0)\,e^{i\hat{\phi}_s(t)}\frac{\beta}{\chi}\sin(\chi t), \tag{2b}$$

where $\chi = \sqrt{q^2v_s^2 + \beta^2}$ and $\hat{\phi}_s(t) = \eta\int_0^t \hat{I}_p(t')\,\mathrm{d}t'$, with $\hat{I}_p(t) = \hat{\Psi}_p^\dagger(0,\tau)\hat{\Psi}_p(0,\tau)$. Thus all the spatial modes $\hat{\psi}_\pm(q,t) = \int \mathrm{d}z\, e^{\mp iqz}\hat{\Psi}_\pm(z,t)$ of the signal polariton acquire the same q-independent phase shift $\hat{\phi}_s(t)$. It follows from eqs. (2) that a signal polariton composed of modes with $|q| \ll \beta/v_s$ will be strongly trapped inside the medium, its wavepacket periodically cycling between the forward and backward components while interacting with the probe polariton, yielding

$$\hat{\Psi}_+(z,t) = \hat{\Psi}_+(z,0)\,e^{i\hat{\phi}_s(t)}\cos(\beta t), \tag{3a}$$

$$\hat{\Psi}_-(z,t) = i\hat{\Psi}_+(z,0)\,e^{i\hat{\phi}_s(t)}\sin(\beta t), \tag{3b}$$

$$\hat{\Psi}_p(z,t) = \hat{\Psi}_p(0,\tau)\,e^{i\hat{\phi}_p(z)}, \tag{3c}$$

where $\hat{\phi}_p(z) = \frac{\eta}{v_p}\int_0^z \hat{I}_s(z')\,\mathrm{d}z'$, with $\hat{I}_s(z) = \Psi_+^\dagger(z,0)\Psi_+(z,0)$, is the probe phase shift.

Let us dwell upon the approximations involved in the derivation of eqs. (3). During the conversion of the signal pulse into a standing-wave polaritonic excitation inside the periodic structure, the nonadiabatic corrections resulting in its dissipation are negligible provided the medium is optically thick, $\varsigma_A \rho_A L \gg 1$, where ς_A is the resonant absorption cross-section for the transition $|b\rangle_A \to |a'\rangle_A$ and $\rho_A = N_A/(SL)$ is the density of atoms A [6, 7]. After the signal pulse has been trapped in the PBG, due to nonzero values of q, it gets spatially distorted (spreads) at a rate $\kappa_d \simeq q^2 v_s^2/(\pi\beta)$ $(0 \le |q| < \beta/v_s)$. We can estimate the bandwidth of the signal pulse from its spatial extent as $\delta q \sim v_s/(cL) < |\Omega_d^{(A)}|^2/(\gamma_a^{(A)} c)$, thus obtaining the upper limit for the distortion rate $\kappa_d \le 2v_s^3/(\pi c L^2 \delta\omega_{\mathrm{PBG}})$. On the other hand, the bandwidth of the probe is limited by the length of the medium via $\delta\omega_p < |\Omega_d^{(B)}|^2[g_B^2 N_B \gamma_a^{(B)} L/c]^{-1/2} = |\Omega_d^{(B)}|^2 k_p[\gamma_a^{(B)}\sqrt{3\pi/2\rho_B L}]^{-1}$, where $\rho_B = N_B/(SL)$ is the density of atoms B [14]. Finally, the interaction time $t_{\mathrm{int}} = L/v_p$ is limited by $t_{\mathrm{int}} \times \max\{\kappa_d, \kappa_s, \kappa_p\} \ll 1$, and so the fidelity of the cross-phase modulation is given by $F = \exp[-(\kappa_d + \kappa_s + \kappa_p)L/v_p]$.

Consider first the classical limit of eqs. (3), where the operators $\hat{\Psi}_{p,\pm}$ and $\hat{I}_{p,s}$ are replaced by c-numbers. Then for two single-photon pulses $\frac{1}{L}\int I_s\,\mathrm{d}z = \frac{v_p}{L}\int I_p\,\mathrm{d}t \simeq 1$, the conditional phase shift accumulated by the probe and signal fields during the interaction is given by

$$\phi_p = \phi_s = \frac{g_B'^2 L \cos^2\theta_A \tan^2\theta_B}{c\Delta_B} \equiv \phi. \tag{4}$$

Note that the phase shift is proportional to the intensity of the photonic component of the signal polariton, as attested by the presence of the $\cos^2\theta_A$ term in the nominator of eq. (4). For realistic experimental parameters, relevant to a doped periodic structure discussed below, one can obtain $\phi \simeq \pi$ (see fig. 1(c)) with the fidelity $F \simeq 1$.

We now turn to the fully quantum treatment of the system. To compare the classical and quantum pictures, we consider first the evolution of input wavepackets composed of the multimode coherent states $|\alpha_p\rangle \otimes |\alpha_+\rangle \otimes |0_-\rangle = \prod_{q_p}|\alpha_p^{q_p}\rangle \otimes \prod_q |\alpha_+^q\rangle \otimes \prod_{q'}|0_-^{q'}\rangle$. States $|\alpha_p\rangle$ and $|\alpha_+\rangle$ are the eigenstates of the input fields operators $\hat{\mathcal{E}}_p(0,t)$ and $\hat{\mathcal{E}}_+(z,0)$ with eigenvalues $\alpha_p(t) = \sum_{q_p}\alpha_p^{q_p}e^{-iq_pct}$ and $\alpha_+(z) = \sum_q \alpha_+^q e^{iqz}$, respectively. The expectation values for the fields are then obtained as

$$\langle\hat{\mathcal{E}}_p(z,t)\rangle = \alpha_p(\tau)\exp\left[\frac{e^{i\phi c/v_s}-1}{L}\int_0^z |\alpha_+(z')|^2\,\mathrm{d}z'\right], \tag{5a}$$

$$\langle\hat{\mathcal{E}}_\pm(z,t)\rangle = \alpha_+(z)\exp\left[\frac{e^{i\phi}-1}{L}c\int_0^t |\alpha_p(\tau')|^2\,\mathrm{d}t'\right] \times \begin{bmatrix}\cos(\beta t)\\ i\sin(\beta t)\end{bmatrix}. \tag{5b}$$

These equations are notably different from those obtained for single-mode [15] and multimode copropagating fields [10, 11] because all parts of the probe pulse interact with the whole signal pulse (and vice versa), as is manifest in the space (time) integration. Similarly to the cases discussed in [10, 11, 15], eqs. (5) reproduce the classical result only in the limit $\phi c/v_s \ll 1$, yielding $\phi_p = \frac{\eta c}{v_p v_s}\int|\alpha_+|^2\,\mathrm{d}z'$ and $\phi_s = \frac{\eta c}{v_p}\int|\alpha_p|^2\,\mathrm{d}t'$. A π phase shift is then obtained for $\frac{c}{Lv_s}\int|\alpha_+|^2\,\mathrm{d}z' = \frac{c}{L}\int|\alpha_p|^2\,\mathrm{d}t' = \pi/\phi$. This restriction on the classical correspondence of coherent states comes about since, for large enough cross-phase modulation rates η, these states exhibit periodic collapses and revivals as ϕ changes from 0 to 2π, which limits their usefulness for quantum information applications.

Consider finally the input state $|\Phi_{\mathrm{in}}\rangle = |1_p\rangle\otimes|1_+\rangle\otimes|0_-\rangle$, consisting of two single-excitation polariton wavepackets $|1_{p,+}\rangle = \sum_q \xi_{p,+}^q |1_{p,+}^q\rangle$, where the Fourier amplitudes, normalized as

$\sum_q |\xi^q_{p,+}|^2 = 1$, define the spatial envelopes $f_{p,+}(z) = \sum_q \xi^q_{p,+} e^{iqz}$ of the probe and forward signal pulses that initially (at $t = 0$) are localized around $z = 0$ and $z = L/2$, respectively (fig. 1(c)). During the evolution, the state of the system evolves according to $|\Phi(t)\rangle = \exp[-\frac{i}{\hbar}\int_0^t H_{\rm int}\,{\rm d}t']\,|\Phi_{\rm in}\rangle$, where $H_{\rm int} = -\frac{\hbar}{L}\int {\rm d}z\,[\eta\hat{\Psi}_p^\dagger\hat{\Psi}_p(\hat{\Psi}_+^\dagger\hat{\Psi}_+ + \hat{\Psi}_-^\dagger\hat{\Psi}_-) + \beta(\hat{\Psi}_+^\dagger\hat{\Psi}_- + \hat{\Psi}_-^\dagger\hat{\Psi}_+)]$ is the effective interaction Hamiltonian whose first and second terms commute. The implicit time dependence of the effective Hamiltonian, due to the propagation of the probe-polariton pulse with the group velocity v_p, is contained in the operator $\hat{\Psi}_p = \hat{\Psi}_p(z - \zeta)$, with $\zeta = v_p t$. After the interaction, at time $t_{\rm out} > L/v_p$, the output state of the system is

$$|\Phi_{\rm out}\rangle = e^{i\eta L/v_p}|1_p\rangle \otimes [\cos(\beta t_{\rm out})\,|1_+\rangle \otimes |0_-\rangle + i\sin(\beta t_{\rm out})\,|0_+\rangle \otimes |1_-\rangle], \tag{6}$$

where $|1_-\rangle \equiv \sum_q \xi^q_+\,|1^{-q}_-\rangle$. Thus, while the signal pulse periodically cycles between the forward and backward modes, the combined state of the system acquires an overall conditional phase shift $\phi = \eta L/v_p$. When $\phi = \pi$, transformation (6) corresponds to the truth table of the *universal* controlled-phase (CPHASE) logic gate between the two photons representing qubits, which can be used to realize arbitrary unitary transformation [1].

Possible experimental realizations. – An x-z 2D-periodic lattice of dielectric rods or semiconductor stacks (fig. 1(b)) [3, 12, 13], with controlled structural defects and mirror confinement in y, appears to be the most suitable structure for realizing polaritonic entanglement of two single photons, since both the signal and the probe pulses may be confined in the x-y directions in the vicinity of a "defect" row forming a PC waveguide, thereby avoiding diffraction losses and focusing the fields to a radius of $\sim 1\,\mu$m [3, 12]. Using the double-Λ dopants (atoms A), the signal pulse can be trapped in the PC, with the localization distance $z_{\rm loc}$ extending over many periods, and interact with the probe pulse via atoms B. Expressing the atom-field coupling constants g through the decay rate γ of the corresponding excited state as $g = 3\pi c\gamma/(2k^2 SL)$, and assuming that $\gamma_a^{(A)} \simeq \gamma_a^{(B)}$ and $g_A^2 N_A \gg |\Omega_d^{(A)}|^2$ ($v_s \ll c$), we have from eq. (4) $\phi \simeq 3\pi\gamma_d\,(2k_p^2\Delta_B S)^{-1}(\Omega_d^{(A)}/\Omega_d^{(B)})^2(\rho_B/\rho_A)$. Among the possible dopants, III-V or II-VI n-doped semiconductor quantum dots, having large dipole moments and level structure conducive to EIT [16], could be the best choice for our scheme, provided high densities can be achieved. In such single-electron doped QDs, the spin degeneracy of the ground and the lowest and higher excited (charged exciton or trion) states can be lifted with a magnetic field [16], realizing the level scheme of fig. 1(a), where atoms A and B can spectroscopically be selected (via optical pumping or spectral hole burning) from the inhomogeneous ensemble of QDs. Thus states $|b\rangle$ and $|c\rangle$ are represented by the Zeeman-split spin-up and spin-down states of the conduction-band electron in the QD. The excited states $|a\rangle_A$ and $|a'\rangle_A$ of atoms A can be the first and the second (or higher) exciton states, while states $|a\rangle_B$ and $|d\rangle_B$ of atoms B are the Zeeman sublevels of the lowest excitonic state. Assuming the parameters $L \simeq 0.1\,$cm, $S \simeq 10^{-8}\,{\rm cm}^2$, $\delta\omega_{\rm PBG} \sim 10^{14}\,$rad/s [3, 12], $\rho_{A,B} \simeq 10^{12}\,{\rm cm}^{-3}$, $\omega_p \simeq 3{\times}10^{15}\,$rad/s, $\Omega_d^{(A)} \simeq 5{\times}10^8\,$rad/s, $\Omega_d^{(B)} \simeq 2{\times}10^7\,$rad/s, $\Delta_B \simeq 10^8\,$rad/s, $\gamma_{a,d} \simeq 10^7\,{\rm s}^{-1}$, and $\gamma_{bc} \simeq 10^4\,{\rm s}^{-1}$ [16], we obtain $\phi \simeq \pi$ with the fidelity $F \geq 0.98\%$, the main limiting factor being the decay of Raman coherence γ_{bc}. Other contenders for observing the proposed effects include periodic structures fabricated from rare-earth doped crystals, such as Pr:YSP, in which high-fidelity EIT has experimentally been demonstrated [17], or cryogenically cooled diamond with high density of nitrogen-vacancy defect centers [18].

Conclusions. – To summarize, we have proposed a new class of multimode quantum-field interactions involving quantized EIT-polaritons in PBG structures. We have shown that such interactions allow efficient cross-phase modulation between a propagating probe pulse and a trapped signal pulse, whose localization is achieved by an adiabatic four-wave mixing process that pulls its frequency into the PBG. This localization allows multiply repeated interaction of the signal with the entire probe pulse. As a result, the combined two-photon state of the system can acquire a conditional π phase shift, which corresponds to the universal cphase logic gate. The phase shift is spatially uniform and the process may have high fidelity. The experimental realization of the predicted effects requires the fabrication of periodic structures with large densities of optically active dopants [16–18], which may also find useful applications in laser technology, optical communication or quantum computation. We note that a similar regime of giant cross-phase modulation with suppressed spectral broadening is also realizable in cold atomic vapors using optically induced PBGs [19], on which we intend to report elsewhere. The proposed scheme may pave the way to quantum information applications such as deterministic all-optical quantum computation, dense coding and teleportation [1].

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V. LONG-RANGE INTERACTION AND ENTANGLEMENT OF SLOW SINGLE-PHOTON PULSES

Long-range interactions and entanglement of slow single-photon pulses

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We show that very large nonlocal nonlinear interactions between pairs of colliding slow-light pulses can be realized in atomic vapors in the regime of electromagnetically induced transparency. These nonlinearities are mediated by strong, long-range dipole-dipole interactions between Rydberg states of the multilevel atoms in a ladder configuration. In contrast to previously studied schemes, this mechanism can yield a homogeneous conditional phase shift of π even for weakly focused single-photon pulses, thereby allowing a deterministic realization of the photonic phase gate.

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Whether or not quantum-information processing and quantum computing [1] become practical technologies crucially depends on the ability to implement high-fidelity quantum logic gates in a scalable way [2]. Among alternative routes to this challenging goal, the schemes operating with photons as qubits [3,4] are of particular interest, since photons are ideal carriers of quantum information in terms of transfer rates, distances, and scalability. A current trend makes use of linear optical elements and photodetectors for the implementation of key components of quantum communications and information processing in a probabilistic way [4]. The desirable objective though is a *deterministic* realization of entangling operations between individual photons, which require sufficiently strong nonlinearities or long interaction times. These are achievable, at the single-photon level, by tight spatial confinement of the photons, in the very demanding regime of strong atom-field coupling in high-Q cavities [5].

A promising alternative is to enhance both the nonlinear susceptibility and interaction time by employing the ultraslow light propagation in resonant media subject to electromagnetically induced transparency (EIT) [6–8]. In a pioneering work, Schmidt and Imamoğlu have suggested the possibility of enhanced, nonabsorptive, cross-phase modulation of two weak fields in the EIT regime [9], provided their interaction time is long enough. However, upon entering the EIT medium light pulses become spatially compressed by the ratio of group velocity v to the vacuum speed of light c [10], so that the interaction time of two colliding pulses is a constant independent of v. In order to maximize this time, copropagating pulses with nearly matched group velocities have been proposed [11,12]. The essential drawback of such an approach is the spatial inhomogeneity of the conditional phase shift, causing spectral broadening of the interacting pulses, thereby preventing the realization of a high-fidelity quantum phase gate. Alternative approaches free of spectral broadening have been suggested [13–15]. In all of them, however, a rather tight transverse confinement through waveguiding or focusing of the pulses, close to the diffraction limit of λ^2, is needed in order to attain a phase shift of π, which is technically challenging.

When the light pulses enter EIT media, photonic excitations are temporarily transferred to atomic excitations through the formation of quasiparticles, the so-called dark-state (or slow-light) polaritons, which are superpositions of light and matter degrees of freedom [16]. The spatial compression of the pulses leads to an *amplification* of the matter components of polaritons. In this paper we propose a hitherto unexplored mechanism for the collisional entanglement of two single-quantum polaritons mediated by the long-range interaction of their matter (atomic) components and demonstrate its effectiveness. In contrast to the previous schemes which employ *local* interactions, namely either two photons interact with the same atom [11–14] or two atoms after absorbing the photons undergo s-wave scattering [15], here the two polaritons interact via the long-range dipole–dipole interactions between their atomic components in the highly excited Rydberg states. In a static electric field, these internal Rydberg states, populated only in the presence of polaritons, possess large permanent dipole moments [17]. We will show that under experimentally realizable conditions, the conditional phase shift accumulated during a collision of two single-quantum polaritons is *spatially homogeneous* and can be sufficiently large for the implementation of the quantum phase gate, even for moderate focusing or transverse confinement of interacting pulses. We note that quantum gates for individual Rydberg atoms, coupled by dipole–dipole interaction, have been proposed in [18], while the manipulation of quantum information with mesoscopic atomic ensembles using the dipole blockade technique, based on long-range interactions of atomic Rydberg states, was discussed in [19].

We consider an ensemble of cold alkali atoms with level configuration as in Fig. 1. All the atoms are initially prepared in the ground state $|g\rangle$. Two weak (quantum) fields $E_{1,2}$ having orthogonal polarizations and propagating in the opposite directions along the z axis resonantly interact with the atoms on the transitions $|g\rangle \to |e_{1,2}\rangle$, respectively. The intermediate states $|e_{1,2}\rangle$ are resonantly coupled by two strong (classical) driving fields with Rabi frequencies $\Omega_{1,2}$ to the highly excited Rydberg states $|d_{1,2}\rangle$. In a static electric field $E_{\mathrm{st}}\mathbf{e}_z$, the Rydberg states $|d\rangle$ possess large permanent dipole moments $\mathbf{p}=\frac{3}{2}nqea_0\mathbf{e}_z$, where n and $q\equiv n_1-n_2$ are, respectively, the (effective) principal and parabolic quantum numbers, e is the electron charge, and a_0 is the Bohr radius [17]. A pair of atoms i and j at positions $\mathbf{r}_i$ and $\mathbf{r}_j$ excited to states $|d\rangle$

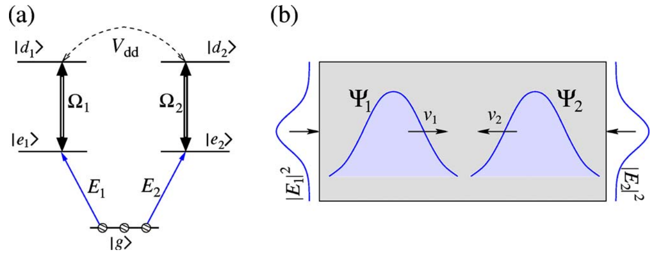

FIG. 1. (Color online) (a) Level scheme of atoms interacting with weak (quantum) fields $E_{1,2}$ on the transitions $|g\rangle \to |e_{1,2}\rangle$ and strong driving fields of Rabi frequencies $\Omega_{1,2}$ on the transitions $|e_{1,2}\rangle \to |d_{1,2}\rangle$, respectively. $V_{\rm dd}$ denotes the dipole–dipole interaction between pairs of atoms in Rydberg states $|d\rangle$. (b) Upon entering the medium, each field having Gaussian transverse intensity profile is converted into the corresponding polariton $\Psi_{1,2}$ representing a coupled excitation of the field and atomic coherence. These polaritons propagate in the opposite directions with slow group velocities $v_{1,2}$ and interact via the dipole–dipole interaction.

interact with each other via the dipole–dipole potential

$$V_{\rm dd} = \frac{\mathbf{p}_i \cdot \mathbf{p}_j - 3(\mathbf{p}_i \cdot \mathbf{e}_{ij})(\mathbf{p}_j \cdot \mathbf{e}_{ij})}{4\pi\epsilon_0 |\mathbf{r}_i - \mathbf{r}_j|^3},$$

where $\mathbf{e}_{ij}$ is a unit vector along the interatomic direction. This dipole–dipole interaction results in an energy shift of the pair of Rydberg atoms, while we assume that the state mixing within the same n manifold is suppressed by the proper choice of parabolic q and magnetic m quantum numbers [17,18]. In the frame rotating with the frequencies of the optical fields, the interaction Hamiltonian has the following form:

$$H = V_{\rm af} + V_{\rm dd}, \tag{1}$$

where the atom-field and dipole–dipole interaction terms are given, respectively, by

$$V_{\rm af} = -\hbar \sum_j^N [g_1^j \hat{\mathcal{E}}_1 \hat{\sigma}^j_{e_1 g} + \Omega_1 \hat{\sigma}^j_{d_1 e_1} + g_2^j \hat{\mathcal{E}}_2 \hat{\sigma}^j_{e_2 g} + \Omega_2 \hat{\sigma}^j_{d_2 e_2} + \text{H.c.}], \tag{2a}$$

$$V_{\rm dd} = \hbar \sum_{i>j}^N \hat{\sigma}^i_{dd} \Delta(\mathbf{r}_i - \mathbf{r}_j) \hat{\sigma}^j_{dd}. \tag{2b}$$

Here $N=\rho V$ is the total number of atoms, ρ being the (uniform) atomic density and V the volume; $\hat{\sigma}^j_{\mu\nu} \equiv |\mu\rangle_{jj}\langle\nu|$ is the transition operator of the jth atom; $\hat{\mathcal{E}}_l$ is the slowly varying operator, corresponding to the electric field E_l $(l=1,2)$, which obeys the commutation relations $[\hat{\mathcal{E}}_l(\mathbf{r}), \hat{\mathcal{E}}^\dagger_{l'}(\mathbf{r}')] = V\delta_{ll'}\delta(\mathbf{r}-\mathbf{r}')$; g_l^j is the corresponding atom-field coupling constant on the transition $|g\rangle_j \to |e_l\rangle_j$; and $\hbar\Delta(\mathbf{r}_i - \mathbf{r}_j) \equiv {}_i\langle d|_j\langle d|V_{\rm dd}|d\rangle_i|d\rangle_j$ is the dipole–dipole energy shift for a pair of atoms i and j, given by

$$\Delta(\mathbf{r}_i - \mathbf{r}_j) = C\frac{1 - 3\cos^2\vartheta}{|\mathbf{r}_i - \mathbf{r}_j|^3},$$

where ϑ is the angle between vectors $\mathbf{e}_z$ and $\mathbf{e}_{ij}$, and $C = \wp_{d_l}\wp_{d_{l'}}/(4\pi\epsilon_0\hbar)$ is a constant proportional to the product of atomic dipole moments $\wp_{d_l} = \langle d_l|\mathbf{p}|d_l\rangle$ assumed the same for both states $|d_{1,2}\rangle$, $\wp_{d_{1,2}} = \wp_d$.

Let us introduce collective atomic operators $\hat{\sigma}_{\mu\nu}(\mathbf{r}) = 1/N_r \Sigma_{j=1}^{N_r} \hat{\sigma}^j_{\mu\nu}$ averaged over the volume element d^3r containing $N_r = \rho\, d^3r \gg 1$ atoms around position $\mathbf{r}$. Then Eqs. (2a) and (2b) can be cast in the continuous form

$$V_{\rm af} = -\hbar\rho \int d^3r \sum_{l=1,2} [g_l \hat{\mathcal{E}}_l \hat{\sigma}_{e_l g}(\mathbf{r}) + \Omega_l \hat{\sigma}_{e_l d_l}(\mathbf{r})] + \text{H.c.}, \tag{3a}$$

$$V_{\rm dd} = \hbar\rho^2 \int\int d^3r\, d^3r'\, \hat{\sigma}_{dd}(\mathbf{r}) \Delta(\mathbf{r}-\mathbf{r}') \hat{\sigma}_{dd}(\mathbf{r}'). \tag{3b}$$

Using Eqs. (3a) and (3b), one can derive a set of Heisenberg-Langevin equations for the atomic operators $\hat{\sigma}_{\mu\nu}$ [7]. When the number of photons in the quantum fields $\hat{\mathcal{E}}_l$ is much smaller than the number of atoms, these equations can be solved perturbatively in the small parameters $g_l\hat{\mathcal{E}}_l/\Omega_l$ and in the adiabatic approximation for all the fields [16], with the result

$$\hat{\sigma}_{ge_l}(\mathbf{r}) = -\frac{i}{\Omega_l}\left[\frac{\partial}{\partial t} + i\hat{\alpha}(\mathbf{r})\right]\hat{\sigma}_{gd_l}(\mathbf{r}), \tag{4a}$$

$$\hat{\alpha}(\mathbf{r}) = \rho \int d^3r' \Delta(\mathbf{r}-\mathbf{r}')[\hat{\sigma}_{d_1 d_1}(\mathbf{r}') + \hat{\sigma}_{d_2 d_2}(\mathbf{r}')], \tag{4b}$$

$$\hat{\sigma}_{gd_l}(\mathbf{r}) = -\frac{g_l\hat{\mathcal{E}}_l}{\Omega_l^*}, \quad \hat{\sigma}_{d_l d_l}(\mathbf{r}) = \hat{\sigma}_{d_l g}(\mathbf{r})\hat{\sigma}_{gd_l}(\mathbf{r}). \tag{4c}$$

Let us assume that the transverse profile of both quantum fields is described by a Gaussian $e^{-r_\perp^2/w^2}$ of width w, where $r_\perp = |\mathbf{r}_\perp|$ is the distance from the field propagation axis, while the Rabi frequencies of classical driving fields Ω_l are uniform over the entire volume V. We may then write $g_l\hat{\mathcal{E}}_l = g_l(\mathbf{r}_\perp)\hat{\mathcal{E}}_l(z)$, where the traveling-wave electric field operators $\hat{\mathcal{E}}_l(z) = \Sigma_k a_l^k e^{ikz}$ are expressed through the superposition of bosonic operators a_l^k for the longitudinal field modes k, while the (transverse-position-dependent) coupling constants are given by $g_l(\mathbf{r}_\perp) = \tilde{g}_l e^{-r_\perp^2/2w^2}$, with $\tilde{g}_l = (\wp_{ge_l}/\hbar)\sqrt{\hbar\omega/2\epsilon_0 V}$, $\wp_{ge_l}$ being the dipole matrix element on the transition $|g\rangle \to |e_l\rangle$, $V = \pi w^2 L$, and L the medium length. Under this approximation, the propagation equations for the slowly varying quantum fields have the form

$$\left(\frac{\partial}{\partial t} \pm c\frac{\partial}{\partial z}\right)\hat{\mathcal{E}}_l(z,t) = i\tilde{g}_l N \hat{\sigma}_{ge_l}(z), \tag{5}$$

the sign "+" or "−" corresponding to $l=1$ or 2, respectively.

Following [16], we introduce new quantum fields $\hat{\Psi}_l$—dark state polaritons—via the canonical transformations

$$\hat{\Psi}_l = \cos\theta_l \hat{\mathcal{E}}_l - \sin\theta_l \sqrt{N}\hat{\sigma}_{gd_l}, \qquad (6)$$

where the mixing angles θ_l are defined through $\tan^2\theta_l = \tilde{g}_l^2 N/|\Omega_l|^2$. These polaritons correspond to coherent superpositions of electric field $\hat{\mathcal{E}}_l$ and atomic coherence $\hat{\sigma}_{gd_l}$ operators. Employing the plane-wave decomposition of the polariton operators, one can show that in the weak-field limit, they obey the bosonic commutation relations $[\hat{\Psi}_l(z), \hat{\Psi}^\dagger_{l'}(z')] \simeq L\delta_{ll'}\delta(z-z')$. Using Eqs. (4a)–(4c) and (5), we obtain the following propagation equations for the polariton operators,

$$\left(\frac{\partial}{\partial t} \pm v_l \frac{\partial}{\partial z}\right)\hat{\Psi}_l(z,t) = -i\sin^2\theta_l \hat{\alpha}(z,t)\hat{\Psi}_l(z,t). \qquad (7)$$

Here $v_l = c\cos^2\theta_l$ is the group velocity, while operator $\hat{\alpha}(z,t)$ is responsible for the self- and cross-phase modulation between the polaritons. It is related to the polariton intensity (excitation number) operators $\hat{\mathcal{I}}_l \equiv \hat{\Psi}_l^\dagger \hat{\Psi}_l$ via

$$\hat{\alpha}(z,t) = \frac{1}{L}\int_0^L dz' \Delta(z-z')[\sin^2\theta_1 \hat{\mathcal{I}}_1(z',t) + \sin^2\theta_2 \hat{\mathcal{I}}_2(z',t)], \qquad (8)$$

where the one-dimensional (1D) dipole–dipole interaction potential $\Delta(z-z')$ is obtained after the integration over the transverse profile of the quantum fields,

$$\Delta(z-z') = \frac{1}{\pi w^2}\int_0^{2\pi} d\varphi' \int_0^\infty dr'_\perp r'_\perp e^{-r'^2_\perp/w^2}\Delta(z\mathbf{e}_z - \mathbf{r}')$$
$$= \frac{2C}{w^3}\left[\frac{2|z-z'|}{w} - \sqrt{\pi}\left(1 + 2\frac{|z-z'|^2}{w^2}\right) \times \exp\left(\frac{|z-z'|^2}{w^2}\right)\mathrm{erfc}\left(\frac{|z-z'|}{w}\right)\right], \qquad (9)$$

and is shown in Fig. 2(a).

It follows from Eq. (7) that the intensity operators $\hat{\mathcal{I}}_l$ are constants of motion: $\hat{\mathcal{I}}_l(z,t) = \hat{\mathcal{I}}_l(z \mp v_l t, 0)$, the upper (lower) sign corresponding to $l=1$ ($l=2$). Then the formal solution for the polariton operators can be written as

$$\hat{\Psi}_l(z,t) = \exp\left[-i\sin^2\theta_l \int_0^t dt' \hat{\alpha}(z \mp v_l(t-t'), t')\right] \times \hat{\Psi}_l(z \mp v_l t, 0). \qquad (10)$$

Equation (10) is our central result. Let us outline the approximations involved in the derivation of this solution. In order to accommodate the pulses in the medium with negligible losses, their duration T should exceed the inverse of the EIT bandwidth $\delta\omega = |\Omega_l|^2(\gamma_{ge_l}\sqrt{\kappa_0 L})^{-1}$, where γ_{ge_l} is the transversal relaxation rate and $\kappa_0 \simeq 3\lambda^2/(2\pi)\rho$ is the resonant absorption coefficient on the transition $|g\rangle \to |e_l\rangle$. This yields the condition $(\kappa_0 L)^{-1/2} \ll T v_l/L < 1$ which requires a medium with large optical depth $\kappa_0 L \gg 1$ [16]. In addition, the dipole–dipole energy shift should lie within the EIT bandwidth $\delta\omega$ for all $|z-z'| \leq L$, which implies that $|\Delta(0)| = 2\sqrt{\pi}C/w^3 < \delta\omega$. Finally, the propagation/interaction time of the two pulses $t_{\text{out}} = L/v_l$ is limited by the relaxation rate γ_{gd_l} of the $\hat{\sigma}_{gd_l}$ coherence via $t_{\text{out}}\gamma_{gd_l} \ll 1$.

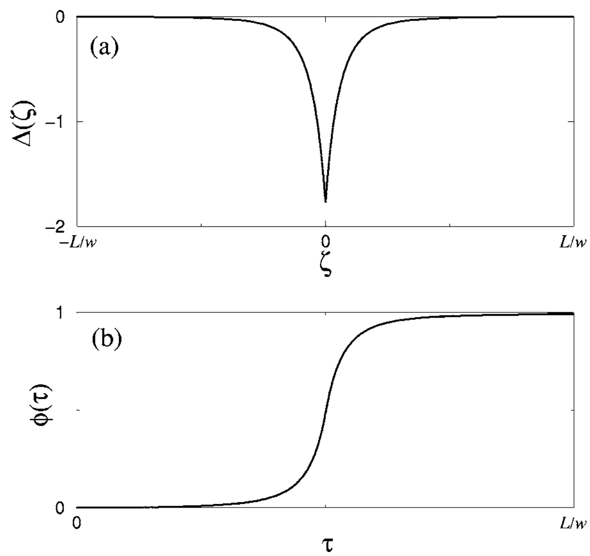

FIG. 2. (a) The 1D dipole–dipole potential $\Delta(\zeta)$ of Eq. (9) as a function of dimensionless distance $\zeta = (z-z')/w$, in units of $2C/w^3$ Hz. (b) The resulting phase shift $\phi(\tau) \equiv \phi(vt, L-vt, t)$ of Eq. (14) as a function of dimensionless time $\tau = vt/w$, in units of $2C/(vw^2)$ rad.

From now on, we assume that $\theta_{1,2} = \theta$, i.e., $\tilde{g}_1^2 N/|\Omega_1|^2 = \tilde{g}_2^2 N/|\Omega_2|^2$, which yields $v_{1,2} = v = c\cos^2\theta$. We are interested in the evolution of input state

$$|\Phi_{\text{in}}\rangle = |1_1\rangle \otimes |1_2\rangle, \qquad (11)$$

composed of two single-excitation polariton wave packets

$$|1_l\rangle = \frac{1}{L}\int dz f_l(z)\hat{\Psi}_l(z)^\dagger|0\rangle,$$

where $f_l(z)$ define the spatial envelopes of the corresponding wave packets $l=1, 2$ which initially (at $t=0$) are localized around $z=0, L$, respectively. For such an initial state, all the relevant information is contained in the expectation values of the polariton intensities $\langle\hat{\mathcal{I}}_l(z,t)\rangle = \langle\Phi_{\text{in}}|\hat{\mathcal{I}}_l(z,t)|\Phi_{\text{in}}\rangle$ and the two-particle wave function [7,11,12]

$$F_{12}(z_1,z_2,t) = \langle 0|\hat{\Psi}_1(z_1,t)\hat{\Psi}_2(z_2,t)|\Phi_{\text{in}}\rangle. \qquad (12)$$

With the above solution, for the polariton intensities we have $\langle\hat{\mathcal{I}}_{1,2}(z,t)\rangle = \langle\hat{\mathcal{I}}_{1,2}(z \mp vt, 0)\rangle = |f_{1,2}(z \mp vt)|^2$, which describes the shape-preserving counterpropagation of the two polaritons with group velocity v. Substituting the operator solution (10) into Eq. (12), after some algebra, we obtain the following expression for the two-particle wave function:

$$F_{12}(z_1,z_2,t) = f_1(z_1 - vt)f_2(z_2 + vt)\exp[i\phi(z_1,z_2,t)], \qquad (13)$$

$$\phi(z_1,z_2,t) = -\sin^4\theta\int_0^t dt'\Delta[z_1 - z_2 - 2v(t-t')], \quad (14)$$

which indicates that the dipole–dipole interaction between the two single-excitation polaritons results in the conditional phase-shift $\phi(z_1,z_2,t)$. We consider a situation in which at time $t=0$, the first pulse is localized at $z_1=0$ and the second pulse is at $z_2=L$, while after the interaction, at time $t_{\mathrm{out}} = L/v$, the coordinates of the two pulses are $z_1=L$ and $z_2=0$, respectively [Fig. 2(b)]. Then the phase shift accumulated during the interaction is spatially uniform, and is given by

$$\phi(L,0,L/v) = -\frac{\sin^4\theta}{v}\int_0^L dz'\Delta(2z'-L) = \frac{2C\sin^4\theta}{vw^2}. \quad (15)$$

This remarkably simple result is obtained upon replacing the variable $(2z'-L)/w \to \zeta'$ and extending the integration limits to $L/w \to \infty$. The main limitation on the phase shift is imposed by the condition $|\Delta(0)| < \delta\omega$. In terms of experimentally relevant parameters, the group velocity is $v \simeq 2|\Omega|^2/(\kappa_0\gamma_{ge}) \ll c(\sin^2\theta \simeq 1)$, and we have

$$\phi < \frac{w}{2}\sqrt{\frac{\kappa_0}{\pi L}}. \quad (16)$$

To relate the foregoing discussion to a realistic experiment, let us assume an ensemble of cold alkali atoms in the ground state $|g\rangle$ with density $\rho \sim 10^{14}$ cm^{-3} confined in a trap of length $L \sim 100$ μm. The resonant quantum fields with $\lambda \sim 0.6$ μm have the transverse width $w \sim 30$ μm. In the presence of driving fields with appropriate frequencies, the single-photon pulses lead to the (two-photon) excitation of single atoms to the Rydberg states $|d\rangle$ with quantum numbers $n \simeq 25$ and $q=n-1$. The corresponding dipole moment is $\wp_d \simeq 900 e a_0$, while the decay rate of $|d\rangle$ is $2\gamma_d \sim 3\times 10^3$ s^{-1} [17]. With $\gamma_{ge} \sim 10^7$ s^{-1} and $\Omega \sim 1.8\times 10^7$ rad/s, the group velocity is $v \simeq 4$ m/s, and the accumulated phase shift is $\phi \simeq \pi$. The corresponding fidelity F of the phase gate is determined by the bandwidth of the transparency window $\delta\omega$ and the two-photon coherence relaxation rate γ_{gd}, as discussed above. For the present parameters, condition (16) is satisfied, the optical depth is large $\kappa_0 L \sim 1.7\times 10^3$, while for a cold atomic gas we have $\gamma_{gd} \simeq \gamma_d$. Therefore the fidelity is mainly limited by the relaxation rate γ_d of Rydberg states and is given by $F = \exp(-\gamma_d L/v) \gtrsim 0.96$.

To summarize, we have studied a highly efficient scheme for cross-phase modulation and entanglement of two counterpropagating single-photon wave packets, employing their ultrasmall group velocities in atomic vapors, under the conditions of electromagnetically induced transparency, and the strong long-range dipole–dipole interactions of the accompanying Rydberg-state excitations in a ladder-type field-atom coupling setup. We have solved, in the weak-field and adiabatic approximations, the effective one-dimensional propagation equations for the polariton operators and have shown that the dipole–dipole interaction leads to a *homogeneous* conditional phase shift that can reach the value of π even if the transverse cross section of the pulses w^2 is much (three orders of magnitude) larger than the diffraction limit λ^2. This is the obvious merit of the present proposal, as compared to previous schemes based on local interactions of photons or slow-light polaritons [9–15], which require the photonic beam cross section to be comparable to the cross section for atomic resonant absorption. Hence our proposal paves the way to the coveted deterministic entanglement of two single-photon pulses and the realization of the universal photonic phase gate [12].

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VI. THIRRING-LIKE SOLITONS IN EIT

We theoretically show that the giant Kerr nonlinearity in the regime of electromagnetically induced transparency (EIT) in vapor may cause the formation of hitherto unobserved 1D and 2D spatial Thirring-like vector solitons, wherein the nonlinear terms are solely due to the cross-phase modulation that couples two parallel light beams.

A. Introduction to optical spatial solitons

Optical beams, having narrow transverse profile, can self-trap themselves via the interaction with the nonlinear homogenous media whenever the natural diffraction-broadening of such a beam is balanced by a nonlinear self-focusing mechanism. Such a self-trapped beam is called an optical spatial soliton with solely self phase modulation (SPM) [52, 63]. The optical beam modifies the refractive index in such a way that it generates an effective positive lens, i.e. the refractive index in the center of the beam becomes larger than at the beam's margins. Hence, the phase velocity in the center of the beam is reduced, thus preventing the natural evolution of the wavefront from planar to quadratic. Effectively, the medium is a graded-index waveguide in the vicinity of the optical beam. Provided that the optical beam that induces the waveguide is at the same time a guided mode of that waveguide, the beam propagation becomes stationary. If, in addition, the propagation is stable under noise, then, the beam is a spatial soliton as the stationary propagation can be physically realized. This perspective is known as the self-consistency principle [64]. One can actually use it to find (numerically) the wave-functions and propagation constants of solitons in NL media which do not yield analytic solutions [65]. The mechanisms underlying the large majority of solitons belong to this type. These include solitons in atomic vapor [66], Optical Kerr media [67–69], photorefractives [70], and many others [52, 63]. In Particular, self-trapped soliton in EIT media was first suggested by Hong et al [71]. In their theoretical proposal, the beam is subject solely to self phase modulation in a gas of N-type level atoms.

1. Vector and composite solitons

A beam consisting of two (or more) field components that mutually self-trap in a nonlinear medium is called a vector solitons. The field components jointly induce a waveguide and trap themselves within it by properly populating its guided modes. Vector solitons, first suggested by Manakov [72], consist of two orthogonally polarized components in a nonlinear Kerr medium where the self-phase modulation (SPM) and cross-phase modulation (XPM) equally contribute to the potentail, that is, each field "feels" its own induced potential (SPM) as well as the other component's induced potential (XPM). As a consequence of the self consistency principle, stationary propagation of a vector soliton is achieved if the field components correspond to guided modes of the waveguide induced by the sum of their intensities. In the degenerate case (Manakov-type solitons), all fields components populate the same mode (typically the fundamental mode) of the waveguide [72, 73]. However, vector solitons can also form when the fields components populate different modes of their jointly induced waveguide, as was predicted in the temporal [74] and spatial [75] domains, and observed in photorefractives [76]. Such solitons are called composite or multi-mode solitons.

2. Dark solitons

Dark soliton [77] is the self-trapping of a void of light, which resides inside a uniform background illumination. A narrow dark notch (stripe) borne on an otherwise-uniform light beam that propagates in the center of a linear medium diffracts. However, when the notched beam propagates in a self-defocusing medium, as a result of the illumination the refractive index decreases in the illuminated region (on both side of the notch), whereas at the center of the notch the index remains unchanged. Due to the fact that the index at the center is now higher than in the illuminated regions, the two portions of the beam (on both side of the notch) expand their inner boundaries and reduce the diffraction of the notch. Under certain conditions, the diffraction of the notch is fully eliminated and the dark notch experiences stationary (non-diffracting) propagation as a dark soliton. "Fundamental" (not higher order) dark solitons must possess an anti-symmetric waveform, i.e., their amplitude must undergo a phase jump in the center of the notch. The self-consistent view of a dark soliton is that a dark soliton is the second guided mode at cut-off of the induced waveguide.

Spatial Thirring-type solitons via electromagnetically induced transparency

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We show that the giant Kerr nonlinearity in the regime of electromagnetically induced transparency in vapor can give rise to the formation of Thirring-type spatial solitons, which are supported solely by cross-phase modulation that couples the two copropagating light beams. © 2005 Optical Society of America

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Coherence between two levels that is induced by a strong (drive) laser field can give rise to absorption cancellation on another transition in a Λ-shaped atomic level configuration for a weak probe field. Absorption cancellation occurs via destructive interference with the drive field. This phenomenon, known as electromagnetically induced transparency (EIT[1,2]), changes the probe-field dispersion, making its group velocity dependent on the drive field, so that by turning the drive field off one can slow a probe pulse down to a complete standstill.[3] Slow-light manifestations of EIT have attracted considerable attention, in view of their possible use for storing and regenerating quantum states of light in atomic quantum networks.[4] Another nonlinear manifestation of EIT is spatial solitons, which were predicted to form when diffraction is balanced by self-phase modulation[5] (SPM). Whereas the foregoing aspects of EIT pertain to a single probe beam, giantly enhanced Kerr nonlinear coupling of two probe beams is not less promising.[6,7] Its highlight is the dramatically enhanced phase shift (compared with similar shifts in other Kerr media), impressed by one ultraweak probe on another (cross-phase modulation, XPM) in the N-shaped atomic level configuration detailed below. This effect may bring about the deterministic entanglement of two single-photon pulses.[8,9]

Despite the extensive discussion of the giant XPM in EIT media and its recent experimental demonstration,[10] its analysis has been restricted mainly to one dimensional (1D) propagation without considering transverse (diffraction) effects of the cross-coupled beams. Here we study unexplored aspects of the giantly enhanced XPM between two beams subject to EIT: the formation of low-power spatial solitons that arise solely from the balance between diffraction and XPM with no contribution from SPM.

This kind of soliton generically conforms to the massive Thirring model.[11–13] In optics, Thirring-type (holographic) solitons were predicted to occur with the XPM arising from the grating induced by two mutually coherent fields,[14,15] having no SPM contribution. However, even though evidence for holographic focusing was reported,[16,17] optical Thirring-type solitons have thus far never been observed. This is because it is very difficult to find optical systems with large XPM but lacking SPM altogether. The system proposed in this Letter offers just that and thus supports the formation of such "exotic" optical Thirring-type solitons.

Consider a cold atomic medium containing two species of atoms, A with a Λ-shaped level configuration and B with an N-shaped level configuration (Fig. 1). All the atoms are optically pumped to the ground states $|b\rangle_{A,B}$. Atoms A and B resonantly interact with two running-wave fields driving the atomic transitions $|c\rangle_{A,B} \to |a\rangle_{A,B}$ with the Rabi frequencies $\Omega_d^{(A,B)}$, respectively. In the absence of level $|d\rangle_B$, this situation corresponds to the usual EIT for the fields $\mathcal{E}_{1,2}$ that are acting on the transitions $|b\rangle_{A,B} \to |a\rangle_{A,B}$: in the vicinity of a frequency corresponding to the two-photon Raman resonances $|b\rangle_{A,B} \to |c\rangle_{A,B}$, the medium becomes transparent for both weak fields.[1,2,18] This transparency is accompanied by a steep variation of the refractive index. The field $\mathcal{E}_1$ dispersively interacts with atoms B via the transition $|c\rangle_B \to |d\rangle_B$ with the detuning $\Delta = \omega_{dc}^{(B)} - \omega_1$. As a result, atoms of species B simultaneously provide EIT for the field $\mathcal{E}_2$ and its cross coupling with the field $\mathcal{E}_1$, known as XPM.[6,7,20] Note that the role of atoms A in Fig. 1 is only to provide EIT for the field $\mathcal{E}_1$. This is necessary to match

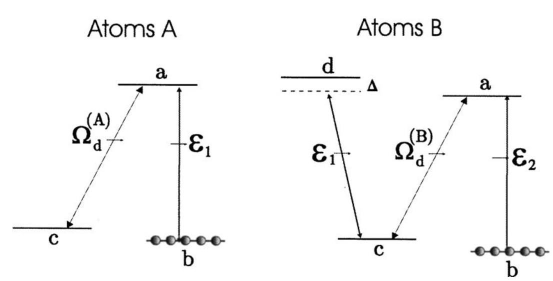

Fig. 1. Atomic level scheme involving two species of atoms A and B, both subject to EIT conditions. The fields $\mathcal{E}_1$, $\mathcal{E}_2$ interact via Kerr-nonlinear XPM. For the case in which $\mathcal{E}_{1,2}$ are cw fields, atoms A and $\Omega_d^{(A)}$ can be ignored, as they are unnecessary.

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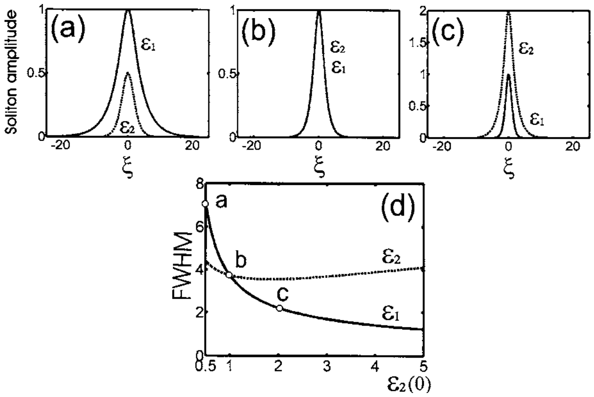

Fig. 2. Fundamental-mode Thirring-type solitons. (a)–(c) Soliton profiles for different ratios between the peak amplitudes. (d) Soliton existence curve: FWHM of each field versus the peak amplitude $\mathcal{E}_2(0)$ at a fixed peak amplitude $\mathcal{E}_1(0)=1$. Points a–c correspond to the solitons of (a)–(c), respectively.

the group velocities of the two copropagating weak pulses and thus maximize their interaction time.[20] In what follows we limit ourselves to continuous-wave (cw) fields, so that EIT conditions for field $\mathcal{E}_1$ are not required, i.e., atoms A, as well as the driving field $\Omega_d^{(A)}$, can be dropped.

Here we study optical beams with narrow transverse profiles, so that diffraction plays a significant role, as opposed to earlier treatments of XPM in EIT.[6–9,20–22] Another distinct feature of this problem is that SPM, caused by the coupling of the field $\mathcal{E}_2$ to the transition $c\rightarrow d$ in atoms B, or its equivalent for the field $\mathcal{E}_1$, is inversely proportional to the detuning of the field from that transition. Hence, assuming that the detuning of field $\mathcal{E}_2$ is much larger than the detuning of field $\mathcal{E}_1$ on the same transition, SPM can be neglected and only XPM survives. These two features are responsible for the ability to form a novel type of spatial soliton. The system is described by the following equations for the slowly varying envelopes, obtained perturbatively under the weak-field adiabatic approximation,[18,19] for cw fields and the standard paraxial conditions (i.e., optical beams much wider than the wavelength):

$$2ik_1\frac{\partial}{\partial z}E_1+\nabla_\perp^2E_1=-k_1^2\eta|E_2|^2E_1, \quad (1a)$$

$$2ik_2\frac{\partial}{\partial z}E_2+\nabla_\perp^2E_2=-k_2^2\eta|E_1|^2E_2, \quad (1b)$$

where $\eta=\mu_{dc}^{(B)2}\mu_{ab}^{(B)2}\rho_B/(\Delta|\Omega_d^{(B)}|^2\epsilon_0\hbar^3)$, ρ_B is the density of atoms B, $\mu_{dc}^{(B)}$ and $\mu_{ab}^{(B)}$ are the $c\rightarrow d$ and $a\rightarrow b$ transition dipoles, and $k_{1,2}$ are the wave vectors of the fields $E_{1,2}$, respectively.

In deriving the above equations we have made the experimentally realistic assumption that absorption of both weak fields is negligible over the propagation length.[6,9,20] Rewriting these equations in dimensionless form yields

$$\nabla_\perp'^2\mathcal{E}_1+i\frac{\partial}{\partial\zeta}\mathcal{E}_1+\sigma|\mathcal{E}_2|^2\mathcal{E}_1=0, \quad (2a)$$

$$a^2\nabla_\perp'^2\mathcal{E}_2+ia\frac{\partial}{\partial\zeta}\mathcal{E}_2+\sigma|\mathcal{E}_1|^2\mathcal{E}_2=0, \quad (2b)$$

where $a=k_1/k_2$ is the asymmetry parameter, $\nabla_\perp'^2=\partial^2/\partial\xi^2+\partial^2/\partial\varsigma^2$; $\xi=k_1x$; $\varsigma=k_1y$; $\zeta=(k_1/2)z$; and $\mathcal{E}_i=\sqrt{|\eta|}E_i$. The sign of the detuning Δ (appearing inside the constant η) is crucial, as it determines the sign of the nonlinearity: $\sigma=+1$ for positive (red) detuning (focusing) and $\sigma=-1$ for negative (blue) detuning (defocusing).

It is important to emphasize the basic difference between Thirring-type solitons and Manakov-like vector solitons.[23] The nonlinearity in Manakov-like systems depends on the sum of the intensities of the individual components; that is, SPM and XPM play a symmetric role. Consequently, the constituents of the Manakov-like solitons are bound states of the potential they jointly induce. For Thirring-type solitons, on the other hand, each component feels, and is guided by, a different potential: $\mathcal{E}_1$ feels the potential induced by $\mathcal{E}_2$ and vice versa. Note also that a must be differ-

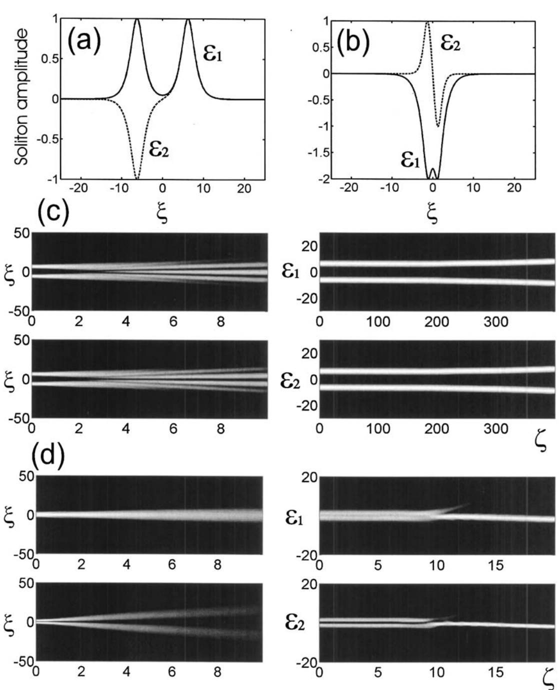

Fig. 3. (a) Composite Thirring-type soliton solutions with beam $\mathcal{E}_1$ in the fundamental (ground-state) mode and beam $\mathcal{E}_2$ in the dipole (first asymmetric) mode. The amplitude ratio is 1 (equal intensities). (b) Same as (a) for amplitude ratio 2. (c) Propagation of the solution shown in (a): beam $\mathcal{E}_1$ (upper) and beam $\mathcal{E}_2$ (lower) without (left) the nonlinearity for one diffraction length and with (right) the nonlinearity for ~10 diffraction lengths. During nonlinear propagation the composite entity splits into two fundamental Thirring-type solitons diverging away from one another. (d) Propagation of the solution shown in (b) in the presence of initial noise. Both components fuse into a single fundamental Thirring-type soliton.

ent from 1: i.e., the fields $\mathcal{E}_1$ and $\mathcal{E}_2$, operating on different transitions, must have different wavelengths. If $a=1$ the SPM term cannot be neglected, and Eqs. (1) are no longer adequate.

The soliton solutions of Eqs. (2) have been found numerically through the self-consistency method and their propagation has been simulated by using the split-step Fourier propagation method. The asymmetry parameter a has been chosen to take the experimentally reasonable value of 1.0005 throughout our calculations. In what follows we discuss the main features of solitons thus obtained, referring to solutions that are bound in one or two transverse directions as 1D and 2D, respectively.

We first discuss the $\sigma=+1$ (focusing) case. We have studied various amplitude ratios of fields $\mathcal{E}_1$ and $\mathcal{E}_2$ in the presence of noise to find that the fundamental (ground-state mode) mode of the 1D system ($\nabla_\perp'^2 = \partial^2/\partial\xi^2$) is stable. Figures 2(a)–2(c) present such fundamental Thirring-type solitons with different amplitude ratios. Figure 2(d) shows the existence curve versus the peak amplitude of the second component, $\mathcal{E}_2(0)$, when $\mathcal{E}_1(0)=1$. Note that the existence curve is governed only by the ratio between the peak amplitudes. Increasing the peak amplitudes of both components by a factor α results in new widths, FWHM_1/α and FWHM_2/α.

When seeking Thirring solitons in two transverse dimensions ($\nabla_\perp'^2=\partial^2/\partial\xi^2+\partial^2/\partial\varsigma^2$), we find that the 2D solitons suffer from a weak instability, similar to 2D Kerr solitons.

Next, we have searched for 1D composite (multimode) Thirring-type solitons for which each field is in a different mode.[24] Specifically, we have looked for solitons in which $\mathcal{E}_1$ and $\mathcal{E}_2$ are in the fundamental and second (dipole-type) modes, respectively, as was found for holographic solitons[25] and the Manakov-like system.[23] However, our system is not saturable (unlike the one in Refs. 14 and 25), and we find these solutions to be unstable. When the amplitude of the dipole mode is equal to or larger than that of the fundamental mode, the composite entity splits into two fundamental Thirring-type solitons diverging away from one another [Fig. 3(c)]. The splitting occurs irrespective of whether we add initial noise to the ideal solution. On the other hand, when the fundamental component is more intense than the dipole component, they fuse (within several diffraction lengths, depending on the noise) into a fundamental-mode Thirring-type soliton [Fig. 3(d)].

The case of defocusing nonlinearity [$\sigma=-1$ in Eq. (2)] might have been expected to yield dark 1D or 2D solitons. However, within our model with $a\neq 1$, we cannot find such solitons. This is related to the fact that a dark soliton (for any local nonlinearity) is the second bound state of the induced potential at cutoff energy.[26] A Thirring-type EIT dark soliton requires both components to be at the cutoff energy of each other's induced potential. This requirement prohibits the existence of a dark Thirring soliton unless $a=1$. Yet, as discussed above, the case $a=1$ does not represent our EIT system anymore.

To conclude, we have shown that the giant Kerr nonlinearity in the regime of EIT in vapor can lead to the formation of spatial Thirring-like vector solitons, supported solely by cross-phase modulation.

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VII. SUMMARY AND CENTRAL CONCLUSIONS

We have studied a novel regime of giant Kerr-nonlinear interaction between two ultra-weak optical fields in which the XPM is not accompanied by spectral broadening of the interacting pulses. This regime is realizable in atomic vapors, when a weak probe pulse, upon propagating through the electromagnetically induced transparency (EIT) medium, interacts with a signal pulse that is dynamically trapped in a photonic band gap created by spatially-periodic modulation of its EIT resonance. This scheme avoids the difficulty posed by copropagating pulses: since the conditional phase-shift of each pulse is proportional to the local intensity of the other pulse, different parts of the interacting pulses acquire different phase shifts, which causes their frequency chirp and spectral broadening. The foregoing difficulties may be overcome via controlled modification of the photonic density of states in gaseous EIT media, by modulating their refractive index with an off-resonant standing light wave [48]. By properly tuning the resulting photonic band structure, a propagating signal pulse can be converted into a standing-wave polaritonic excitation inside the photonic band gap (PBG). The trapped signal polariton, having an appreciable photonic component, can impress a large phase shift that is spatially-uniform (across the pulse) upon the propagating probe, at the single-photon level. The advantageous features of the present scheme pave the way for possible QI applications based on deterministic photon-photon entanglement, without the limitations associated with traveling wave configurations [36] and without invoking cavity QED techniques [19, 21, 22, 62].

Another possible scheme for giantly enhanced XPM with suppressed spectral broadening between optically-induced dark-state polaritons in spatially periodic media with multilevel dopants was suggested by us [25]. This mechanism can produce photon-photon entanglement with high fidelity, by combining the advantages of their dispersion in PBG structures and of the strongly enhanced nonlinear optical coupling achievable via EIT in an appropriately doped medium. To this end we have proposed a new class of multimode quantum-field interactions involving quantized EIT-polaritons in PBG structures. We have shown that such interactions allow efficient XPM between a propagating probe pulse and a trapped signal pulse, whose localization is achieved by an adiabatic four-wave mixing process that pulls its frequency into the PBG. This localization allows multiply repeated interaction of the signal with the entire probe pulse. As a result, the combined two-photon state of the system can acquire a conditional phase-shift, which corresponds to the universal cphase logic gate. The phase shift is spatially-uniform and the process may have high fidelity. The proposed schemes may therefore pave the way to quantum information applications, such as *deterministic all-optical quantum computation, dense coding and teleportation* [28].

In the foregoing schemes [24, 25] a rather strong transverse confinement of the pulses close to the minimum of λ^2 is needed in order to attain a phase shift of π , which is technically challenging. We have therefore considered a different approach [26], which involves induced dipole-dipole interactions in the EIT medium, and leads to a conditional phase shift of between two single-photon pulses subject to moderate transverse confinement. Instead of resorting to the rather weak short-range collisional interactions, as in [59], we have made use of long-range dipole-dipole interactions between atoms in an internal state which is populated only in the presence of a polariton. If these internal states are Rydberg states, their induced dipole moments in dc fields are large and their interaction can be rather strong. The nonlocal character of the interaction further enhances the effective interaction time. In [26]we have discussed a novel scheme for a strong cross-phase modulation of two counterpropagating single-photon wavepackets, making use of their ultra-small group velocities in atomic vapors, under EIT conditions of, and of their long-range dipole-dipole interactions. We have shown that the dipole-dipole interaction leads to a homogeneous conditional phase shift. The magnitude of this phase shift can reach the value of π even if the transverse cross section of the pulses is much larger than the diffraction limit λ^2. This is in contrast to the previous schemes based on local interactions of photons or slow-light polaritons[24, 25, 34, 36, 46], in which the photonic beam cross section has to be comparable to the cross section for atomic resonant absorption. Thus the present scheme could be used to deterministically entangle two single-photon pulses or realize the controlled-phase universal logic gate [24, 25] under practical conditions.

A more classical venue that has emerged in the course of the research, is concerned with vector solitons formed by two optical beams via XPM within the EIT media. Despite the extensive discussion of the giant XPM in EIT media, and its recent experimental demonstration [38], its analysis has been mainly restricted to one dimensional (1D) propagation, without considering transverse (diffraction) effects of the cross-coupled beams. We have shown [78] that the giant Kerr nonlinearity in the EIT regime in vapor can give rise to the formation of Thirring-type spatial solitons, by two co-propagating light beams. They represent an unexplored aspect of the giantly-enhanced XPM between two beams subject to EIT: the formation of low-power spatial solitons that arise from the balance between transverse diffraction and XPM, with no contribution from self-phase modulation (SPM). This is, to our knowledge, the first physical system supporting spatial solitons with solely XPM interaction (each field "feels" only the potential induced by the other field).

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