Electromagnetically Induced Transparency
with Rydberg Atoms

David Petrosyan

Outline

- Background:
  - Electromagnetically induced transparency (EIT)
  - Rydberg atoms: dipole-dipole (DD) & van der Waals (VdW) interactions

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  - Theoretical model
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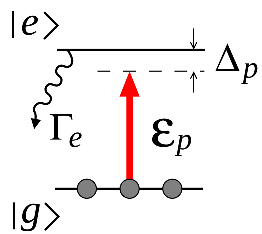
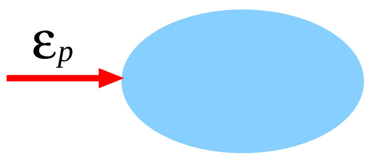
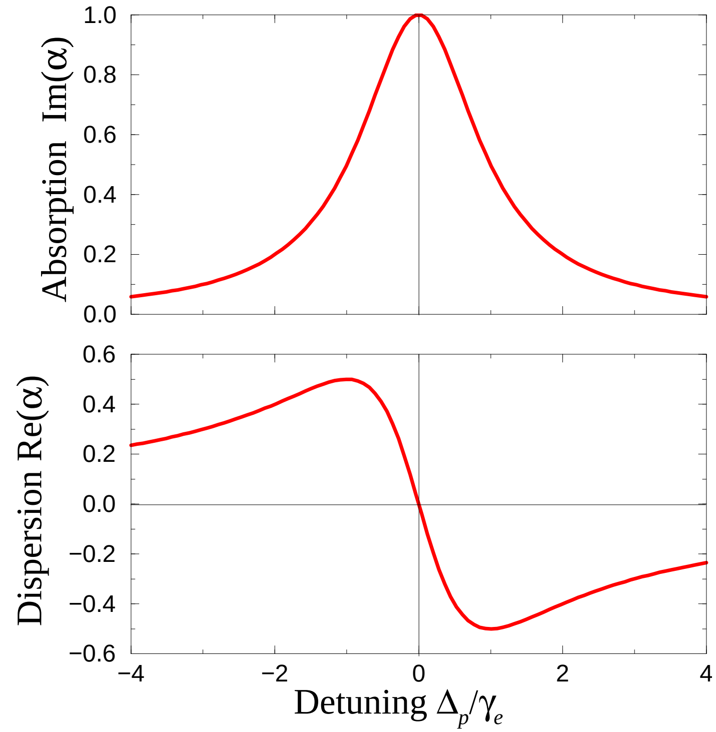
  - Theoretical model
  - Numerical simulations
- Conclusions

Electromagnetically Induced Transparency



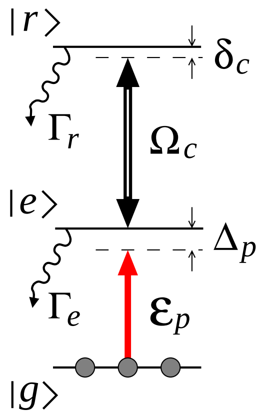


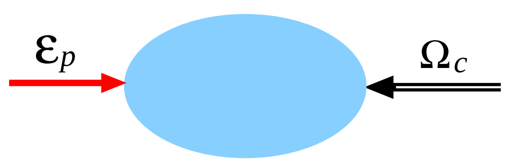


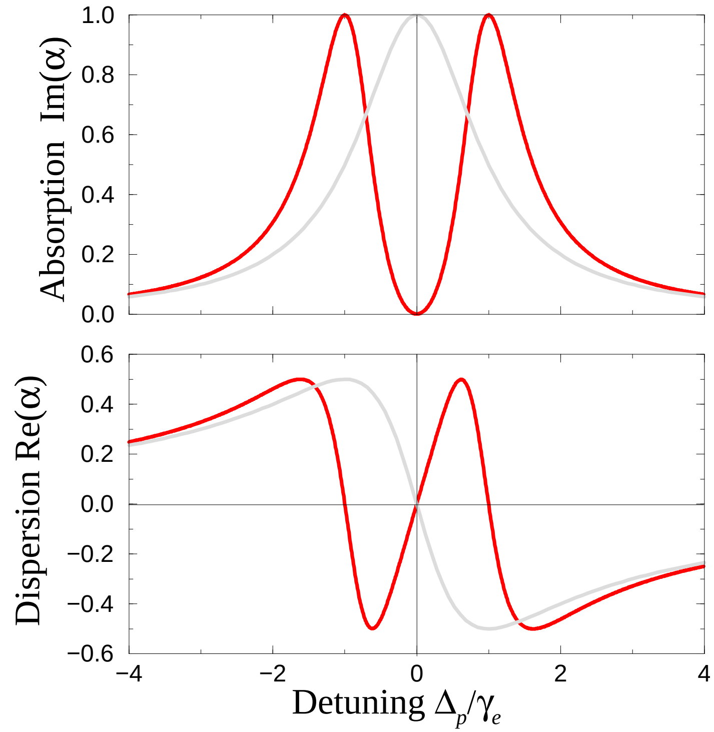
Stationary propagation $$\boxed{\partial_z \mathcal{E}_p = i\frac{\kappa}{2}\alpha\mathcal{E}_p}$$ with $\kappa = \varsigma_0 \rho \quad [\rho \gg \rho_{\text{phot}}]$

2LA Polarizability $$\alpha = \frac{i\gamma_e}{\gamma_e - i\Delta_p} \equiv \alpha_{\text{TLA}}$$

Electromagnetically Induced Transparency







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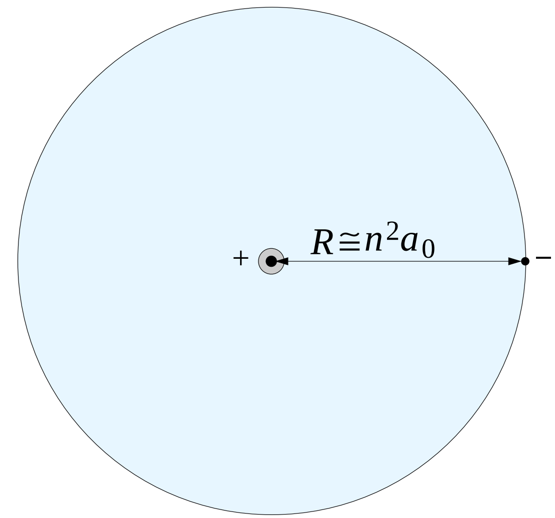
3LA (EIT) Polarizability $$\alpha = \frac{i\gamma_e}{\gamma_e - i\Delta_p + \frac{|\Omega_c|^2}{\gamma_r - i(\Delta_p + \delta_c)}} \equiv \alpha_{\text{EIT}}$$

Fleischhauer, Imamoglu, Marangos, RMP **77**, 633 (2005)

Rydberg Atoms

High principal quantum number

$$\boxed{n \gg 1}$$ (H-like)



Rydberg Atoms

High principal quantum number

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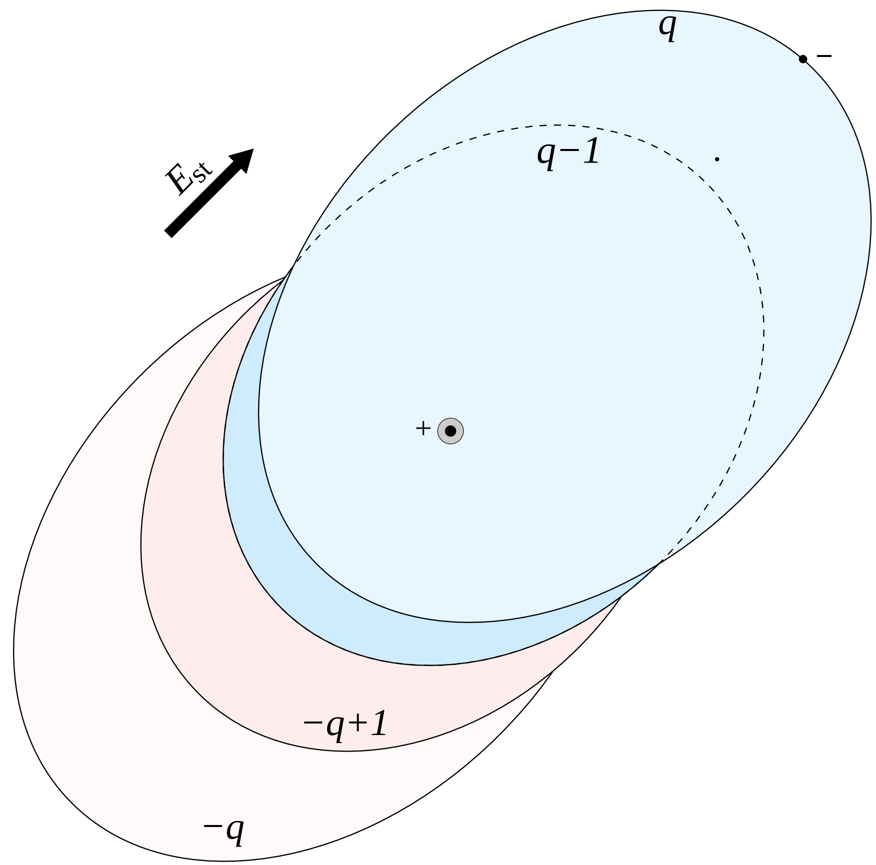
Static electric field E_{st}

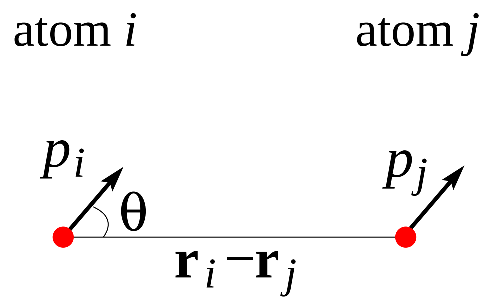
⇒ Stark eigenstates with permanent dipole moments

$$\boxed{\wp_r = \frac{3}{2} n q e a_0} \quad q \in [-n+1, n-1]$$



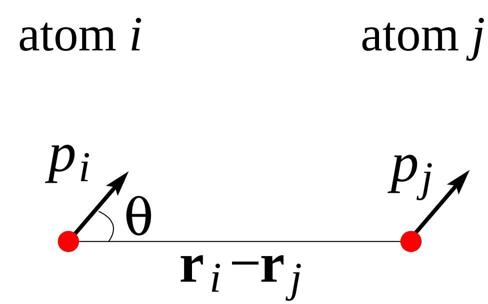
Gallagher, *Rydberg Atoms* (Cambridge 1994)

Static Dipole-Dipole Interaction



Atoms i, j in state $|r\rangle$ possess permanent dipole moments $\mathbf{e}_z \wp_{i,j}$

Static Dipole-Dipole Interaction



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⇒ Static DDI

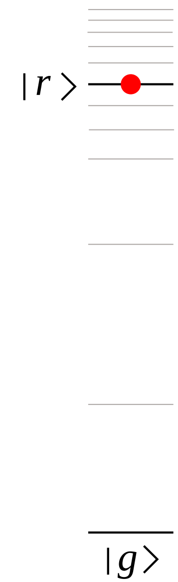
$$\mathcal{V}_{\mathrm{SDD}} = \hbar \sigma_{rr}^{i} D_{ij} \sigma_{rr}^{j}$$

$D_{ij} \equiv D(\mathbf{r}_i - \mathbf{r}_j) \propto \frac{\wp_i \wp_j (1 - 3\cos^2\theta)}{|\mathbf{r}_i - \mathbf{r}_j|^3} \propto n^4$ — SDDI strength

Resonant Dipole-Dipole Interaction

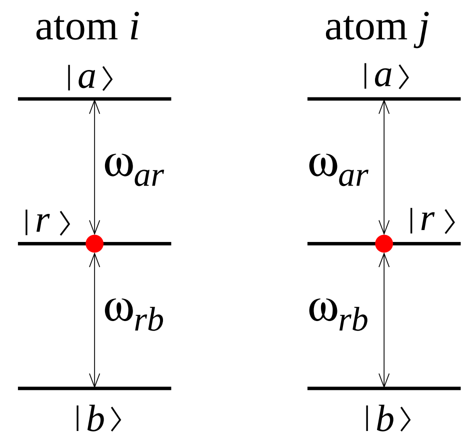
Energy $\boxed{E_r = -\frac{Ry}{n^{*2}}}$

effective PQN $n^* = n - \delta_l$ (δ_l quantum defect)



Resonant Dipole-Dipole Interaction

$$\boxed{\omega_{ar} = \omega_{rb}}$$



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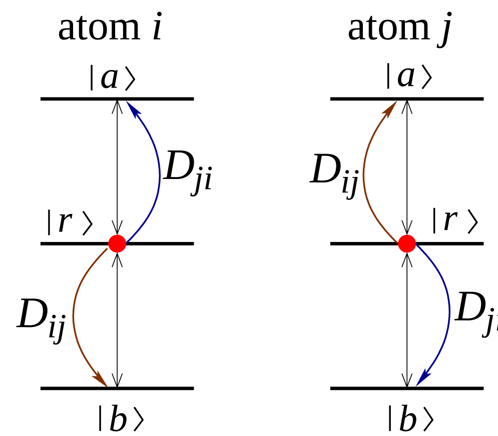


$\Rightarrow$ $|r_i\rangle\,|r_j\rangle \to |a_{i,j}\rangle\,|b_{j,i}\rangle$: Resonant exchange (Förster process)

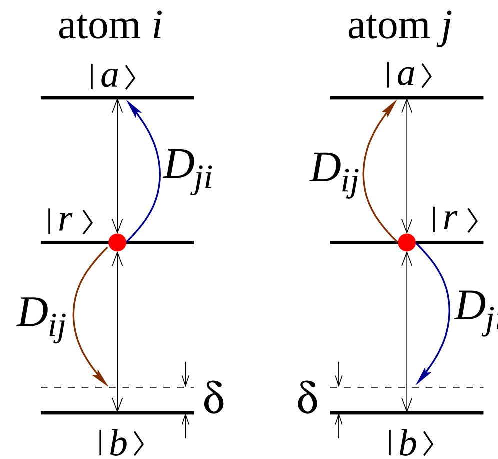
$$\mathcal{V}_{\mathrm{RDD}} = \hbar\big(\sigma^i_{br} D_{ij}\sigma^j_{ar} + \sigma^j_{br} D_{ji}\sigma^i_{ar}\big) + \mathrm{H.c}$$

$D_{ij} \equiv D(\mathbf{r}_i - \mathbf{r}_j) \propto \frac{\wp_{br}\wp_{ar}}{|\mathbf{r}_i - \mathbf{r}_j|^3} \propto n^4$ — RDDI strength

Van der Waals Interaction

$$\boxed{\omega_{rb} - \omega_{ar} = \delta \gg D}$$

$(\delta \propto n^{-3})$



Van der Waals Interaction

$$\boxed{\omega_{rb} - \omega_{ar} = \delta \gg D}$$

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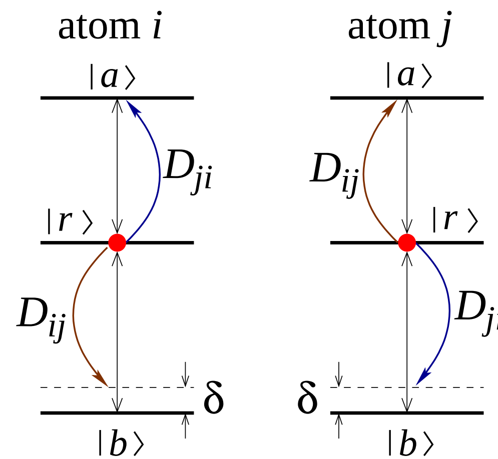

$\Rightarrow$ $|r_i\rangle\,|r_j\rangle \nrightarrow |a_{i,j}\rangle\,|b_{j,i}\rangle$: Non-Resonant DDI (Adiabatic elim. $|a_{i,j}\rangle\,|b_{j,i}\rangle$)

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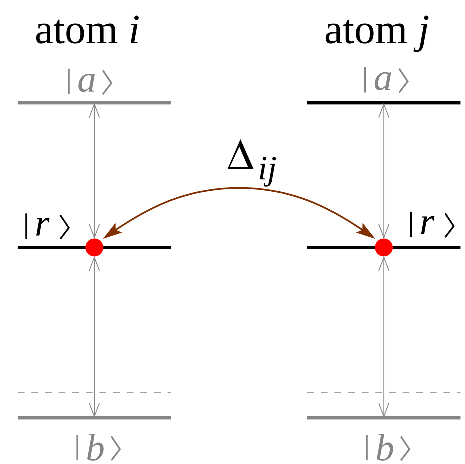


$\Rightarrow$ $|r_i\rangle\,|r_j\rangle \nrightarrow |a_{i,j}\rangle\,|b_{j,i}\rangle$: Non-Resonant DDI (Adiabatic elim. $|a_{i,j}\rangle\,|b_{j,i}\rangle$)

$\Rightarrow$ Energy shift of $|r_i\rangle\,|r_j\rangle$ (2nd-order in D/δ)

$$\mathcal{V}_{\mathrm{VdW}} = \hbar \sigma_{rr}^{i} \Delta_{ij} \sigma_{rr}^{j}$$

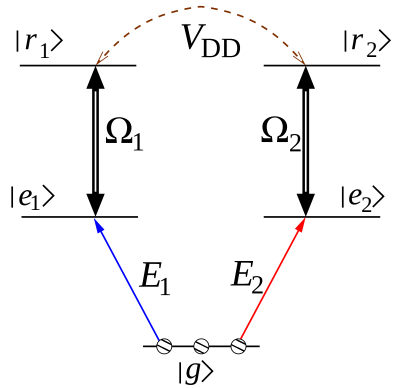
$\Delta_{ij} \equiv \Delta(\mathbf{r}_i - \mathbf{r}_j) = 2\frac{|D(\mathbf{r}_i - \mathbf{r}_j)|^2}{\delta} = \frac{C_6}{|\mathbf{r}_i - \mathbf{r}_j|^6} \propto n^{11}$ — VdWI strength

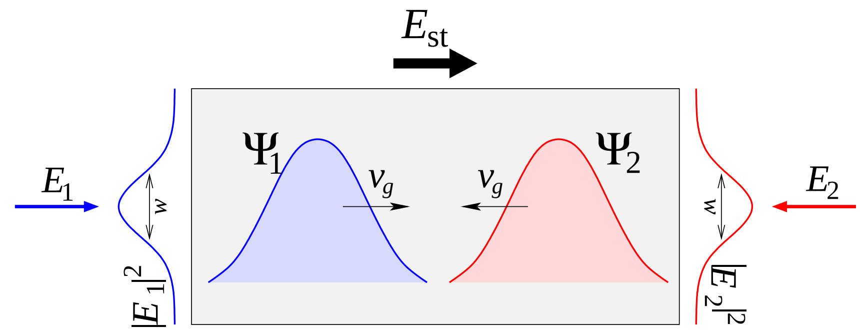
Saffman, Walker, Mølmer, RMP **82**, 2313 (2010)

Photonic phase gate with SDDI

Friedler, Petrosyan, Fleischhauer, Kurizki, PRA **72**, 043803 (2005)
Shahmoon, Kurizki, Fleischhauer, Petrosyan, PRA **83**, 033806 (2011)

Photon-Photon Interaction





Static $E_{\rm st}\mathbf{e}_z \Rightarrow$ Stark eigenstates $|r_i\rangle$ with SDMs $\wp_r \mathbf{e}_z = \frac{3}{2} n q e a_0 \mathbf{e}_z$

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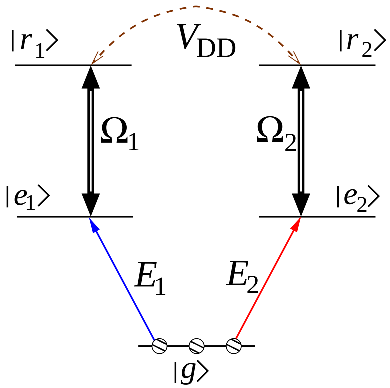

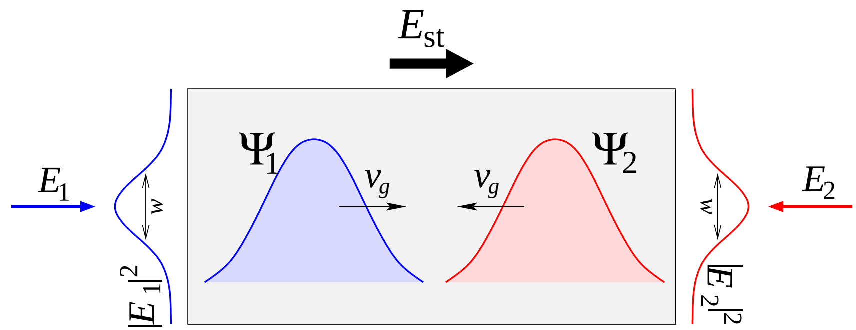

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$E_i \to \hat{\Psi}_i = \cos\theta \hat{\mathcal{E}}_i - \sin\theta \sqrt{N} \hat{\sigma}_{g r_i}$ $(i = 1, 2)$ propagate with $\pm v_g = c \cos^2\theta$

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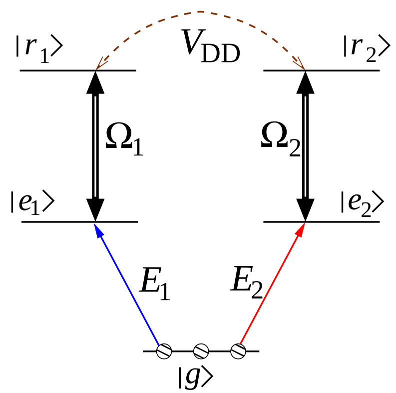

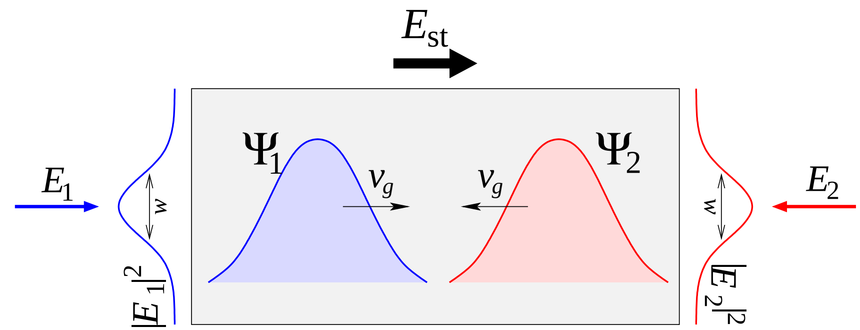

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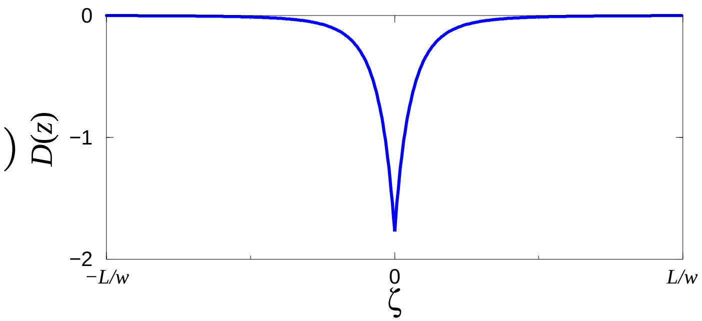
Atomic components of $\hat{\Psi}_i$ interact via $\mathcal{V}_{\mathrm{SDD}} \Rightarrow$ induces XPM

Resonant DDI (state mixing) is suppressed for $q = n - 1$, $m = 0$

Cross-Phase Modulation

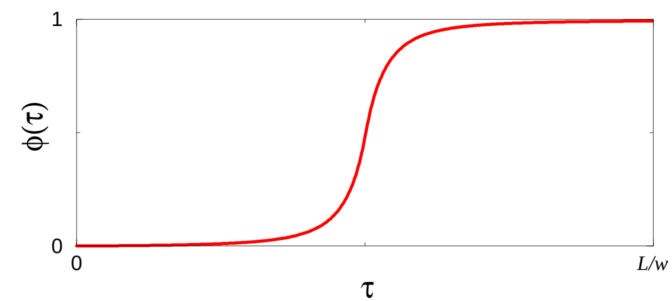
- DD level shift [vs. $\zeta = (z - z')/w$]

$$D(z - z') = \frac{1}{\pi w^2} \int_0^{2\pi} \mathrm{d}\varphi' \int_0^{\infty} \mathrm{d}r'_\perp \, r'_\perp \, e^{-r'^2_\perp / w^2} D(z\mathbf{e}_z - \mathbf{r}')$$



- Phase shift [vs. $\tau = v_g t/w$]

$$\phi(z_1, z_2, t) = -\sin^4\theta \int_0^t \mathrm{d}t' D(z_1 - z_2 - 2v_g(t - t'))$$



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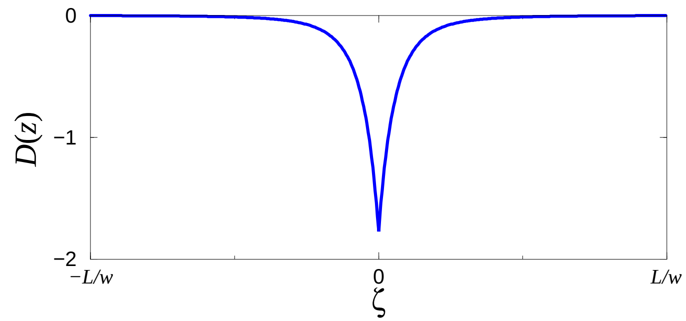

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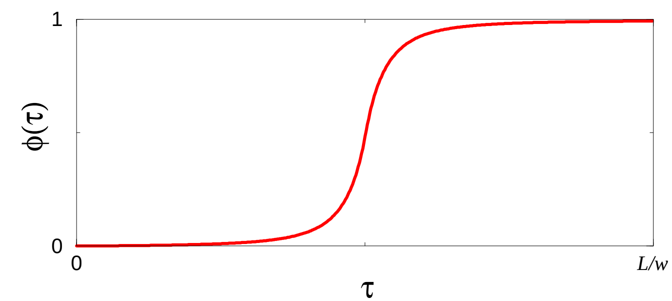

$$\phi(z_1, z_2, t) = -\sin^4\theta \int_0^t \mathrm{d}t' D(z_1 - z_2 - 2v_g(t - t'))$$

Initially $t = 0$, $z_1 = 0$ & $z_2 = L \Rightarrow \phi(0, L, 0) = 0$

After the interaction $t = L/v_g$, $z_1 = L$ & $z_2 = 0$

$$\phi(L, 0, L/v) = -\frac{\sin^4\theta}{v_g} \int_0^L dz' D(2z' - L) = \frac{2C}{v_g w^2}$$

Cross-Phase Modulation

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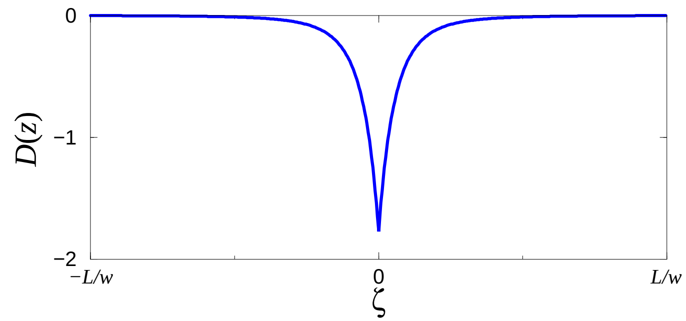

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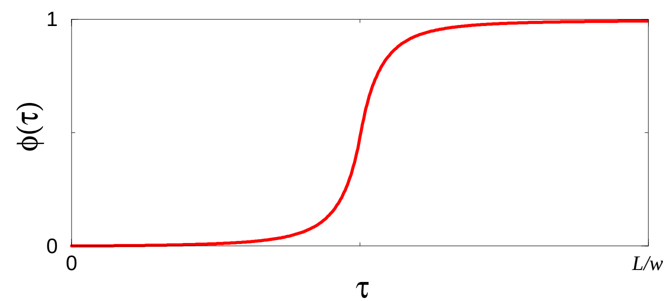

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- Phase shift $\phi = \pi$ [spatially uniform!]

$\Rightarrow$ Universal CPHASE gate between SPh pulses $\hat{\mathcal{E}}_1$ & $\hat{\mathcal{E}}_2$

$$|x\rangle_1 |y\rangle_2 \to (-1)^{xy} |x\rangle_1 |y\rangle_2 \qquad (x, y \in [0, 1])$$

EIT with strong VdWI

Petrosyan, Otterbach, Fleischhauer, arXiv:1106.1360 [quant-ph]

Experiment

PRL **105**, 193603 (2010) PHYSICAL REVIEW LETTERS week ending 5 NOVEMBER 2010

Cooperative Atom-Light Interaction in a Blockaded Rydberg Ensemble

J. D. Pritchard,* D. Maxwell, A. Gauguet, K. J. Weatherill, M. P. A. Jones, and C. S. Adams†
Department of Physics, Durham University, Rochester Building, South Road, Durham DH1 3LE, United Kingdom
(Received 21 June 2010; published 5 November 2010)

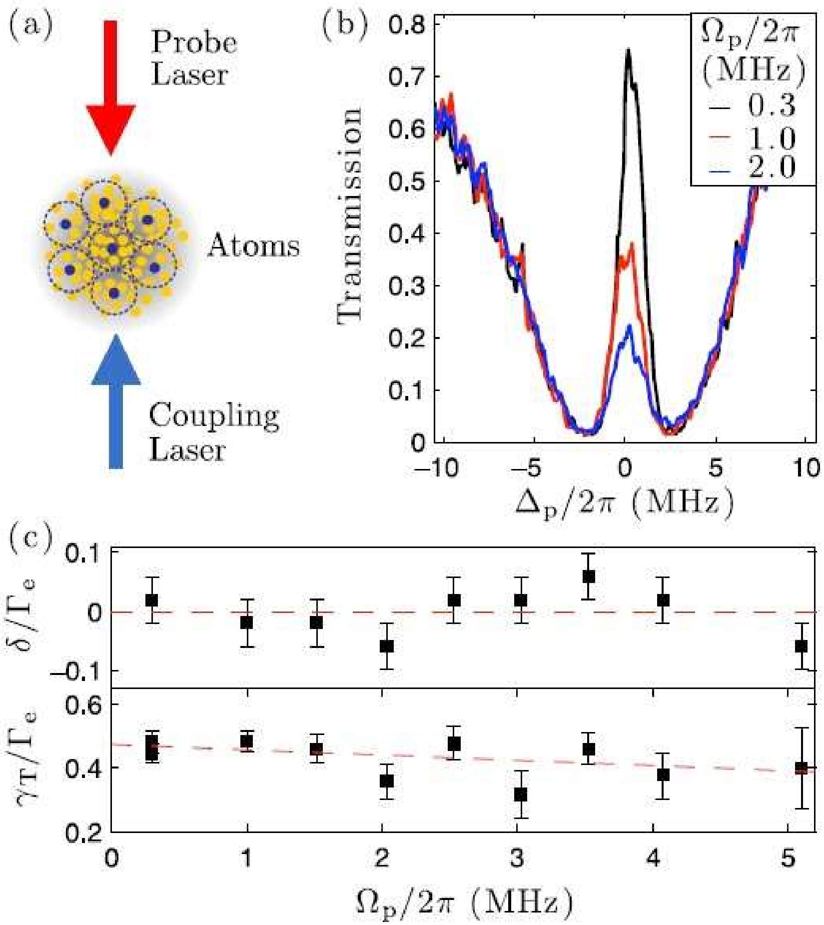


FIG. 1 (color online). (a) Schematic of experiment. EIT spectroscopy is performed on an ultracold ^{87}Rb atom cloud. (b) Suppression of transparency on resonance for coupling to $60S_{1/2}$ for increasing probe Rabi frequency Ω_p at a density $\rho = 1.2 \pm 0.1 \times 10^{10}$ cm^{-3}. (c) Detuning, δ, and width, γ_T, of EIT resonance as a function of Ω_p in units of the excited state width $\Gamma_e = 2\pi \times 6$ MHz, showing no dephasing and no resonance shift.

Theoretical Model

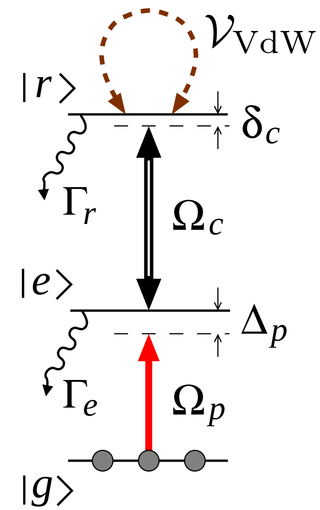
Hamiltonian $\mathcal{H} = \mathcal{H}_{\rm a} + \mathcal{V}_{\rm af} + \mathcal{V}_{\rm VdW}$

$$\mathcal{H}_{\rm a} = -\hbar \sum_j^N [\Delta_p \sigma_{ee}(\mathbf{r}_j) + (\Delta_p + \delta_c)\sigma_{rr}(\mathbf{r}_j)]$$

$$\mathcal{V}_{\rm af} = -\hbar \sum_j^N [\Omega_p(\mathbf{r}_j)\sigma_{eg}(\mathbf{r}_j) + \Omega_c \sigma_{re}(\mathbf{r}_j) + \text{H.c.}]$$

$$\mathcal{V}_{\rm VdW} = \hbar \sum_{i<j}^N \sigma_{rr}(\mathbf{r}_i)\Delta(\mathbf{r}_i - \mathbf{r}_j)\sigma_{rr}(\mathbf{r}_j)$$

$$\Downarrow$$



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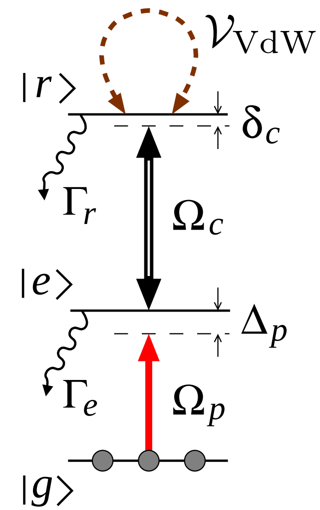

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$$\Downarrow$$

Stationary probe-field propagation $[\Omega_p \equiv \eta \mathcal{E}_p]$

$$\partial_z \langle \mathcal{E}_p^\dagger(\mathbf{r})\mathcal{E}_p(\mathbf{r})\rangle = -\kappa(\mathbf{r})\langle \mathcal{E}_p^\dagger(\mathbf{r})\text{Im}[\alpha(\mathbf{r})]\mathcal{E}_p(\mathbf{r})\rangle$$

Polarizability $\alpha(\mathbf{r}) = \dfrac{i\gamma_e}{\gamma_e - i\Delta_p + \frac{|\Omega_c|^2}{\gamma_r - i[\Delta_p + \delta_c - S(\mathbf{r})]}}$

with $\boxed{S(\mathbf{r}) \equiv \sum_j^N \Delta(\mathbf{r} - \mathbf{r}_j)\sigma_{rr}(\mathbf{r}_j)}$ **total VdW shift of $|r\rangle$ at position r**

Rydberg Excitation Blockade (3LA)

Population of $|r\rangle$: $\langle\sigma_{rr}(\Delta_2)\rangle \approx \frac{\langle\Omega_p^\dagger\Omega_p\rangle}{|\Omega_c|^2+\Delta_2^2\frac{\gamma_e^2}{|\Omega_c|^2}}$ $[\Delta_2 = \Delta_p + \delta_c]$

$\Rightarrow \langle\sigma_{rr}(0)\rangle = \frac{\langle\Omega_p^\dagger\Omega_p\rangle}{|\Omega_c|^2}$ & $\langle\sigma_{rr}(w)\rangle = \frac{1}{2}\langle\sigma_{rr}(0)\rangle$ $\boxed{w \equiv \frac{|\Omega_c|^2}{\gamma_e}}$

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An atom in $|r\rangle$ blocks Rydberg excitations for $\Delta(R) \gtrsim w$ $[\Delta_2 \to \Delta_2 - \Delta(R)]$

$\Rightarrow$ **Blockade radius** $R_{\rm sa} \simeq \sqrt[6]{\frac{|C_6|}{w}}$ & **(superatom) volume** $V_{\rm sa} = \frac{4\pi}{3}R_{\rm sa}^3$

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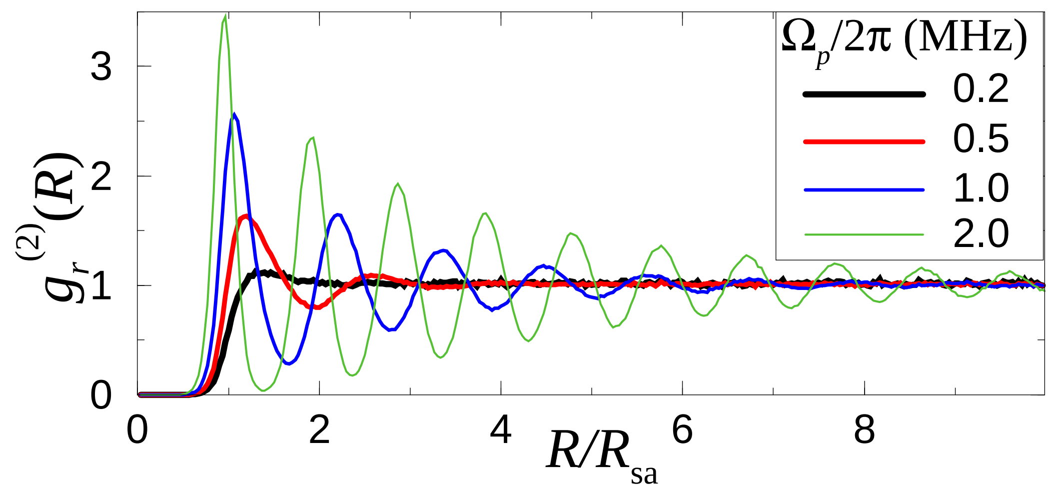
$$g_r^{(2)}(R) \equiv \frac{\langle\sigma_{rr}(0)\sigma_{rr}(R)\rangle}{\langle\sigma_{rr}(0)\rangle\langle\sigma_{rr}(R)\rangle}$$



$$\sigma_{rr}(R) = \frac{|\Omega_c|^2\Omega_p^\dagger\Omega_p}{|\Omega_c|^2\Omega_p^\dagger\Omega_p+[|\Omega_c|^2-\Delta_p\Delta_2(R)]^2+\Delta_2^2(R)\gamma_e^2}$$

$[\Delta_2(R) \equiv \Delta_2 - S(R)]$

Superatom

$$\boxed{n_{\rm sa} = \rho V_{\rm sa}}$$

$$|G\rangle = |g_1, g_2, \ldots, g_{n_{\rm sa}}\rangle$$

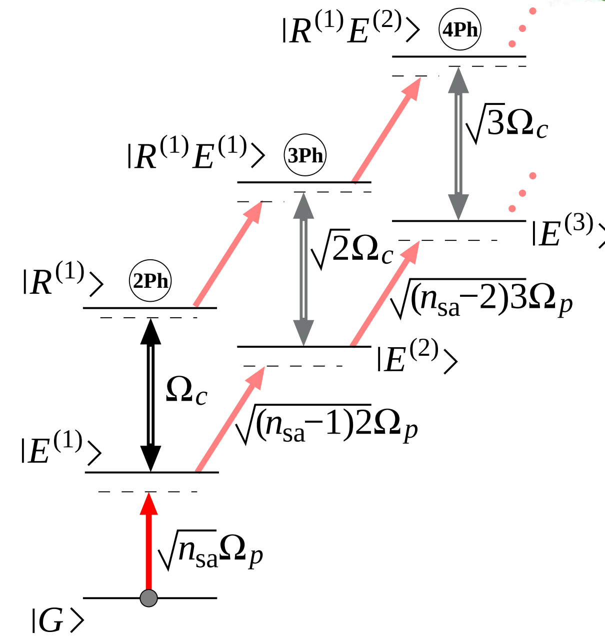
$$|E^{(1)}\rangle = \frac{1}{\sqrt{n_{\rm sa}}} \sum_j^{n_{\rm sa}} |g_1, g_2, \ldots, e_j, \ldots, g_{n_{\rm sa}}\rangle$$

$$|R^{(1)}\rangle = \frac{1}{\sqrt{n_{\rm sa}}} \sum_j^{n_{\rm sa}} |g_1, g_2, \ldots, r_j, \ldots, g_{n_{\rm sa}}\rangle$$

$$|E^{(2)}\rangle = \frac{1}{\sqrt{n_{\rm sa}(n_{\rm sa}-1)}} \sum_{i<j}^{n_{\rm sa}} |g_1, \ldots, e_i, \ldots, e_j, \ldots, g_{n_{\rm sa}}\rangle$$

$$|R^{(1)}E^{(1)}\rangle = \frac{1}{\sqrt{n_{\rm sa}(n_{\rm sa}-1)2}} \sum_{i,j}^{n_{\rm sa}} |g_1, \ldots, r_i, \ldots, e_j, \ldots, g_{n_{\rm sa}}\rangle$$

etc.



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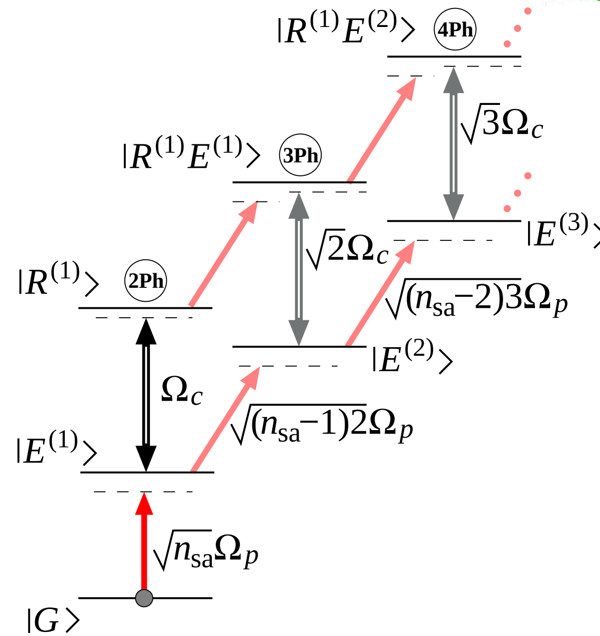


- Adiabatic elimination of $|E^{(k)}\rangle$ $\quad [\gamma_e = \frac{1}{2}\Gamma_e \gg \Omega_{p,c}]$
- while $\Sigma_{GG} + \Sigma_{RR} = \mathbb{1}$ $\quad$ [saturation]

$\Rightarrow$

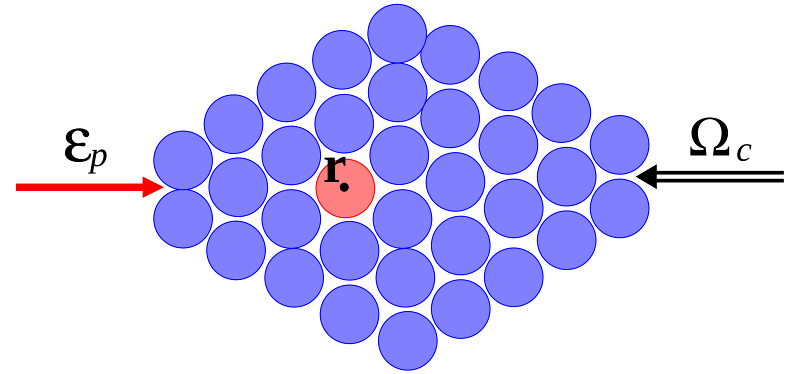
$$\Sigma_{RR} = \frac{|\Omega_c|^2 n_{\rm sa} \Omega_p^\dagger \Omega_p}{|\Omega_c|^2 n_{\rm sa} \Omega_p^\dagger \Omega_p + [|\Omega_c|^2 - \Delta_p \Delta_2]^2 + \Delta_2^2 \gamma_e^2}$$

Total VdW shift at position r

$N_{\rm sa} = V/V_{\rm sa}$ superatoms

$$S(\mathbf{r}) \approx \sum_j^{N_{\rm sa}} \Delta(\mathbf{r}-\mathbf{r}_j)\Sigma_{RR}(\bar{\mathbf{r}}_j)$$

$$= \boxed{\Delta(0)\Sigma_{RR}(\bar{\mathbf{r}})} + \boxed{s(\mathbf{r})}$$



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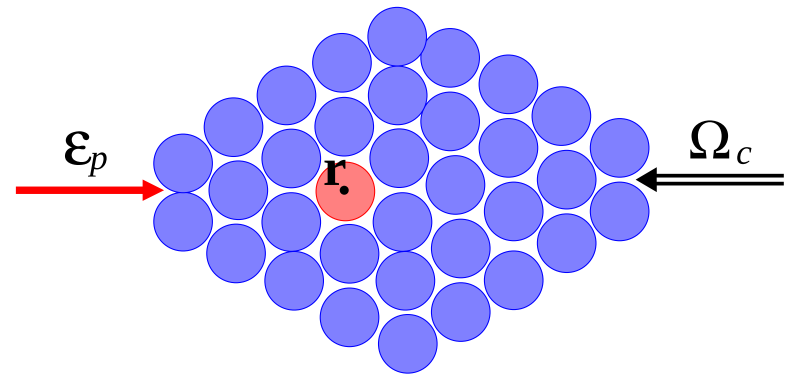

MF shift $$s(\mathbf{r}) \equiv \rho_{\rm sa}\int_{V\backslash V_{\rm sa}^{(\mathbf{r})}} \Delta(\mathbf{r}-\mathbf{r}')\Sigma_{RR}(\bar{\mathbf{r}}')d^3r' \simeq \frac{w}{8}\Sigma_{RR}(\tilde{\mathbf{r}})$$

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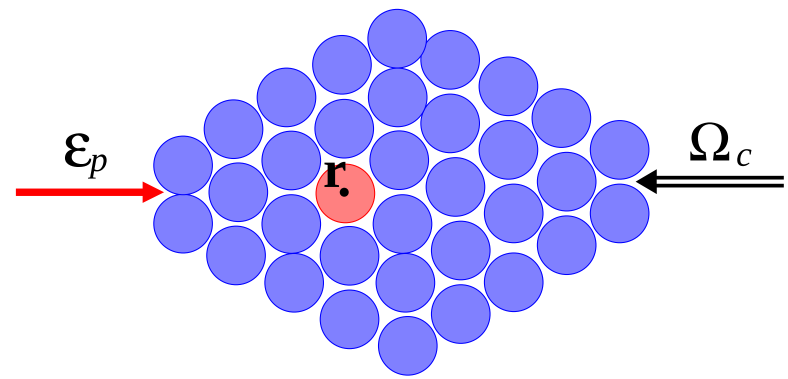

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Blockade $\quad \Delta(0) = \frac{1}{V_{\rm sa}}\int_{V_{\rm sa}} \Delta(\mathbf{r}')d^3r' = \frac{3C_6}{R_{\rm sa}^3}\int_0^{R_{\rm sa}} \frac{dr'}{r'^4} \to \infty \quad (\gg \gamma_e)$

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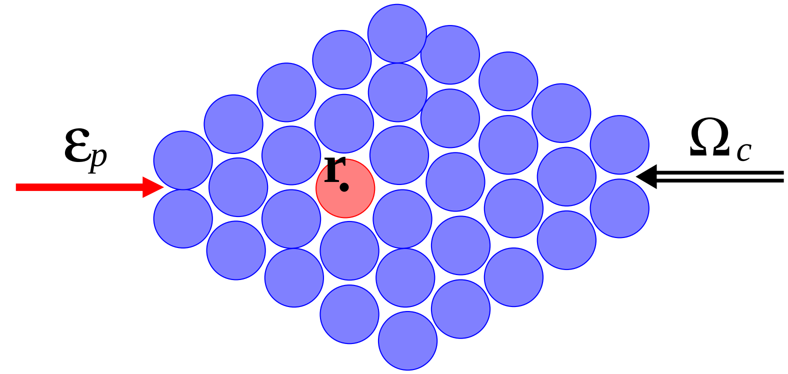

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$\Sigma_{RR}(\bar{\mathbf{r}})$ — **Projector**

$\langle\Sigma_{RR}(\bar{\mathbf{r}})\rangle \in [0,1]$ — **Blockade probability**: $\Delta(0) \gg \gamma_e$

Probe field intensity evolution

$$\partial_z \langle \Omega_p^\dagger(\mathbf{r}) \Omega_p(\mathbf{r}) \rangle = -\kappa(\mathbf{r}) \langle \Omega_p^\dagger(\mathbf{r}) \mathrm{Im}[\alpha(\mathbf{r})] \Omega_p(\mathbf{r}) \rangle \Rightarrow -\kappa(\mathbf{r}) \mathrm{Im}[\langle \alpha(\mathbf{r}) \rangle_{\mathbf{r}}] \langle \Omega_p^\dagger(\mathbf{r}) \Omega_p(\mathbf{r}) \rangle$$

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$$\langle \alpha(\mathbf{r})\rangle_{\mathbf{r}} = \langle \Sigma_{RR}(\bar{\mathbf{r}})\rangle_{\mathbf{r}} \underbrace{\frac{i\gamma_e}{\gamma_e - i\Delta_p}}_{\alpha_{\mathrm{TLA}}} + [1 - \langle \Sigma_{RR}(\bar{\mathbf{r}})\rangle_{\mathbf{r}}] \underbrace{\frac{i\gamma_e}{\gamma_e - -i\Delta_p + \frac{|\Omega_c|^2}{\gamma_r - i[\Delta_p + \delta_c - \langle s(\mathbf{r})\rangle]}}}_{\alpha_{\mathrm{EIT}}}$$

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Intensity correlation within $V_{\mathrm{sa}}^{(\mathbf{r})}$:

$$g_p^{(2)}(\mathbf{r}, \bar{\mathbf{r}}) \equiv \frac{\langle \mathcal{E}_p^\dagger(\mathbf{r})\mathcal{E}_p^\dagger(\bar{\mathbf{r}})\mathcal{E}_p(\bar{\mathbf{r}})\mathcal{E}_p(\mathbf{r})\rangle}{\langle \mathcal{E}_p^\dagger(\mathbf{r})\mathcal{E}_p(\mathbf{r})\rangle\langle \mathcal{E}_p^\dagger(\bar{\mathbf{r}})\mathcal{E}_p(\bar{\mathbf{r}})\rangle} \equiv g_p^{(2)}(\mathbf{r})$$

$$\partial_z g_p^{(2)}(\mathbf{r}) = -\kappa(\mathbf{r})\mathrm{Im}[\langle \alpha(\mathbf{r})\rangle - \alpha_{\mathrm{EIT}}] g_p^{(2)}(\mathbf{r})$$

Numerical Simulations: comput. procedure

FORTH IESL

Stochastic (Monte-Carlo)

- Divide the propagation distance L into $\frac{L}{2R_{\rm sa}}$ intervals (superatoms)
- For each $z \in \mathsf{SA}_j$ generate uniform random number $p_z \in [0,1]$
  - if $p_z \leq \langle \Sigma_{RR}(\bar{\mathbf{r}}) \rangle_{\mathbf{r}} \quad \Rightarrow \quad \langle \alpha(\mathbf{r}) \rangle_{\mathbf{r}} = \alpha_{\rm TLA}$
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Continuous limit

- Infinitely many realizations $\Rightarrow \partial_z I_p(\mathbf{r}) = -\kappa(\mathbf{r}) \text{Im}[\langle \alpha(\mathbf{r}) \rangle_{\mathbf{r}}] I_p(\mathbf{r})$

with $$\langle \alpha(\mathbf{r}) \rangle_{\mathbf{r}} = \langle \Sigma_{RR}(\bar{\mathbf{r}}) \rangle_{\mathbf{r}} \alpha_{\rm TLA} + [1 - \langle \Sigma_{RR}(\bar{\mathbf{r}}) \rangle_{\mathbf{r}}] \alpha_{\rm EIT}$$

and $$\partial_z g_p^{(2)}(\mathbf{r}) \simeq -\kappa(\mathbf{r}) \langle \Sigma_{RR}(\bar{\mathbf{r}}) \rangle \text{Im}[\alpha_{\rm TLA} - \alpha_{\rm EIT}] g_p^{(2)}(\mathbf{r})$$

Numerical Simulations: exper. parameters

Atoms: ^{87}Rb at $T = 20\ \mu$K

$|g\rangle \equiv 5S_{1/2}\,|F=2, m_F=2\rangle \quad |e\rangle \equiv 5P_{3/2}\,|F=3, m_F=3\rangle \quad |r\rangle \equiv 60S_{1/2}$

$\Gamma_e = 3.8 \times 10^7$ s^{-1}, $\delta\omega_1 \simeq 2\pi \cdot 5.7 \times 10^4$ s^{-1} $\qquad \gamma_e = \frac{1}{2}\Gamma_e + \delta\omega_1$

$\Gamma_r = 5 \times 10^3$ s^{-1}, $\quad \delta\omega_2 \simeq 2\pi \cdot 1.1 \times 10^5$ s^{-1} $\qquad \gamma_r = \frac{1}{2}\Gamma_r + \delta\omega_2$

$C_6/2\pi = 1.4 \times 10^{11}$ s$^{-1}\mu$m^6

$\rho(z) = \rho_0 \exp[-(z-z_0)^2/2\sigma_\rho^2]; \quad \rho_0 = 1.32 \times 10^7$ mm^{-3} $\quad \sigma_\rho = 0.7$ mm

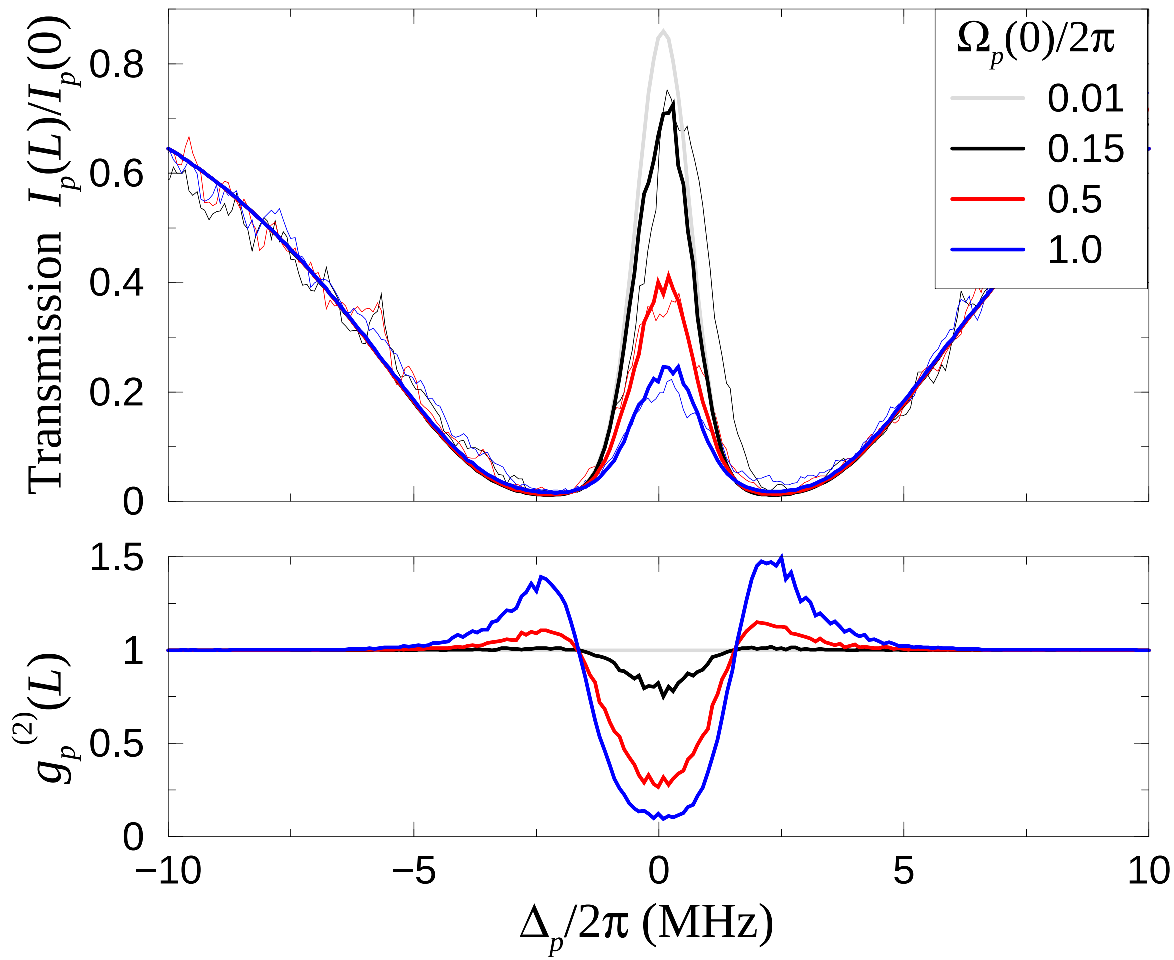
[$\bar{\rho} = 1.2 \times 10^7$ mm^{-3}, $L = 1.3$ mm $\quad \Rightarrow \quad \bar{\kappa}L = 4.524$]

$\Omega_c = 2\pi \cdot 2.25 \times 10^6$ s^{-1} ($\frac{\delta_c}{2\pi} = -10^5$ s^{-1}) $\Rightarrow$ $R_{\rm sa} \simeq 6.6\ \mu$m & $\bar{n}_{\rm sa} \simeq 14.7$

$v_g(\Delta_2 \simeq 0) = \frac{2|\Omega_c|^2}{\bar{\kappa}\gamma_e} \simeq 6000$ m/s

Pritchard *et al.*, PRL **105**, 193603 (2010); Singer *et al.*, JPB **38**, S295 (2005)

Numerical Simulations [stochastic MC]

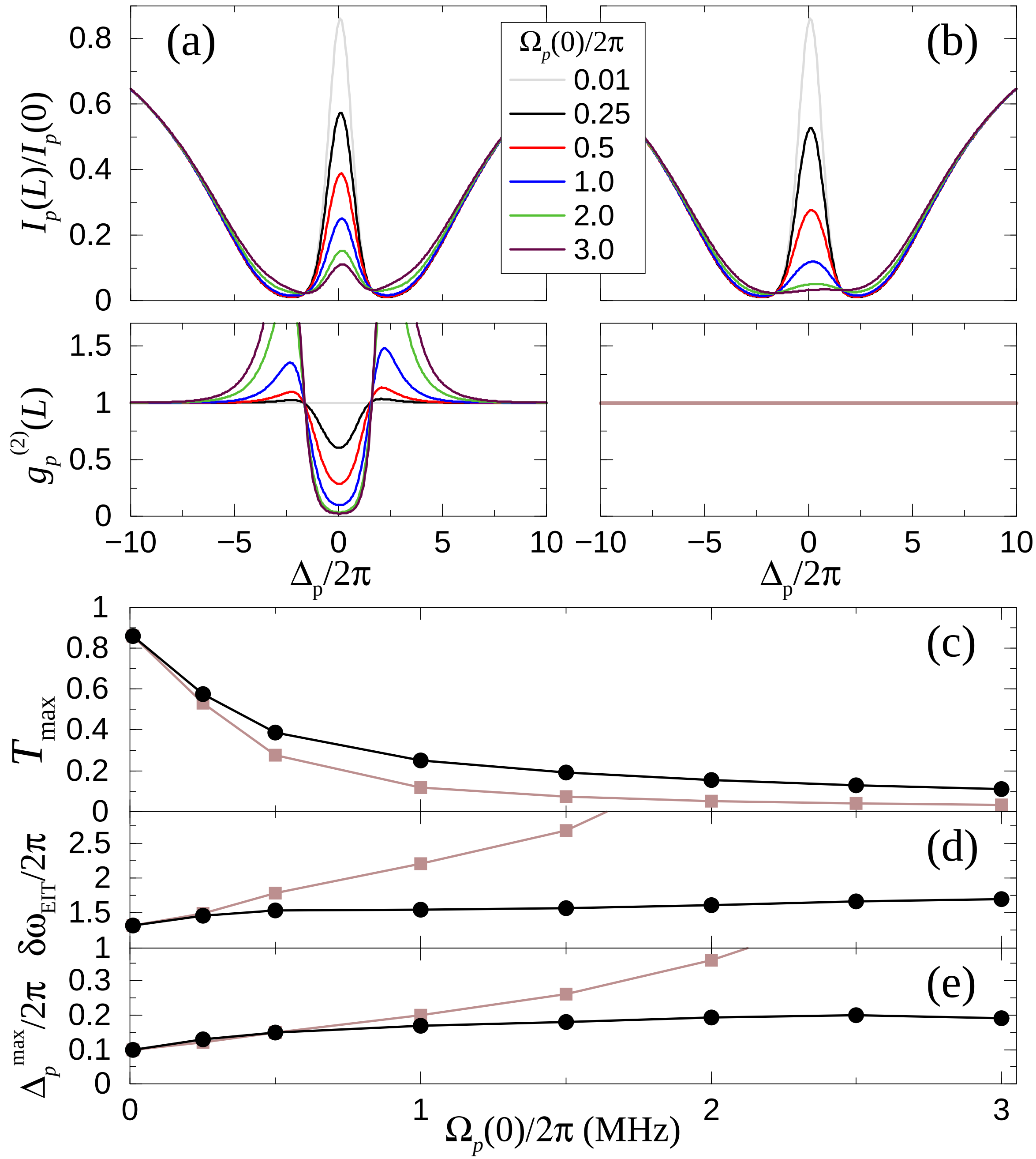


averaged over

10 realizations

Experiment ⇔ **Theory**: negligible shift & broadening of EIT line

Numerical Simulations [continuous]



averaged over

inf. realizations

Conclusions

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  - All the other superatoms induce a small mean-field shift of $|r\rangle$: $\delta_c \to \delta_c + \langle s\rangle$

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