# State transfer in noisy spin chains

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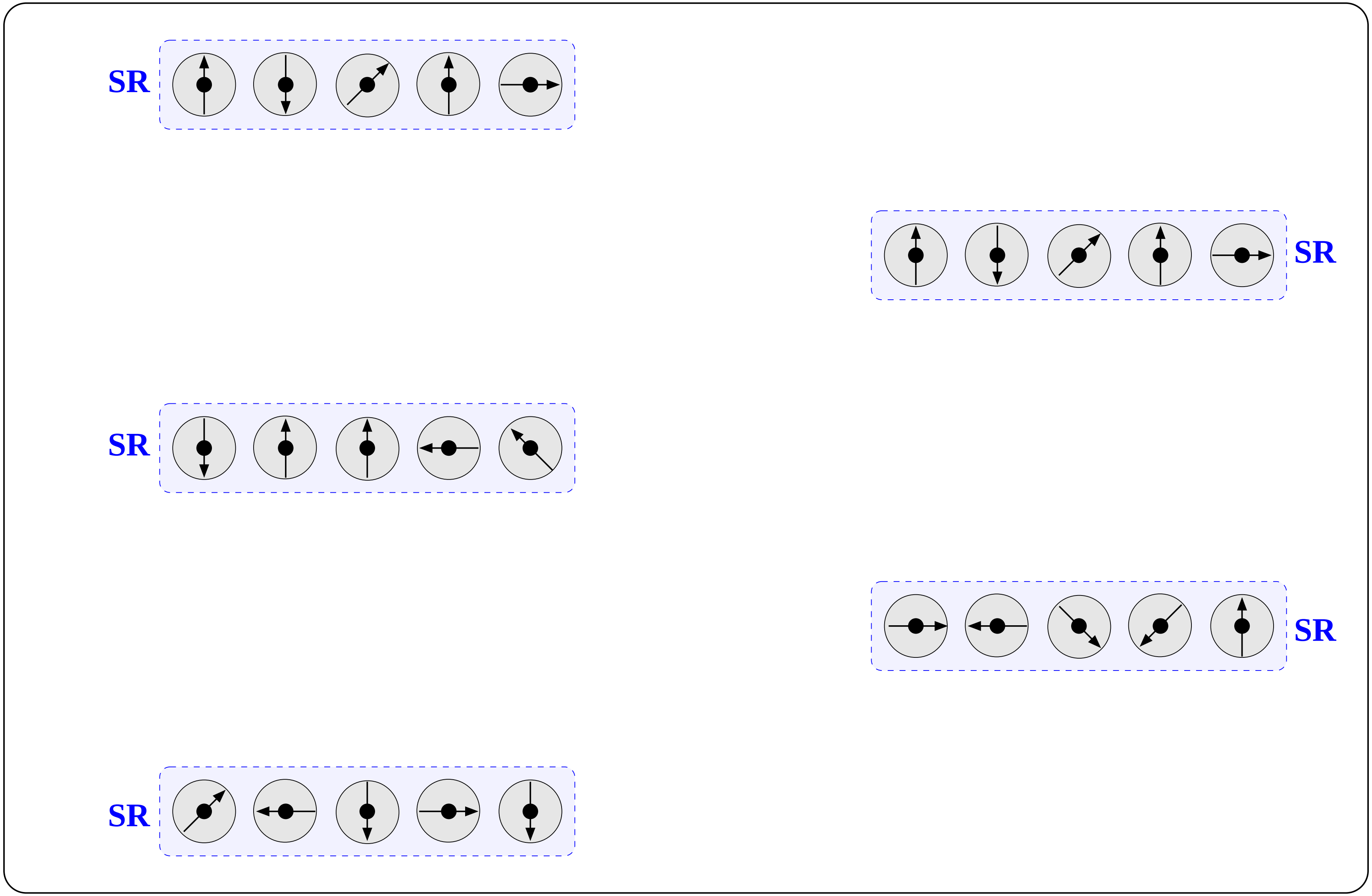
Greece

# Integrated Quantum Register




**Finite-range qubit-qubit interactions**

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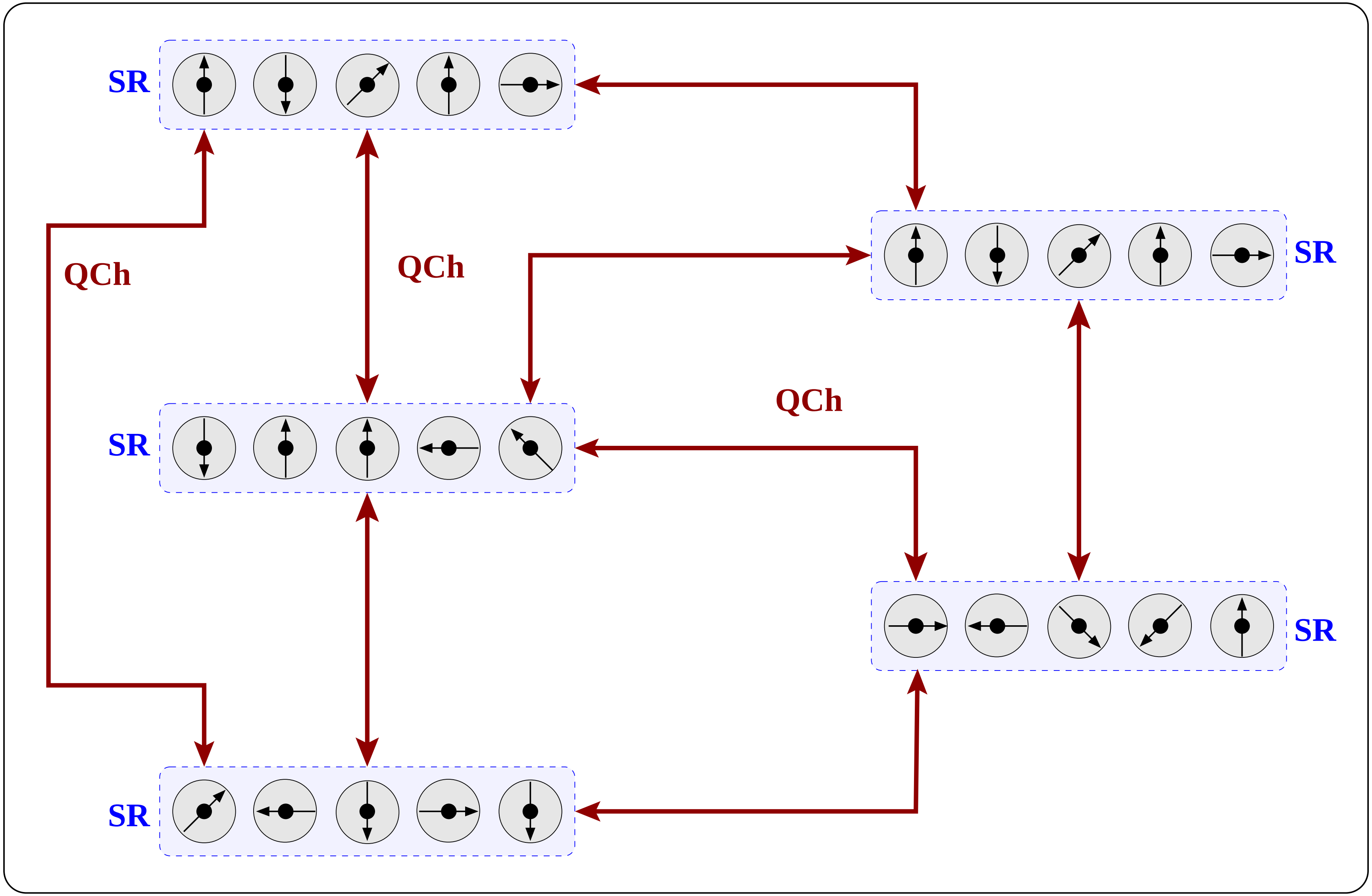


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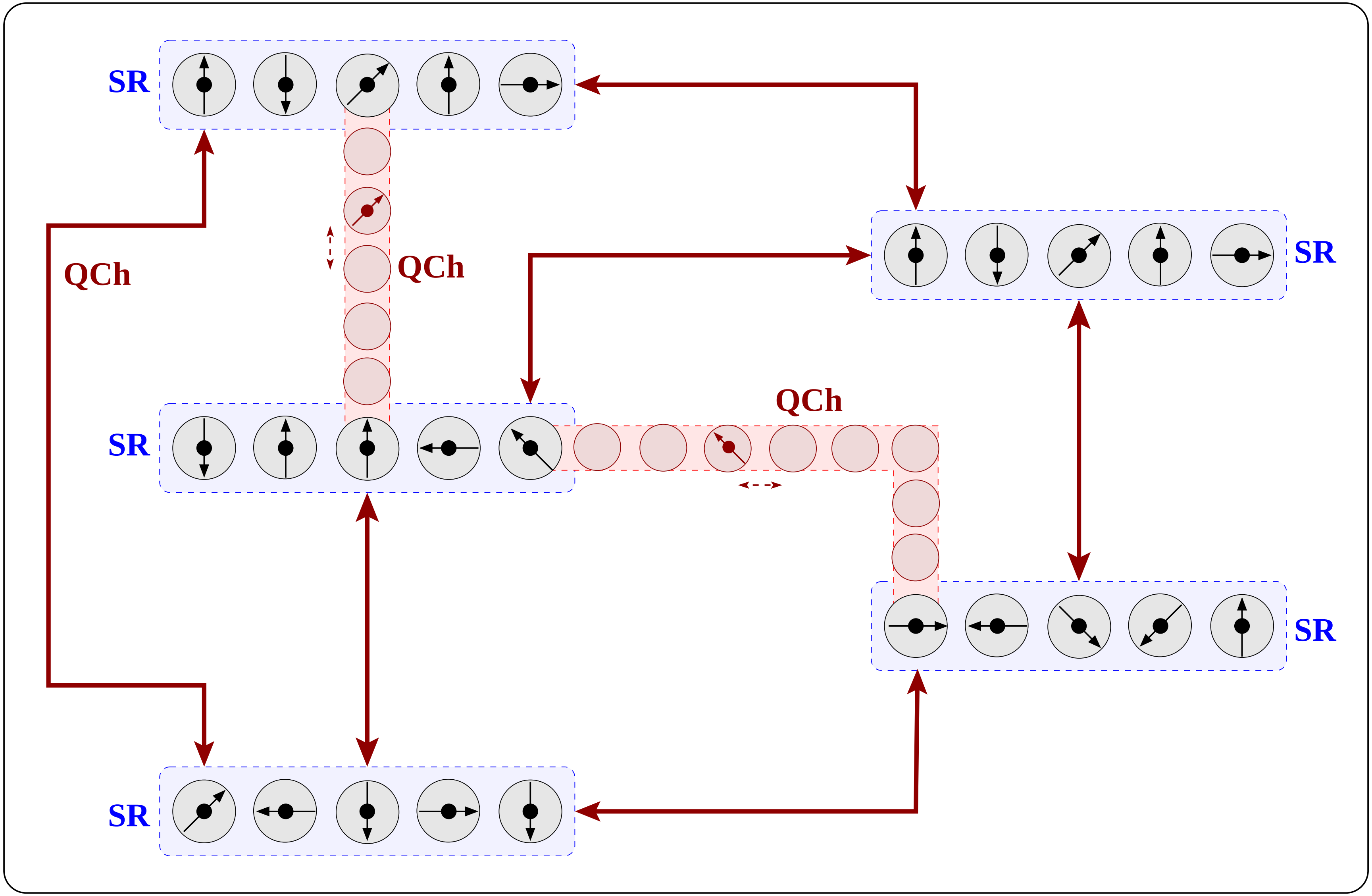


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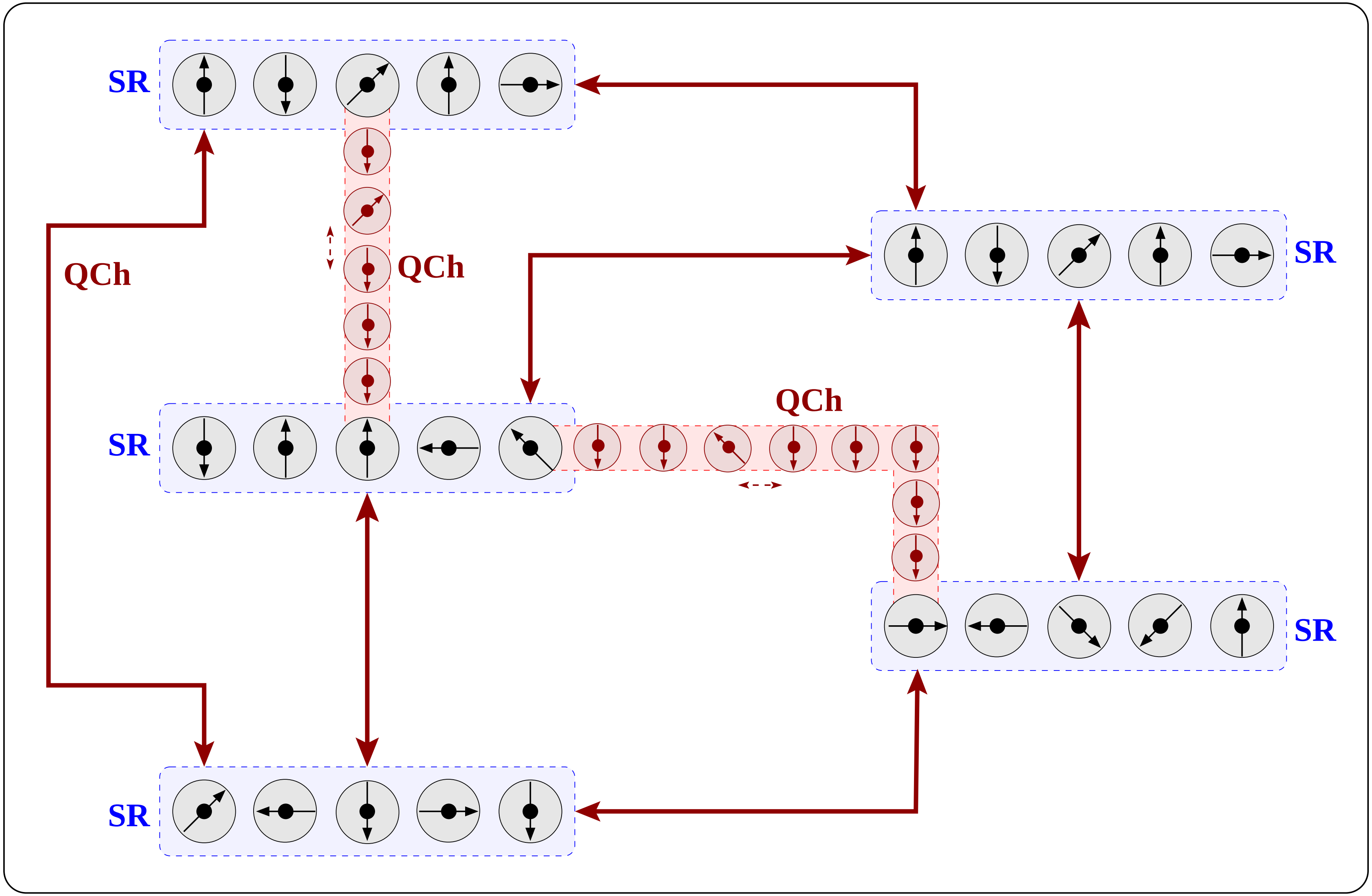


**Finite-range qubit-qubit interactions**

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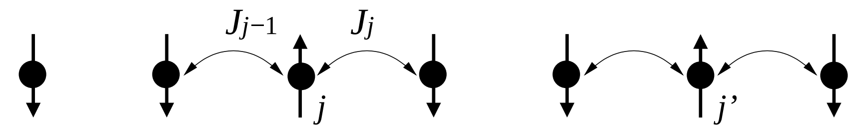
- Conclusions

# Spin Chain Hamiltonian

$$H = \frac{1}{2}\sum_{j=1}^{N} h_j \hat{\sigma}_j^z - \frac{1}{2}\sum_{j=1}^{N-1} J_j (\hat{\sigma}_j^x \hat{\sigma}_{j+1}^x + \hat{\sigma}_j^y \hat{\sigma}_{j+1}^y + \Delta \hat{\sigma}_j^z \hat{\sigma}_{j+1}^z)$$




$h_j$: local magnetic field at site $j$

$\hat{\sigma}_j^{x,y,z}$: Pauli spin operators

$J_j(t)$: inter-spin couplings ($J_j \in [0, J_{\max}]$)

$\Delta\ (=0)$: anisotropy ($XX$ model)

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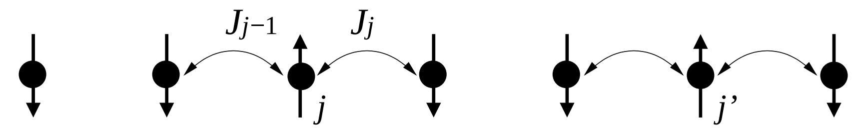


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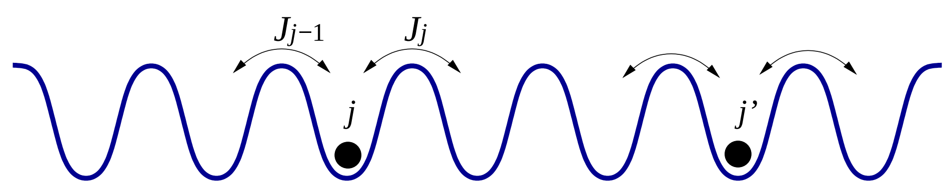
$\Delta$ $(= 0)$: anisotropy ($XX$ model)

**Isomorphic to the Hubbard Hamiltonian**: $|\downarrow\rangle_j \equiv |0\rangle_j$ & $|\uparrow\rangle_j \equiv |1\rangle_j$ $\Rightarrow$

$$H = \sum_{j=1}^{N} h_j \hat{a}_j^\dagger \hat{a}_j - \sum_{j=1}^{N-1} J_j(\hat{a}_j^\dagger \hat{a}_{j+1} + \hat{a}_{j+1}^\dagger \hat{a}_j)$$




$h_j$: single-particle energy at site $j$

$\hat{a}_j, \hat{a}_j^\dagger$: fermion or hard-core boson operators $[(\hat{a}_j^\dagger)^2 = 0]$

$J_j(t)$: inter-site hopping ($J_j \in [0, J_{\max}]$)

# Qubit State Transfer

Qubit state: $|\psi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle$ $\qquad [\,|\downarrow\rangle \equiv |0\rangle \;\&\; |\uparrow\rangle \equiv |1\rangle]$

Transfer state $|\psi\rangle$ through the Spin Chain (of length $N$):

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- Time *evolution* due to $H(t)$ $\quad [U(t) = \mathcal{T}\exp\left[\frac{1}{i\hbar}\int_0^t H(t')dt'\right]]$:
  $|\Psi(t)\rangle = U(t)\,|\Psi_{\rm in}\rangle = \alpha\,|\mathbf{0}\rangle + \beta\sum_{j=1}^{N} A_j(t)\,|\boldsymbol{j}\rangle$

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**Perfect state transfer**: $|\psi\rangle_N = \alpha\,|0\rangle + \mathrm{e}^{\mathrm{i}\phi_0} \beta\,|1\rangle \Rightarrow$

$$|A_N(t_{\mathrm{out}})| = 1 \quad \& \quad \phi = \arg(A_N) = \phi_0 = \mathrm{const}$$

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**Imperfect state transfer**: $\rho_N \equiv {\rm Tr}_{\not N}(\,|\Psi\rangle\langle\Psi|\,) \Rightarrow$

$$F \equiv \frac{1}{4\pi}\int \langle\psi|\,\rho_N\,|\psi\rangle {\rm d}\Omega_\psi = \frac{1}{2} + \frac{|A_N|^2}{6} + \frac{|A_N|\cos(\phi-\phi_0)}{3}$$

Bose, PRL **91**, 207901 (2003)

# State Transfer Protocols

**Idealized spin chain**: *No disorder*

$$h_j := 0 \; \forall \; j \in [1, N] \qquad J_j = J_j(t) \leq J_{\max}$$

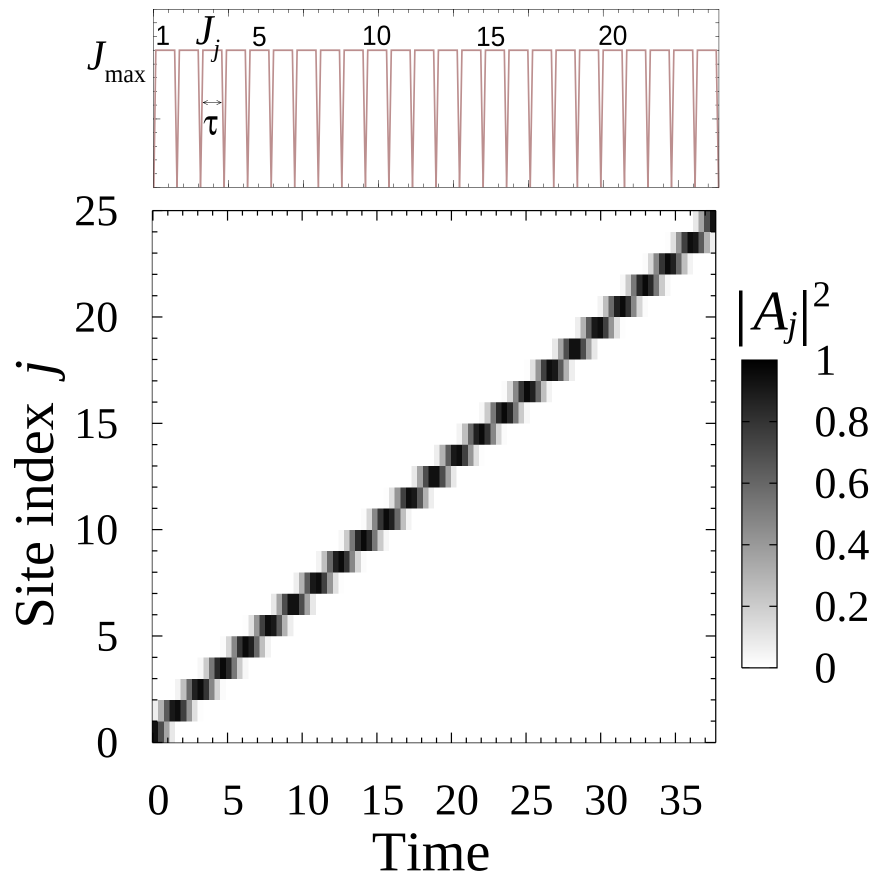
**Number of** [spin or particle] **excitations is preserved**:

$$\sum \hat{\sigma}_j^z = \mathrm{const} \in [0, 1] \qquad |\Psi(t)\rangle \in [\,|\mathbf{0}\rangle, \{\,|\boldsymbol{j}\rangle\}]$$

# State Transfer Protocol: Sequential SWAP

**Sequence of $\pi$ pulses:** $J_1(t_1), J_2(t_2), \ldots, J_{N-1}(t_{N-1}) = J_{\max}$

$$\int J_j(t')\mathrm{d}t' = J_{\max}\tau = \pi/2 \quad \Rightarrow \quad A_j(t_{j-1}) = -i\sin(\pi/2)\, A_{j-1}(t_{j-2}) = (-i)^{j-1}$$



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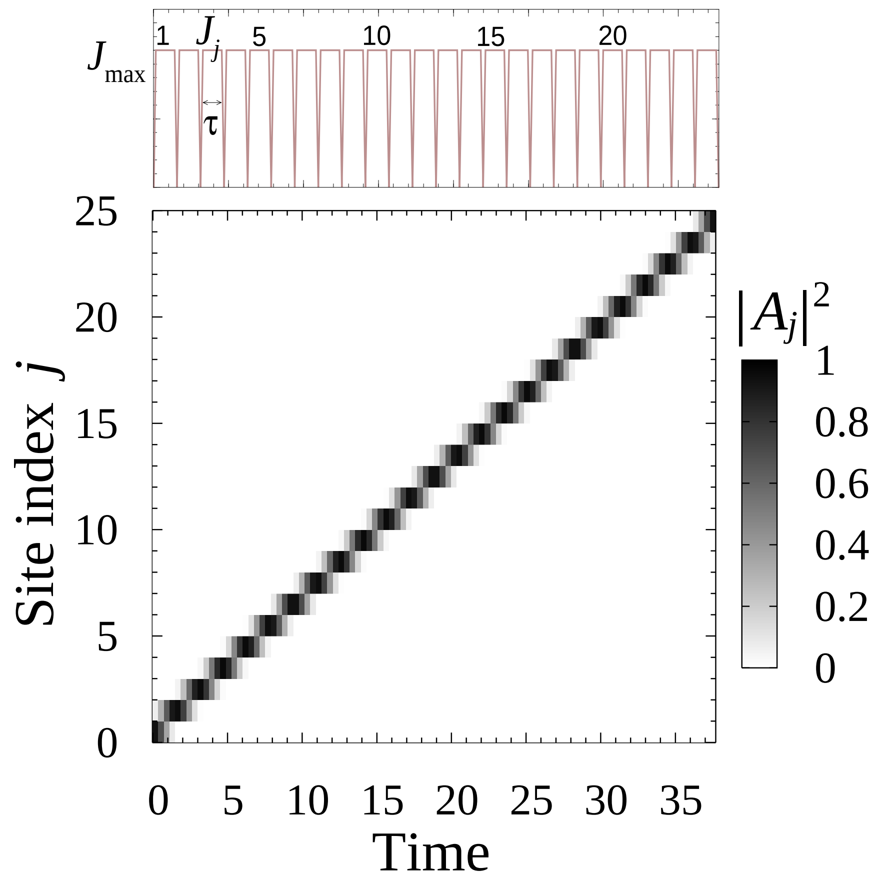
- $A_N(t_{\mathrm{out}}) = (-i)^{N-1}$

@ $t_{\mathrm{out}} = \frac{(N-1)\pi}{2J_{\max}} \simeq \frac{\pi}{2}\frac{N}{J_{\max}} \quad (N \gg 1)$

$\Rightarrow |A_N(t_{\mathrm{out}})| = 1$

$\phi_0 = -\frac{\pi}{2}(N-1) \pmod{2\pi}$

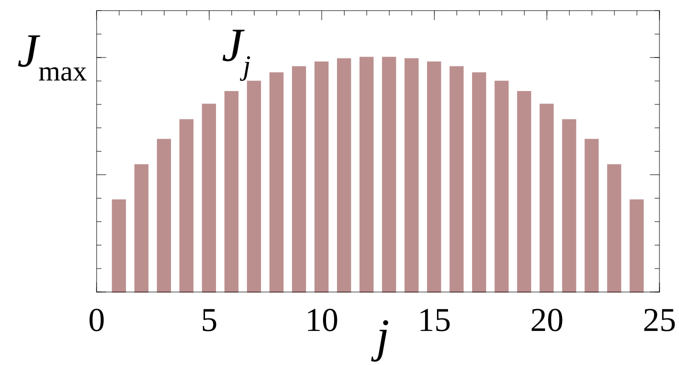


# State Transfer Protocol: Spin-coupling

**Static couplings:** $J_j = J_0\sqrt{(N-j)j} \Rightarrow$

Equidistant spectrum of $H_1$:

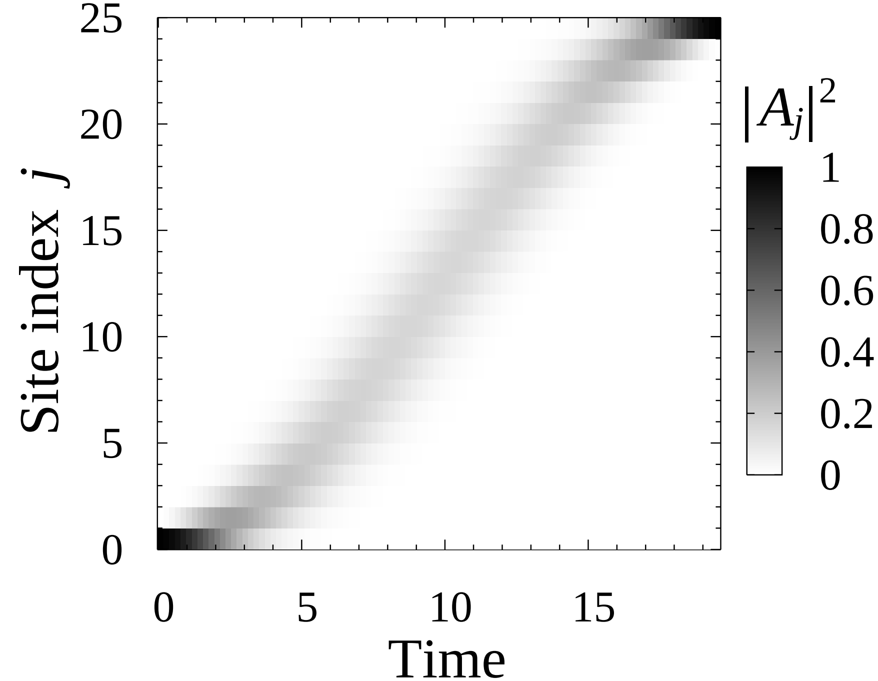
$\lambda_k = 2J_0 k - J_0(N+1)$ ($k = 1, \ldots, N$)




Equivalent to spin-$\mathcal{J}$ in $B_x$:

$\langle \mathcal{J}, m| \hat{J}_x |\mathcal{J}, m+1\rangle$
$= \frac{1}{2}\sqrt{(\mathcal{J}-m)(\mathcal{J}+m+1)}$

[$N = 2\mathcal{J}+1$ & $j = \mathcal{J}+m+1$]



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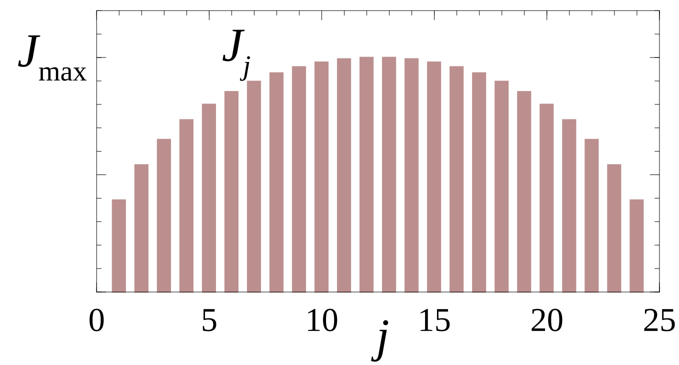


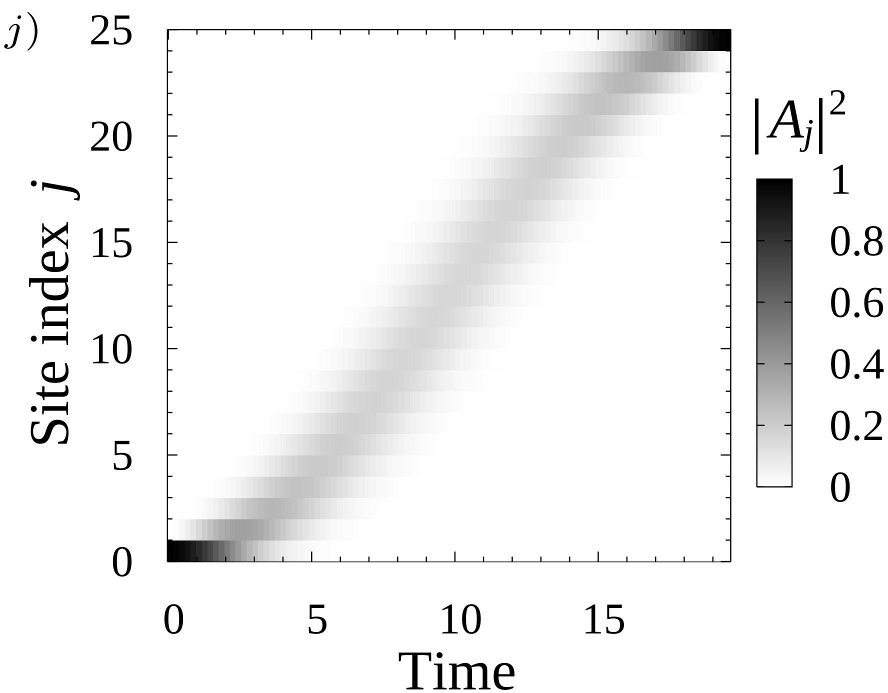

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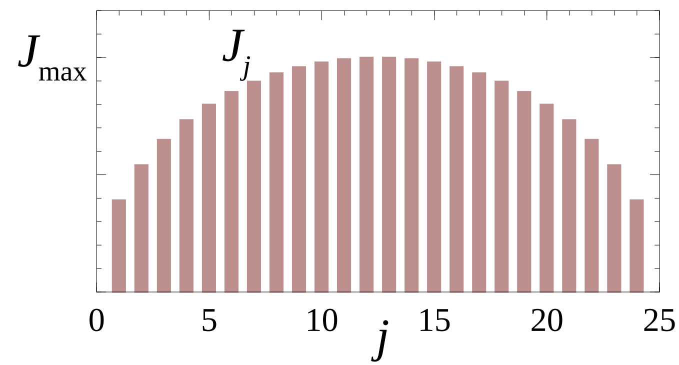


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- $A_N(t_{\rm out}) = (-i)^{N-1}$

@ $t_{\rm out} = \frac{\pi}{2J_0} = \frac{\pi}{4}\frac{N}{J_{\max}}$ **[Optimal!]**

$[J_{\max} = J_{N/2} = \frac{1}{2}J_0 N]$

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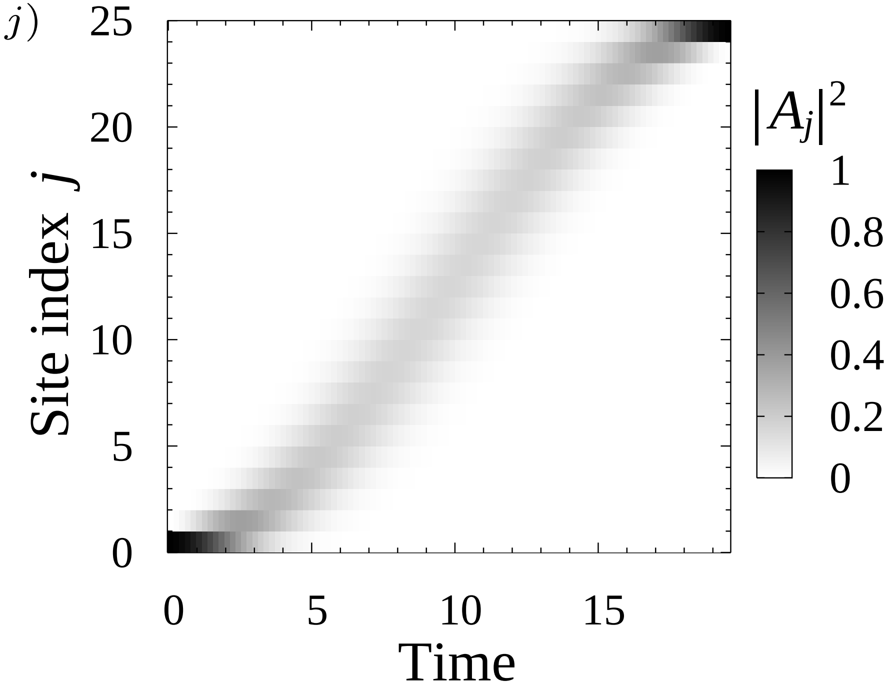


Nikolopoulos, Petrosyan, Lambropoulos, EPL **65**, 297 (2004); JPCM **16**, 4991 (2004)
Christandl *et al.*, PRL **92**, 187902 (2004); Yung, PRA **74**, 030303(R) (2006)

# State Transfer Protocol: Adiabatic transfer

Zero energy $\lambda^{(0)} = 0$ (**dark**) eigenstate of $H_1$ ($N$ - odd):

$$|\Psi^{(0)}\rangle = \frac{1}{\sqrt{\mathcal{N}_0}}[J_2 J_4 \ldots J_{N-1}\,|\mathbf{1}\rangle + (-1)J_1 J_4 \ldots J_{N-1}\,|\mathbf{3}\rangle +$$

$$\ldots + (-1)^{\mathcal{J}} J_1 J_3 \ldots J_{N-2}\,|\boldsymbol{N}\rangle] \qquad [\mathcal{J} \equiv \tfrac{1}{2}(N-1)]$$

Bergmann, Theuer, Shore, RMP **70**, 1003 (1998); Greentree *et al.*, PRB **70**, 235317 (2004)
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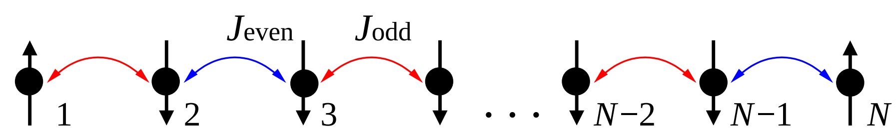
If $J_2, J_4, \ldots, J_{N-1} = J_{\text{even}}(t)$
$J_1, J_3, \ldots, J_{N-2} = J_{\text{odd}}(t)$




$$\Rightarrow \quad A_1(t) = \frac{[J_{\text{even}}(t)]^{\mathcal{J}}}{\sqrt{\mathcal{N}_0(t)}} \qquad A_N(t) = (-1)^{\mathcal{J}} \frac{[J_{\text{odd}}(t)]^{\mathcal{J}}}{\sqrt{\mathcal{N}_0(t)}}$$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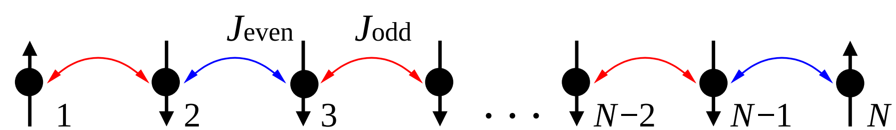


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- $J_{\rm even}(t_{\rm in}) \gg J_{\rm odd}(t_{\rm in}) \quad \Rightarrow \quad A_1(t_{\rm in}) = 1 \quad \& \quad A_N(t_{\rm in}) = 0$
- $J_{\rm odd}(t_{\rm out}) \gg J_{\rm even}(t_{\rm out}) \quad \Rightarrow \quad A_1(t_{\rm out}) = 0 \quad \& \quad A_N(t_{\rm out}) = (-1)^{\mathcal{J}}$

@ $t_{\rm out} \gg \frac{N}{2\pi J_{\rm max}}$ **[Adiabatic transfer!]**

$$\Rightarrow |A_N(t_{\rm out})| = 1 \quad \& \quad \phi_0 = -\tfrac{\pi}{2}(N-1) \pmod{2\pi}$$


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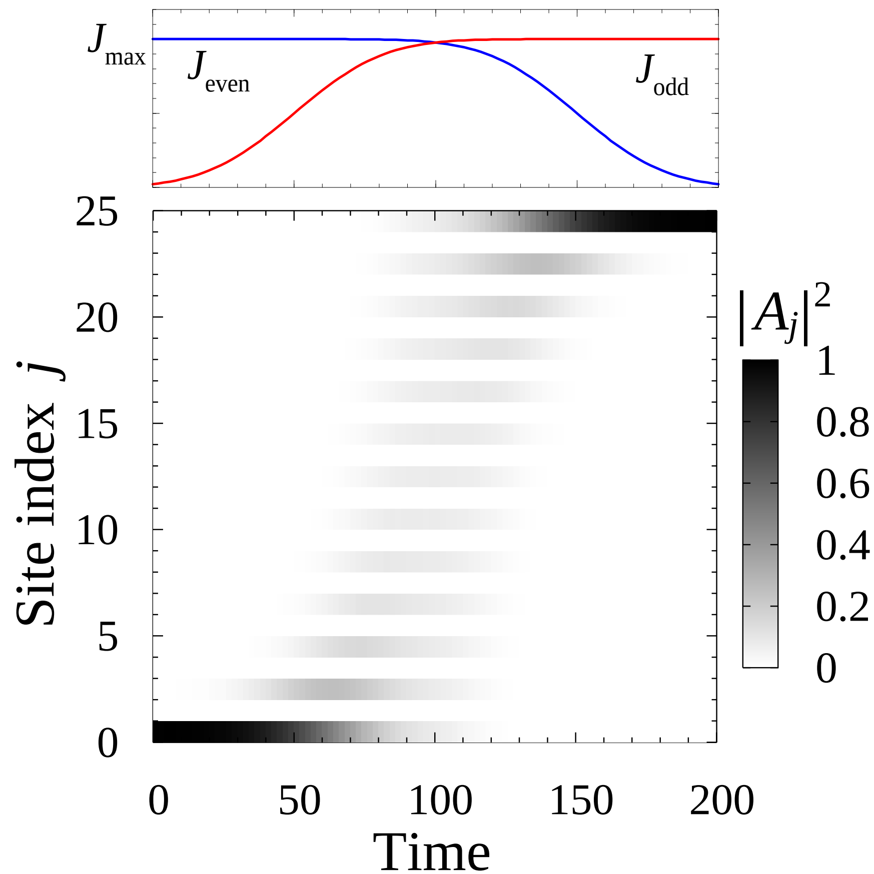
# State Transfer Protocol: Adiabatic transfer

- **Smooth coupling functions**

$$J_{\text{even}}(t) = J_{\max}\frac{1}{2}\left[1 - \text{erf}\left(\frac{t - \frac{1}{2}t_{\text{out}} \pm 2\sigma_t}{\sqrt{2}\sigma_t}\right)\right]$$

$$J_{\text{odd}}(t) = J_{\max}\frac{1}{2}\left[1 + \text{erf}\left(\frac{t - \frac{1}{2}t_{\text{out}} \pm 2\sigma_t}{\sqrt{2}\sigma_t}\right)\right]$$

$\sigma_t = \frac{1}{8}t_{\text{out}} \qquad t_{\text{out}} = 8\frac{N}{J_{\max}}$ [Slow]

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# Noisy Spin Chains

## Physical origins of disorder

- Fabrication imperfections and inhomogeneities
- Noise of the external controls [slowly varying during $t_{\mathrm{out}} - t_{\mathrm{in}}$]

$\Rightarrow$ *Static disorder during each run of State Transfer Protocol*

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**Diagonal disorder:**

$h_j$ Random Gaussian variable with variance $\sigma_h^2$

$$P(h_j) = \frac{1}{\sqrt{2\pi\sigma_h^2}} \exp\left(-\frac{h_j^2}{2\sigma_h^2}\right) \qquad \langle h_j \rangle = 0$$

**Off- diagonal disorder:** $J_j \to J_j(1+\delta J_j)$

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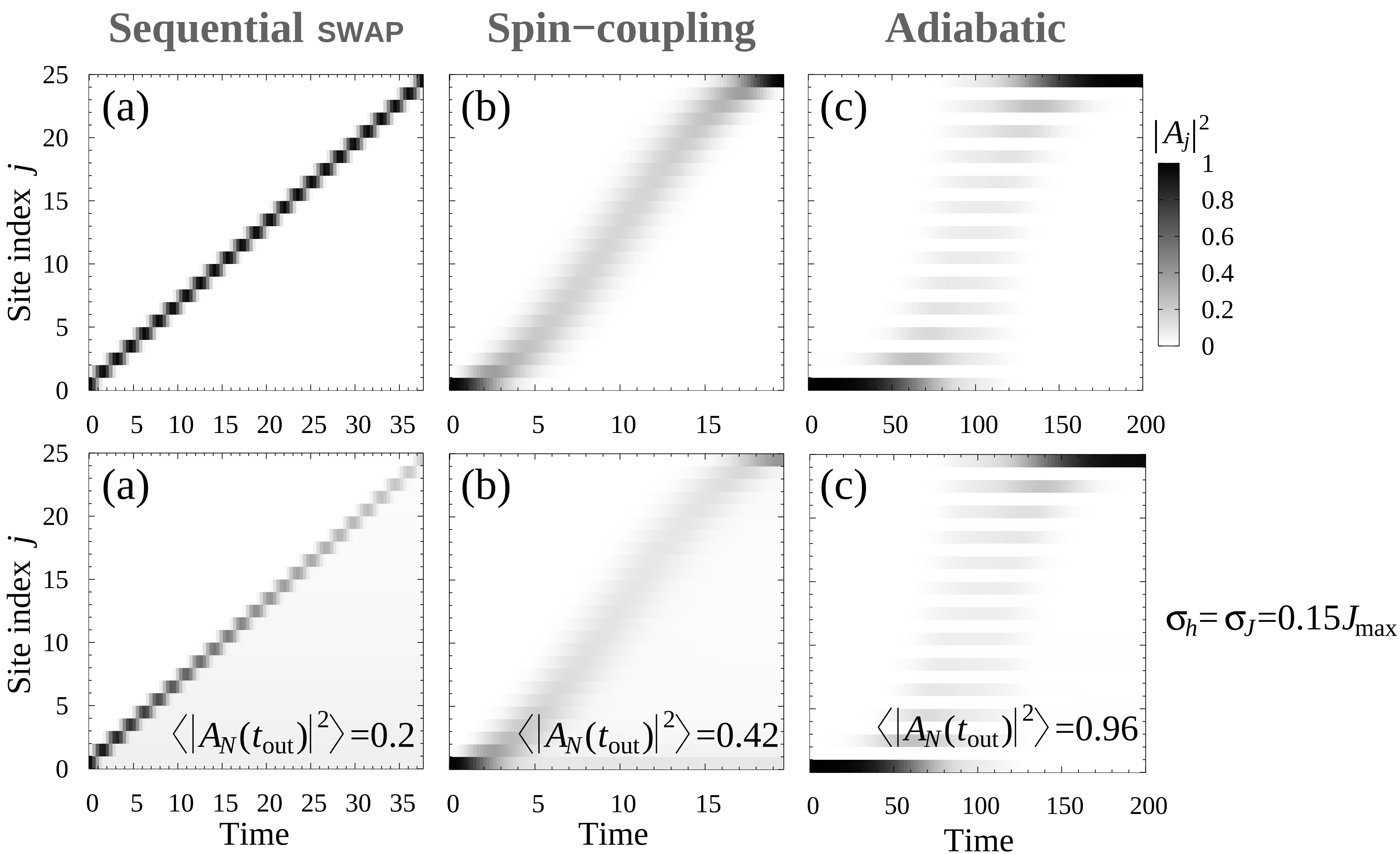
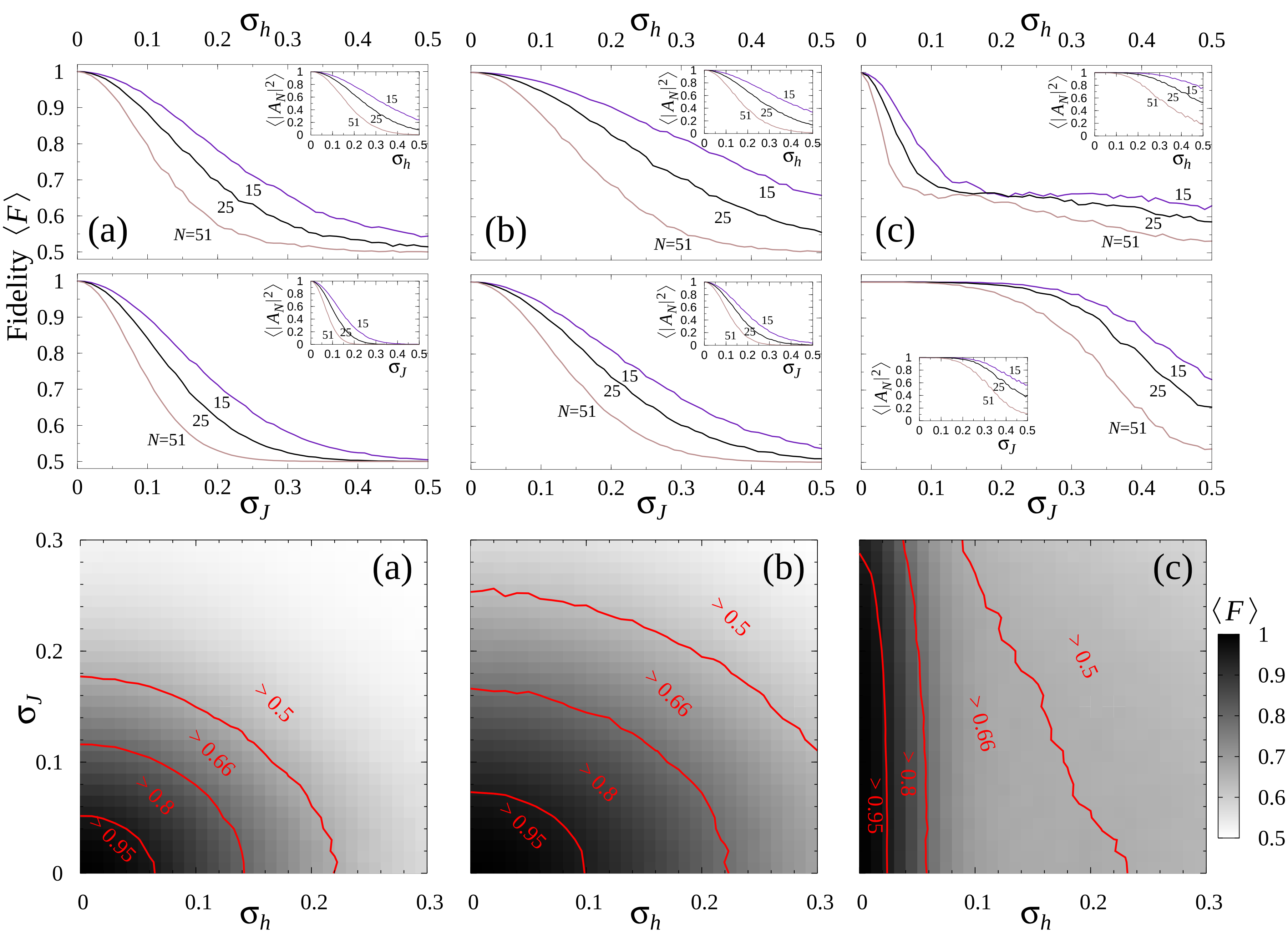
**Average over many (1000) independent realizations**

# Population Transfer Dynamics



# State Transfer Fidelities



# Conclusions

## Advantageous and Disadvantages of State-transfer Protocols

(a) Sequential SWAP protocol

- susceptible to diagonal and off-diagonal disorder $(\sigma_h, \sigma_J)$
- requires precisely pulsed couplings $J_j(t)$

(b) Spin-coupling protocol

- is the fastest [*optimal*]; twice faster than Sequential SWAP
- tolerates well diagonal disorder $(\sigma_h)$

(c) Adiabatic transfer protocol

- very tolerant to off-diagonal disorder $(\sigma_J)$ and pulse uncertainties
- is slow $\Rightarrow$ very susceptible to diagonal disorder $(\sigma_h)$: phase drift $\sigma_\phi \simeq \sigma_h t_{\mathrm{out}}$

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- Arrays of coupled quantum dots
- Arrays of coupled superconducting qubits
- Atoms in optical lattices
- ...

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Petrosyan, Nikolopoulos, Lambropoulos, PRA **81**, 042307 (2010)