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# Quantum Information Processing with single photons & atomic ensembles

**David Petrosyan**

# Outline

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- Background: Electromagnetically Induced Transparency

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- Atoms in Microwave Coplanar Waveguide Resonator
- Summary

# Photonic Qubit

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Single-photon wavepacket in the polarization state

$$|\psi\rangle = \alpha |V\rangle + \beta |H\rangle \quad \text{with} \quad |\alpha|^2 + |\beta|^2 = 1$$

$|V\rangle \equiv |0\rangle$  &  $|H\rangle \equiv |1\rangle$  form the computational basis  $\{|0\rangle, |1\rangle\}$

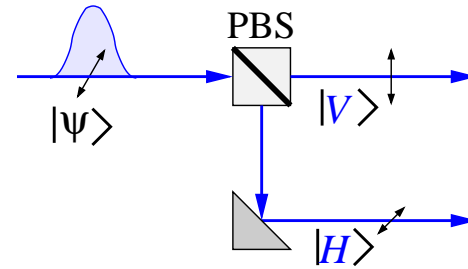
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Single-rail  $\leftrightarrow$  Dual-rail conversion



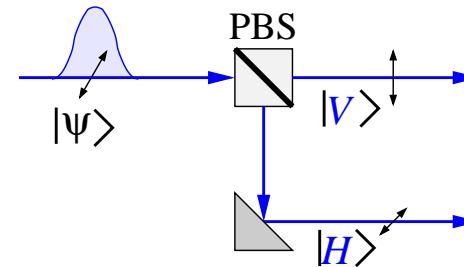
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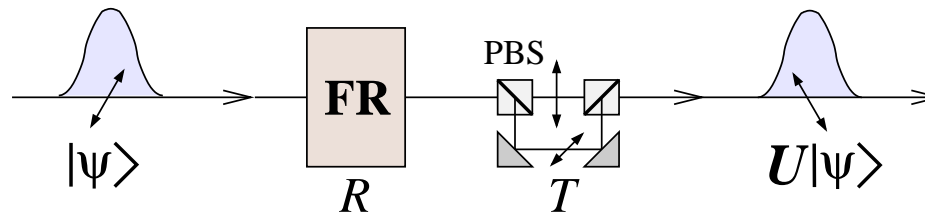
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Single-qubit unitary transformations  $U \propto R(\vartheta) \otimes T(\phi)$  can be implemented with linear-optics operations



# Universal Two-Qubit Logic Gates



## Controlled-NOT gate

$$W_{\text{CNOT}} |a\rangle |b\rangle \longrightarrow |a\rangle |a \oplus b\rangle$$

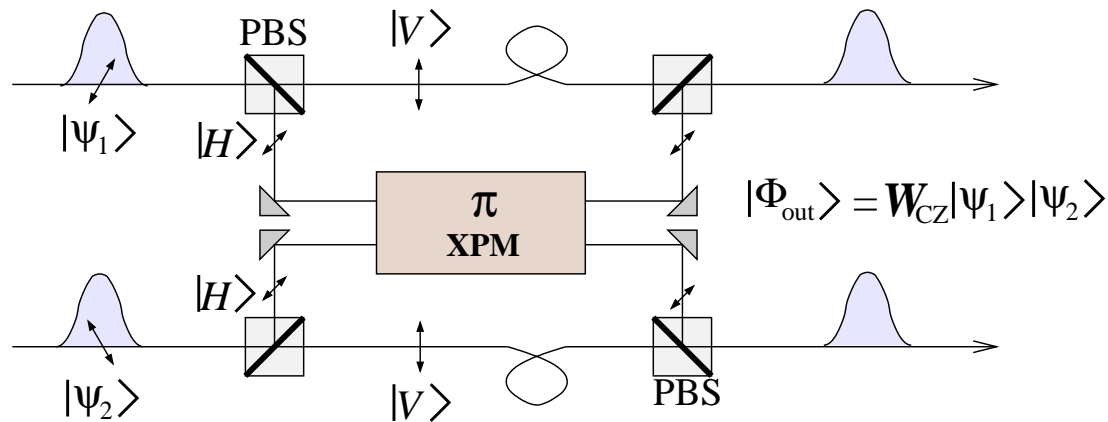
$$a, b \in \{0, 1\}$$

## Controlled-Z (PHASE) gate

$$W_{\text{CZ}} |a\rangle |b\rangle \longrightarrow (-1)^{ab} |a\rangle |b\rangle$$

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$W_{\text{CZ}}$  requires nonlinear (Kerr) photon-photon interaction – XPM



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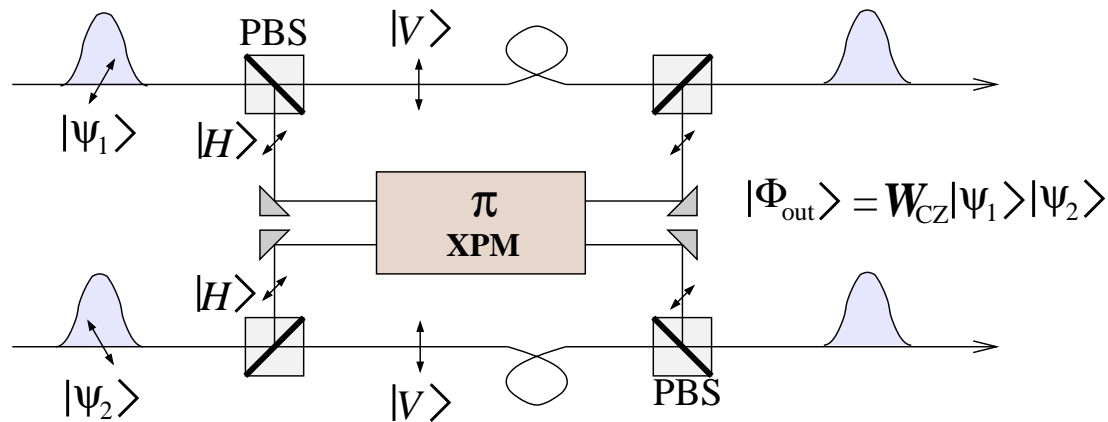
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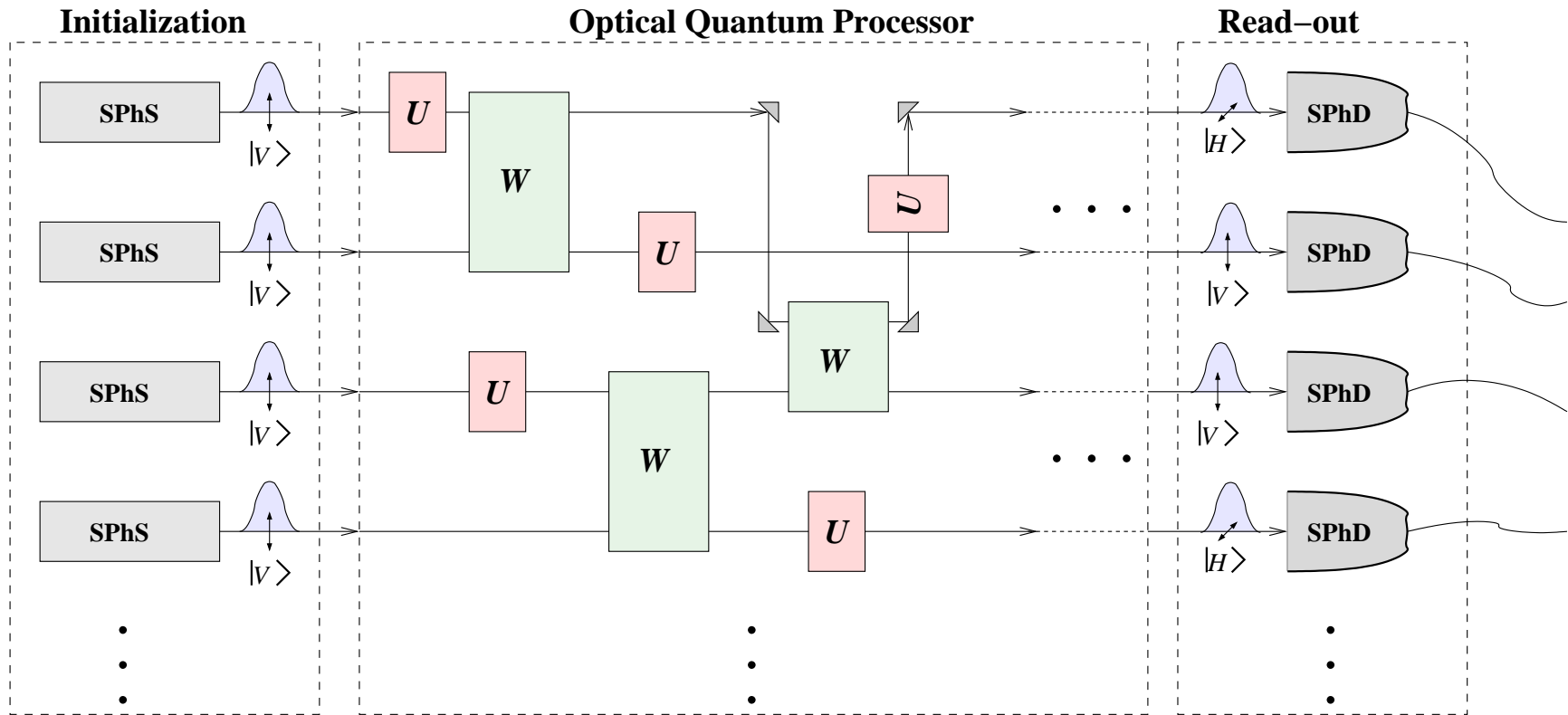
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- Any multiqubit transformation can be decomposed into single-qubit  $U$  and two-qubit  $W_{\text{CNOT}}$  or  $W_{\text{CZ}}$  transformations  
 $\Rightarrow U$  and  $W$  are Universal

# Optical Quantum Computer

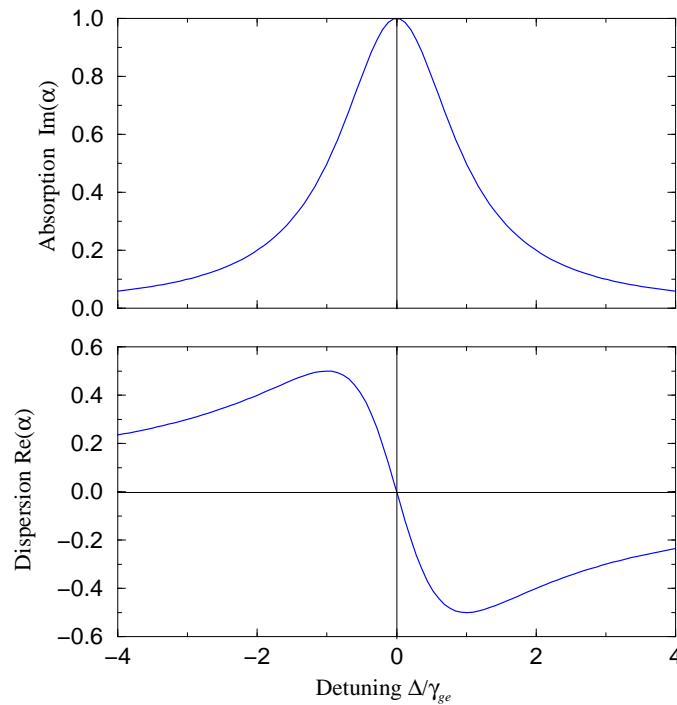
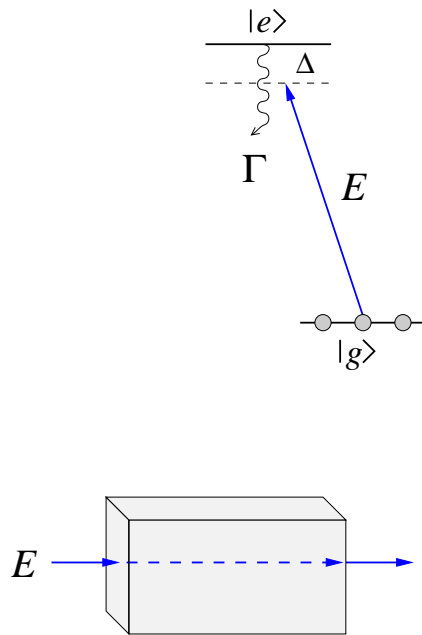


- SPhS: Single Photon Sources
- $W=XPM$ : Cross Phase Modulation
- SPhD: Single Photon Detectors

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# Electromagnetically Induced Transparency

# Spectral Properties

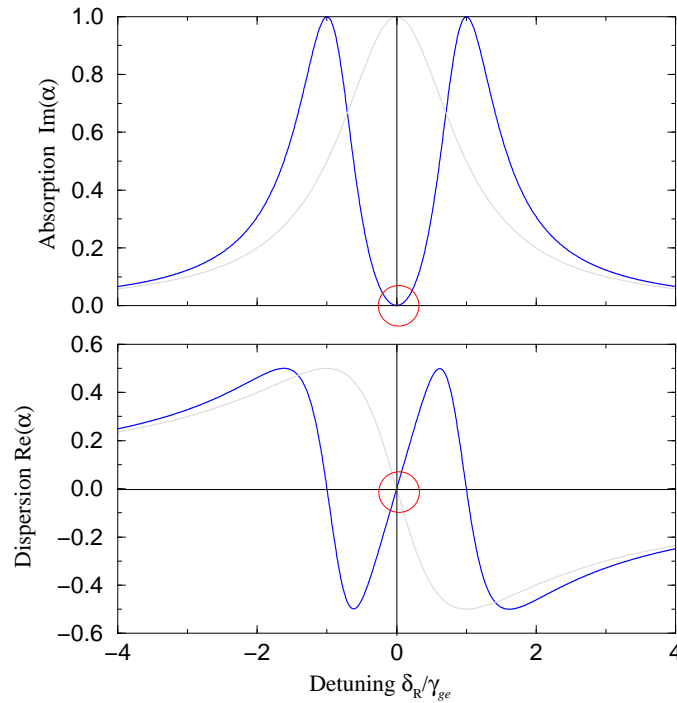
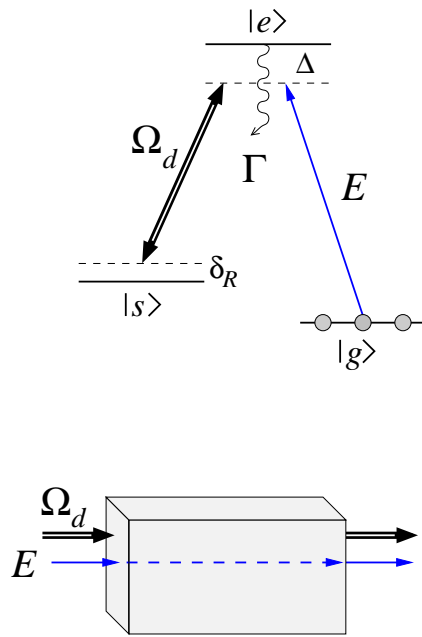


$$E(z, t) = E\left(0, t - \frac{z}{c}\right) e^{i\alpha z}$$

Medium Polarizability  $\alpha(\Delta) = \kappa_0 \frac{i\gamma_{ge}}{\gamma_{ge} - i\Delta} \Rightarrow$

**Absorption**  $\text{Im}(\alpha) \simeq \kappa_0 \quad (\Delta \ll \gamma_{ge})$

# Spectral Properties



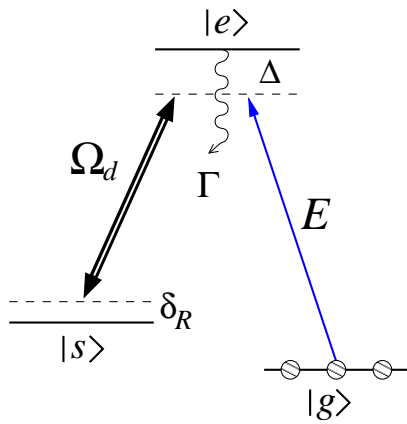
$$E(z, t) = E\left(0, t - \frac{z}{v_g}\right) e^{i\alpha z}$$

$$\text{Medium Polarizability } \alpha(\Delta) = \kappa_0 \frac{i\gamma_{ge}}{\gamma_{ge} - i\Delta + \frac{|\Omega_d|^2}{\gamma_R - i\delta_R}} \Rightarrow$$

$$\text{Absorption } \text{Im}(\alpha) = \frac{1}{v_g} \left[ \gamma_R + \frac{\delta_R^2}{|\Omega_d|^2} \right] \ll \kappa_0 \quad v_g = \frac{c}{1 + c \frac{\partial}{\partial \omega} \text{Re}(\alpha)} \simeq \frac{|\Omega_d|^2}{\kappa_0 \gamma_{ge}} \ll c$$

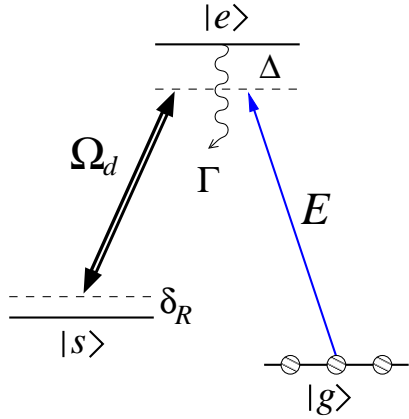
$$\text{EIT conditions: } |\Omega_d|^2 \gg (\gamma_{ge} + |\Delta|)(\gamma_R + \delta_R)$$

# Hamiltonian



$$H = \hbar \sum_{j=1}^N \begin{bmatrix} 0 & g\hat{\mathcal{E}}^\dagger(z_j)e^{-ikz_j} & 0 \\ g\hat{\mathcal{E}}(z_j)e^{ikz_j} & \Delta & \Omega_d(t)e^{ik_dz_j} \\ 0 & \Omega_d^*(t)e^{-ik_dz_j} & \delta_R \end{bmatrix}_j$$

with  $\hat{\mathcal{E}}(z, t) = \sum_q a^q(t)e^{iqz}$  and  $g = \frac{\wp_{ge}}{\hbar} \sqrt{\frac{\hbar\omega}{2\epsilon_0 V}}$



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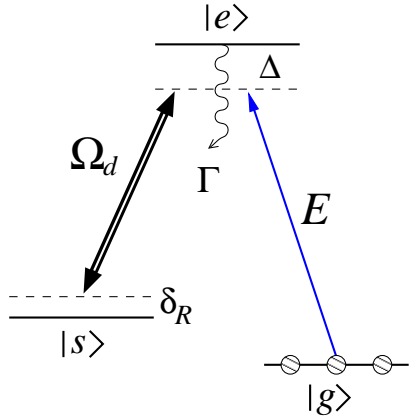
$\delta_R = 0 \Rightarrow$  **Dark states** ( $H |D_1^q\rangle = 0 |D_1^q\rangle$ ) for single-photons  $|1^q\rangle = a^{q\dagger} |0\rangle$

$$\boxed{|D_1^q\rangle = \cos\theta |1^q, s^{(0)}\rangle - \sin\theta |0^q, s^{(1)}\rangle} \quad \tan^2\theta(t) = \frac{g^2 N}{|\Omega_d(t)|^2}$$

with collective atomic states

$$|s^{(0)}\rangle \equiv |g_1, g_2, \dots, g_N\rangle$$

$$|s^{(1)}\rangle \equiv \frac{1}{\sqrt{N}} \sum_{j=1}^N e^{i(k+q-k_d)z_j} |g_1, \dots, s_j, \dots, g_N\rangle$$



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- $\theta = 0$  ( $|\Omega_d|^2 \gg g^2 N$ )  $\Rightarrow |D_1^q\rangle = |1^q\rangle |s^{(0)}\rangle$  purely photonic excitation
- $\theta = \pi/2$  ( $|\Omega_d|^2 \ll g^2 N$ )  $\Rightarrow |D_1^q\rangle = |0^q\rangle |s^{(1)}\rangle$  purely atomic excitation

# Dark-State Polariton

Define operator

$$\hat{\Psi}(z, t) = \cos \theta(t) \hat{\mathcal{E}}(z, t) - \sin \theta(t) \sqrt{N} \hat{\sigma}_{gs}(z, t)$$

$$\text{with } \hat{\sigma}_{gs}(z, t) = \frac{1}{N_z} \sum_{j=1}^{N_z} |g_j\rangle \langle s_j|, \quad N_z = \frac{N}{L} dz \gg 1$$

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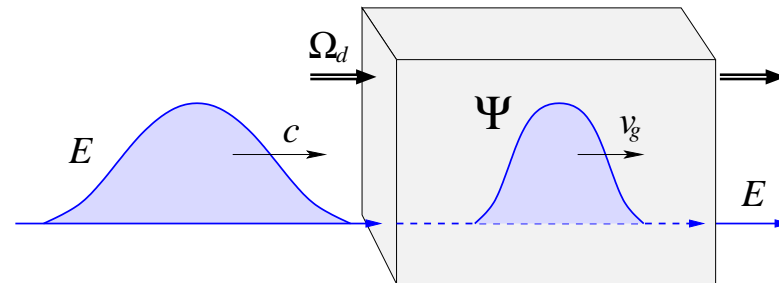
Equation of motion

$$\left( \frac{\partial}{\partial t} + v_g(t) \frac{\partial}{\partial z} \right) \hat{\Psi}(z, t) = 0 \quad \Rightarrow \quad \hat{\Psi}(z, t) = \hat{\Psi} \left( z - \int_0^t v_g(t') dt', 0 \right)$$

$$v_g(t) = c \cos^2 \theta(t) = c \frac{|\Omega_d(t)|^2}{g^2 N + |\Omega_d(t)|^2} \quad \text{group velocity (time-dependent)}$$

$\Rightarrow$  One can decelerate/accelerate the propagation of  $\hat{\Psi}(z, t)$

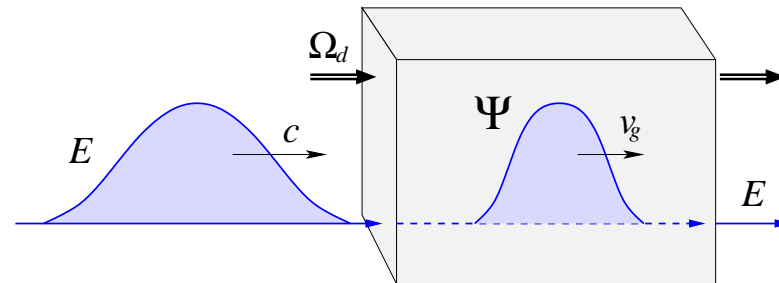
# Stopping of Light



$$\text{Single-photon WP: } |1\rangle = \sum_q \xi^q |1^q\rangle = \frac{1}{L} \int dz f(z) \hat{\mathcal{E}}^\dagger(z) |0\rangle$$

- i)* At the entrance  $0 < \theta(0) \lesssim \pi/2$  ( $0 < \Omega_d(t) \ll g^2 N$ )  
 $\Rightarrow$  Pulse is spatially compressed by  $\frac{v_g(0)}{c} = \cos^2 \theta(0) \ll 1$

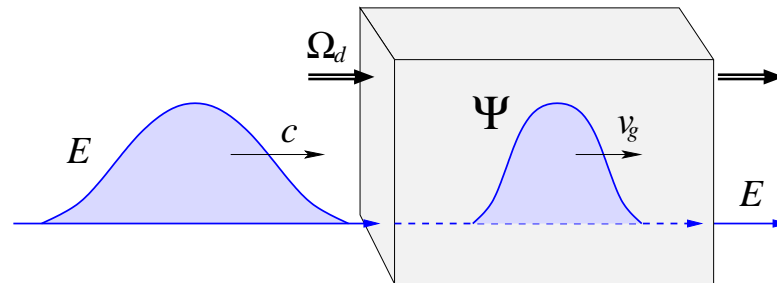
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- ii)* Rotate  $\theta(t) \rightarrow \pi/2$  ( $\Omega_d(t) \rightarrow 0$ )  
 $\Rightarrow$  Pulse is stopped  $v_g(t) = 0$   
 and stored as atomic excitation  $|D_1^q\rangle = |0^q\rangle |s^{(1)}\rangle$

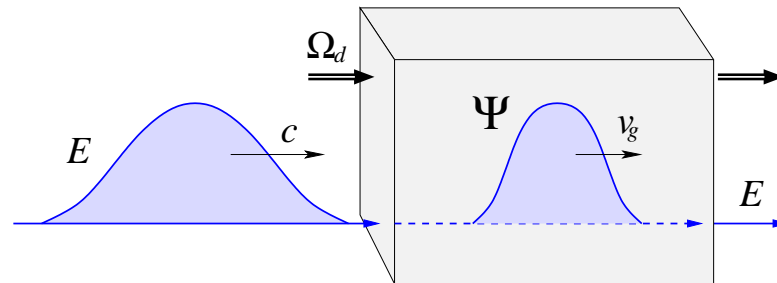
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- iii)* Rotate  $\theta(t') < \pi/2$  ( $\Omega_d(t') > 0$ )  
 $\Rightarrow$  Pulse is released  $v_g(t') > 0$

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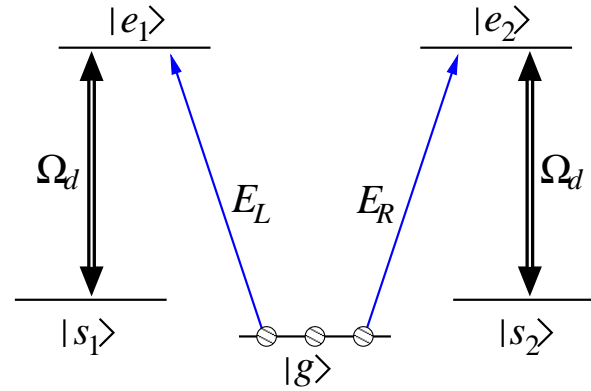
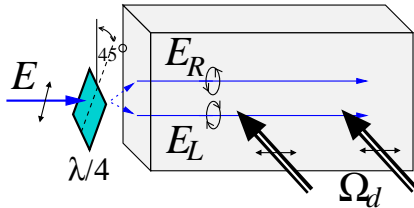


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Requires  $T v_g(0) < L$  &  $T^{-1} < \delta\omega_{\text{tw}} \Rightarrow$  large optical depth  $2\kappa_0 L \gg 1$

# Photonic Memory

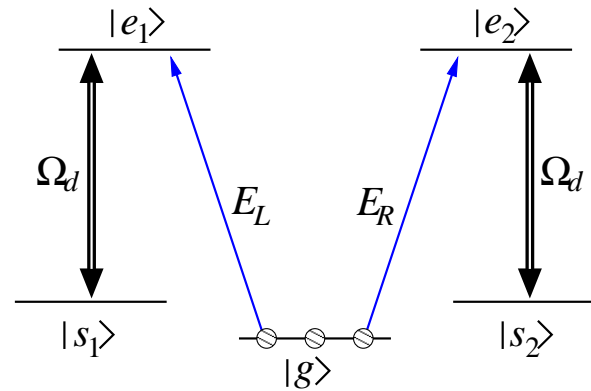
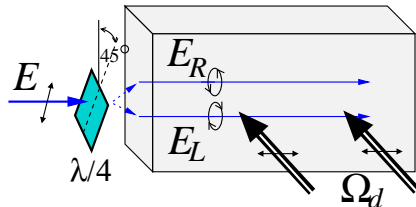


- $\lambda/4$  plate oriented at  $45^\circ$

$$|V\rangle \rightarrow |R\rangle = \frac{1}{\sqrt{2}}(|V\rangle + i|H\rangle) \quad |H\rangle \rightarrow |L\rangle = \frac{1}{\sqrt{2}}(|V\rangle - i|H\rangle)$$

$$\Rightarrow |\psi(0)\rangle \rightarrow \alpha |R\rangle + \beta |L\rangle$$

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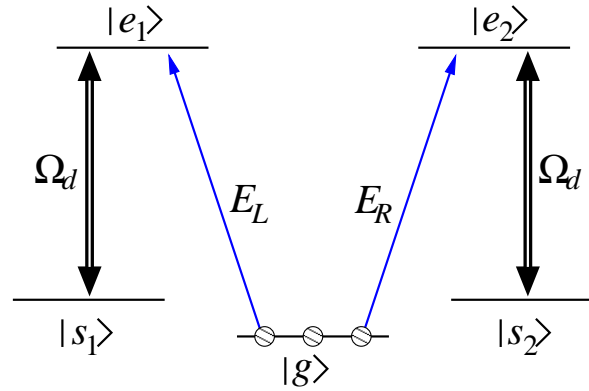
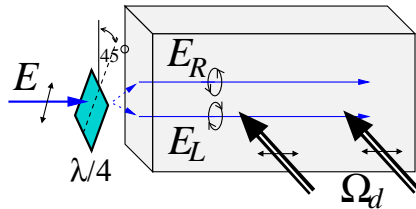
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- Rotate  $\theta(t) \rightarrow \pi/2$  ( $\Omega_d(t) \rightarrow 0$ )

$\Rightarrow$  Photon is stopped and stored as superposition of  $|s_1^{(1)}\rangle$  &  $|s_2^{(1)}\rangle$

$$\boxed{|\psi(t)\rangle = \alpha |s_1^{(1)}\rangle + \beta |s_2^{(1)}\rangle} \quad \text{With low decoherence!}$$

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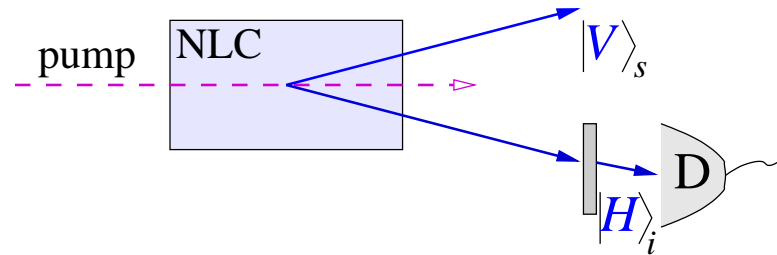
$\Rightarrow$  Photon is released  $|\psi(t')\rangle \rightarrow \alpha |R\rangle + \beta |L\rangle$



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# Single-Photon Sources

# Parametric Down-Conversion

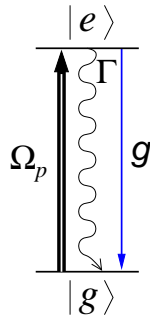
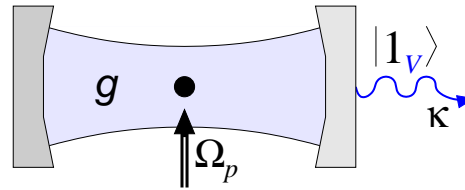


Pairs of P&M entangled photons

$$|\Phi\rangle = \frac{1}{\sqrt{2}} (|V\rangle_s |H\rangle_i + |H\rangle_s |V\rangle_i)$$

Detect *idler* in  $|H\rangle_i \Rightarrow$  project *signal* onto  $|V\rangle_s$

# Cavity QED

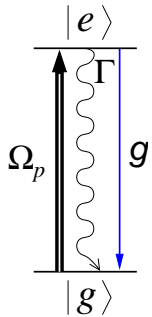
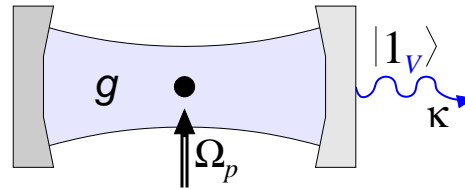


**Two-Level System** (atom, QD ...) with  $\Gamma < g < \kappa$

Apply  $\Omega_p T = \pi$  pulse  $\Rightarrow$  single photon is emitted  
(**Purcell effect**)

Khitrova et al, Nature Physics **2**, 81 (2006)

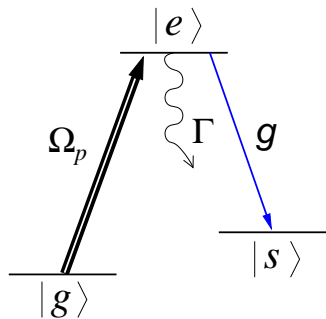
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**Three-Level System** with  $g > \kappa, \Gamma$

$$|D\rangle = \cos \theta |g, 0\rangle - \sin \theta |s, 1\rangle \quad \tan \theta = \frac{\Omega_p}{g}$$

Apply  $\Omega_p$  adiabatic pulse  $\Rightarrow$  single photon is emitted  
(intracavity **STIRAP**)

Kuhn, Henrich, Rempe, PRL **89**, 067901 (2002); McKeever et al, Science **303**, 1992 (2004)

Requires high- $Q$  optical cavities

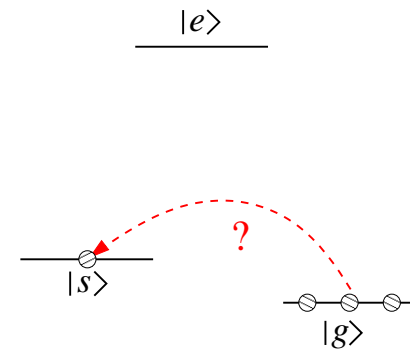
# EIT Based Single-Photon Sources



i) **Create** symmetric spin (Raman) excitation state

$$|s^{(1)}\rangle = \frac{1}{\sqrt{N}} \sum_{j=1}^N e^{i\delta k z_j} |g_1, \dots, s_j, \dots, g_N\rangle$$

$$\Rightarrow |D_1(0)\rangle = |0\rangle |s^{(1)}\rangle$$



# EIT Based Single-Photon Sources

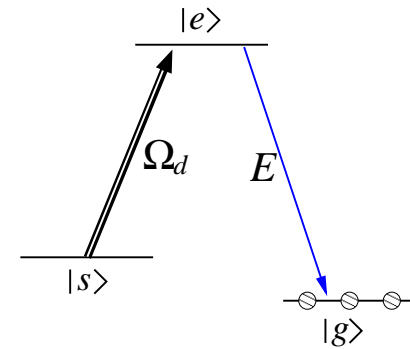
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$$|s^{(1)}\rangle = \frac{1}{\sqrt{N}} \sum_{j=1}^N e^{i\delta k z_j} |g_1, \dots, s_j, \dots, g_N\rangle$$

$$\Rightarrow |D_1(0)\rangle = |0\rangle |s^{(1)}\rangle$$

*ii)* **Apply**  $\Omega_d \neq 0$  ( $\theta < \pi/2$ ) & **Release SPh WP**

$$\Rightarrow |D_1(t)\rangle \rightarrow |1\rangle |s^{(0)}\rangle$$



# EIT Based Single-Photon Sources

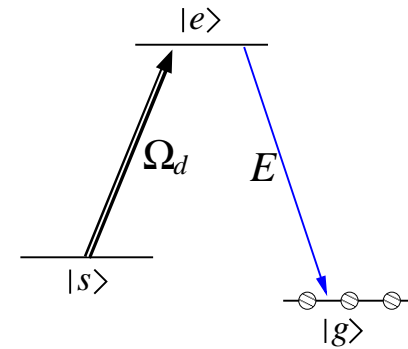
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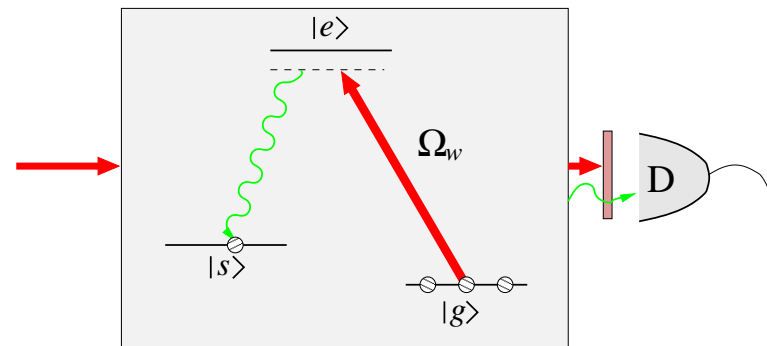
$$\Rightarrow |D_1(t)\rangle \rightarrow |1\rangle |s^{(0)}\rangle$$



## Spontaneous Raman Scattering

*i)* Apply *write* pulse  $\Omega_w$  & detect forward scattered Stokes photon

$\Rightarrow$  Click of **D** corresponds to  $|s^{(1)}\rangle$   
(with  $\delta k = k_w - k_s$ )



Go to *ii)*

Duan et al, Nature **414**, 413 (2001); Kuzmich et al, Nature **423**, 731 (2003);  
van der Wal et al, Science **301**, 196 (2003); Chou et al, PRL **92**, 213601 (2004);  
Eisaman et al, PRL **93**, 233602 (2004); Eisaman et al, Nature **438**, 837 (2005)

# EIT Based Single-Photon Sources



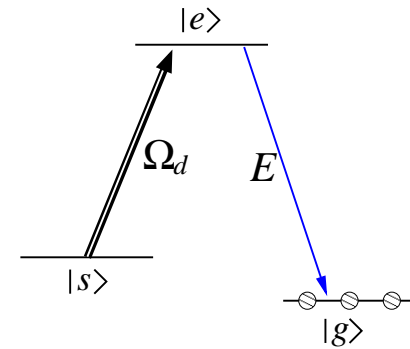
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$$|s^{(1)}\rangle = \frac{1}{\sqrt{N}} \sum_{j=1}^N e^{i\delta k z_j} |g_1, \dots, s_j, \dots, g_N\rangle$$

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ii) **Apply**  $\Omega_d \neq 0$  ( $\theta < \pi/2$ ) & **Release SPh WP**

$$\Rightarrow |D_1(t)\rangle \rightarrow |1\rangle |s^{(0)}\rangle$$



## Dipole Blockade

Pair of atoms  $i, j$  in Rydberg states  $|r\rangle$

$\Rightarrow$  **Dipole-Dipole Interaction** (anisotropic)

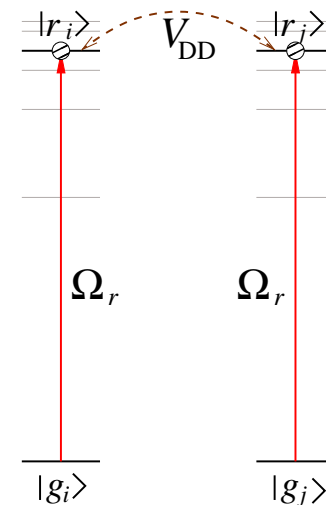
$$V_{DD} = \hbar D(\mathbf{r}_i - \mathbf{r}_j) |r_i r_j\rangle \langle r_i r_j|$$

Resonant DDI (Förster process) + Static DDI (in dc E-field)

$$D(\mathbf{r}_i - \mathbf{r}_j) \approx -\frac{n^4 e^2 a_0^2}{\pi \hbar \epsilon_0 |\mathbf{r}_i - \mathbf{r}_j|^3}$$

Double-excitation is nonresonant

$$P_{\text{double}} \sim \frac{|\Omega_r|^2}{D^2} \ll 1 \text{ if } \Omega_r < D$$



# EIT Based Single-Photon Sources



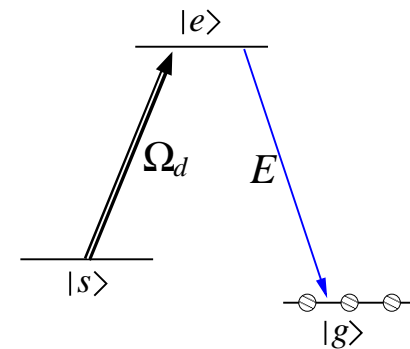
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## Dipole Blockade

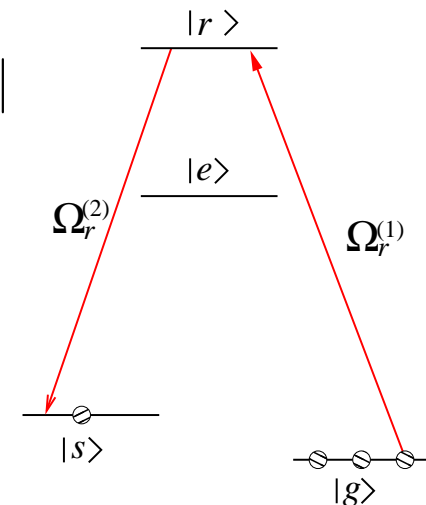
Atomic ensemble  $V_{DD} = \hbar \sum_{ij} D(\mathbf{r}_i - \mathbf{r}_j) |r_i r_j\rangle \langle r_i r_j|$

**i) Apply**  $\Omega_r^{(1)}$  for  $\sqrt{N}\Omega_r^{(1)}T_1 = \pi$  (collective  $\pi$  pulse)

$$\Rightarrow |s^{(0)}\rangle \equiv |g_1, g_2, \dots, g_N\rangle \rightarrow$$

$$\rightarrow \frac{1}{\sqrt{N}} \sum_j e^{ik_r^{(1)} z_j} |g_1, \dots, r_j, \dots, g_N\rangle \equiv |r^{(1)}\rangle$$

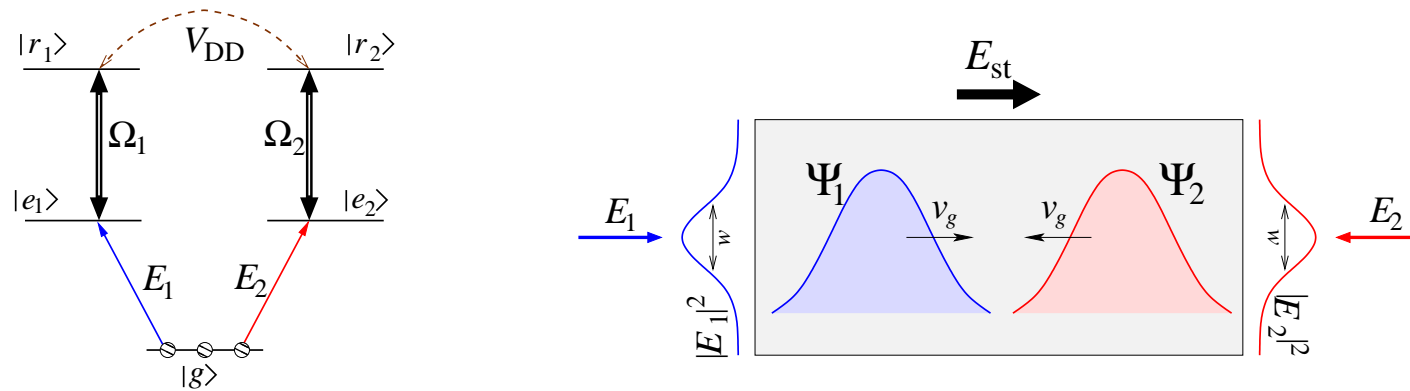
Single collective Rydberg excitation ( $\sqrt{N}\Omega_r^{(1)} < \bar{D}$ )



**i') Apply**  $\Omega_r^{(2)}T_2 = \pi \Rightarrow |r^{(1)}\rangle \rightarrow |s^{(1)}\rangle$  (with  $\delta k = k_r^{(1)} - k_r^{(2)}$ )

Go to **ii)**

# Cross-Phase Modulation via Static DDI



Static  $E_{st} \mathbf{e}_z \Rightarrow$  Rydberg states  $|r_i\rangle$  have large permanent dipole moments

$$\wp_r \mathbf{e}_z = \frac{3}{2} n q e a_0 \mathbf{e}_z \quad (\text{Stark eigenstates})$$

$E_i \rightarrow \hat{\Psi}_i = \cos \theta \hat{\mathcal{E}}_i - \sin \theta \sqrt{N} \hat{\sigma}_{gr_i}$  ( $i = 1, 2$ ) propagate with  $\pm v_g = c \cos^2 \theta$

Atomic components of  $\hat{\Psi}_i$  interact via Static DDI  $\Rightarrow$  induces **XPM**

$$V_{DD} = \hbar \rho^2 \iint d^3 r d^3 r' \hat{\sigma}_{rr}(\mathbf{r}) D(\mathbf{r} - \mathbf{r}') \hat{\sigma}_{rr}(\mathbf{r}')$$

$$D(\mathbf{r} - \mathbf{r}') = C \frac{1 - 3 \cos^2 \vartheta}{|\mathbf{r} - \mathbf{r}'|^3} \quad \text{with} \quad C = \frac{\wp_{r_i} \wp_{r_{i'}}}{4\pi \epsilon_0 \hbar}$$

Resonant DDI (state mixing) is suppressed for  $q = n - 1, m = 0$

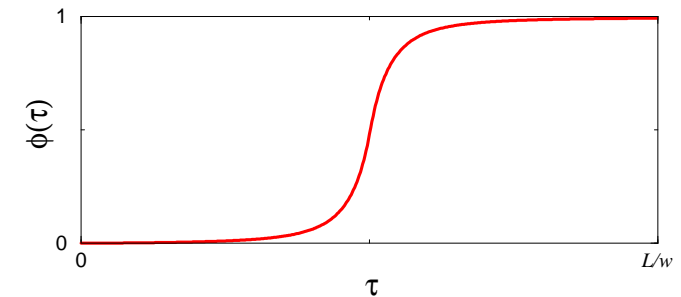
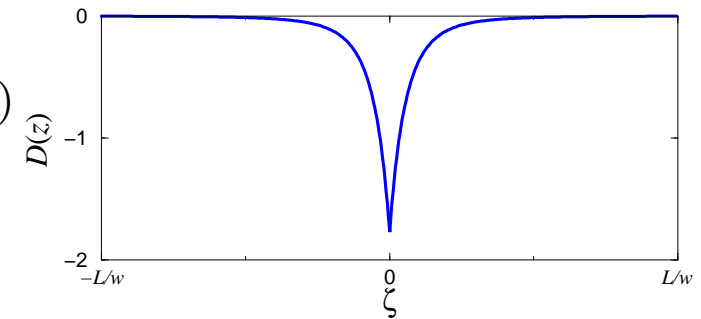
# Cross-Phase Modulation via Static DDI

- DD level shift [vs.  $\zeta = (z - z')/w$ ]

$$D(z - z') = \frac{1}{\pi w^2} \int_0^{2\pi} d\varphi' \int_0^\infty dr'_\perp r'_\perp e^{-r'^2_\perp/w^2} D(z\mathbf{e}_z - \mathbf{r}')$$

- Phase shift [vs.  $\tau = v_g t/w$ ]

$$\phi(z_1, z_2, t) = -\sin^4 \theta \int_0^t dt' D(z_1 - z_2 - 2v_g(t - t'))$$



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Initially  $t = 0$ ,  $z_1 = 0$  &  $z_2 = L \Rightarrow \phi(0, L, 0) = 0$

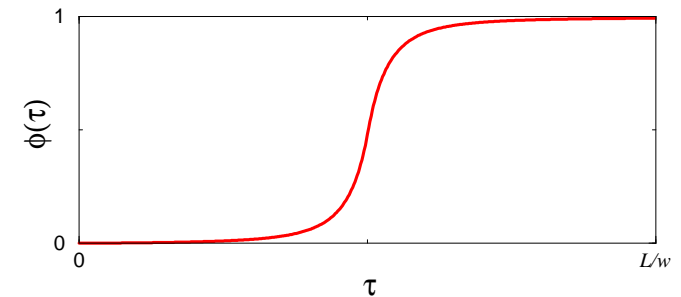
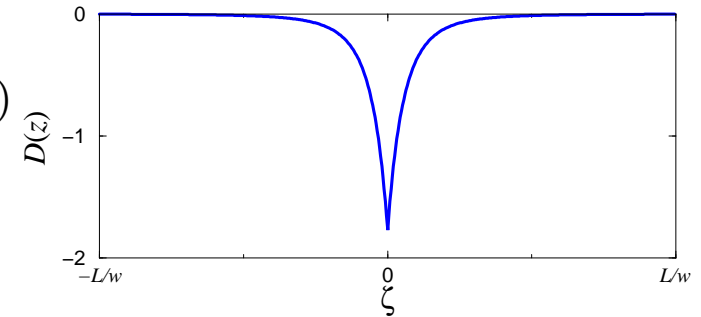
After the interaction  $t = L/v_g$ ,  $z_1 = L$  &  $z_2 = 0$

$$\phi(L, 0, L/v) = -\frac{\sin^4 \theta}{v_g} \int_0^L dz' D(2z' - L) = \frac{2C}{v_g w^2}$$

- Phase shift  $\phi = \pi$

$\Rightarrow$  Universal CPHASE gate between SPh pulses  $\hat{\mathcal{E}}_1$  &  $\hat{\mathcal{E}}_2$

$$|x\rangle_1 |y\rangle_2 \rightarrow (-1)^{xy} |x\rangle_1 |y\rangle_2 \quad (x, y \in [0, 1])$$



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- DD level shift [vs.  $\zeta = (z - z')/w$ ]

$$D(z - z') = \frac{1}{\pi w^2} \int_0^{2\pi} d\varphi' \int_0^\infty dr'_\perp r'_\perp e^{-r'^2_\perp/w^2} D(z\mathbf{e}_z - \mathbf{r}')$$

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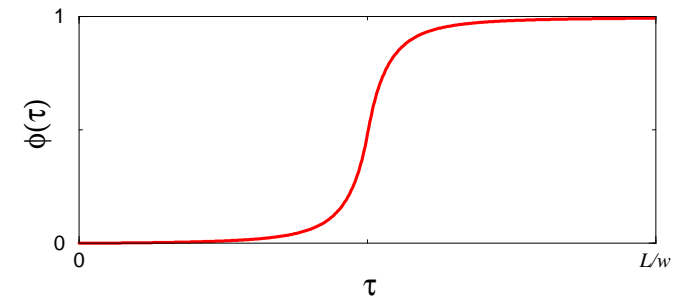
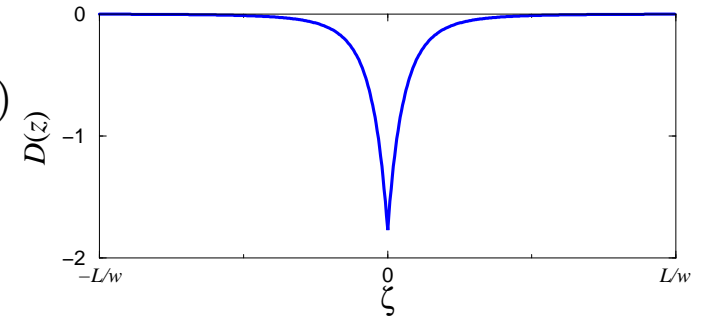
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$$|x\rangle_1 |y\rangle_2 \rightarrow (-1)^{xy} |x\rangle_1 |y\rangle_2 \quad (x, y \in [0, 1])$$

**Advantages:** Weak focusing ( $w \sim 30\mu\text{m}$ ) & Uniform phase-shift



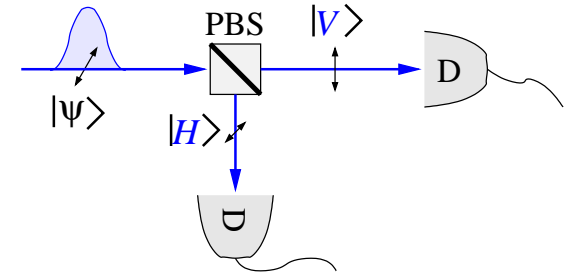
# Single Photon Detection

Single photon WP  $|\psi\rangle = \alpha |V\rangle + \beta |H\rangle$   
 passes through PBS

$\Rightarrow$   $|V\rangle$  and  $|H\rangle$  polarization components  
 are directed into Photodetectors D



Qubit Measurement Requires High-Efficiency Photodetectors



# Single Photon Detection

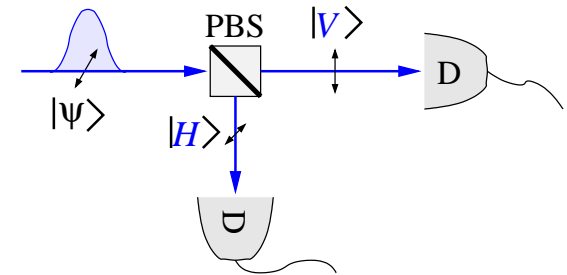
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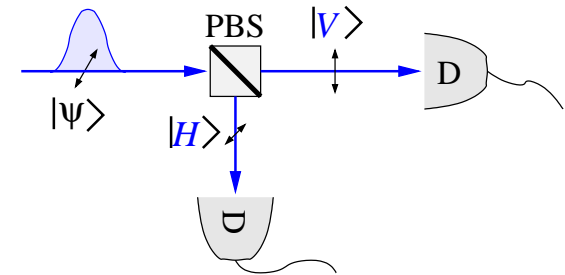
- Avalanche Photodetectors — quantum efficiency  $\eta \lesssim 70\%$



# Single Photon Detection

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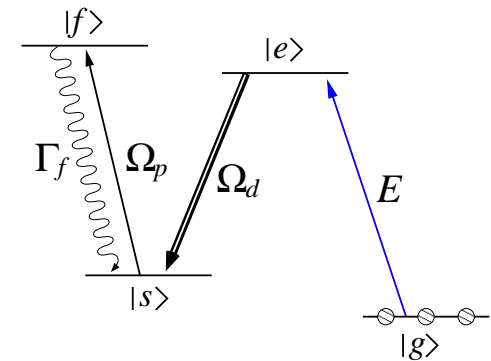
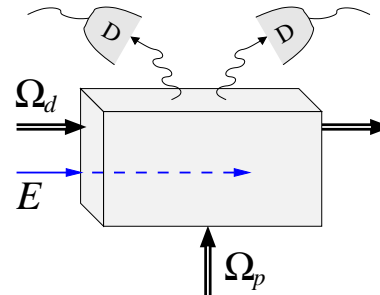
## Qubit Measurement Requires High-Efficiency Photodetectors

- Avalanche Photodetectors — quantum efficiency  $\eta \lesssim 70\%$
- EIT based Photodetection — quantum efficiency  $\eta \rightarrow 100\%$

i) Rotate  $\theta(t) \rightarrow \pi/2$  ( $\Omega_d(t) \rightarrow 0$ )

$\Rightarrow$  Pulse is stopped

$$|1^q\rangle |s^{(0)}\rangle \rightarrow |0^q\rangle |s^{(1)}\rangle$$



ii) Apply pump  $\Omega_p$  to

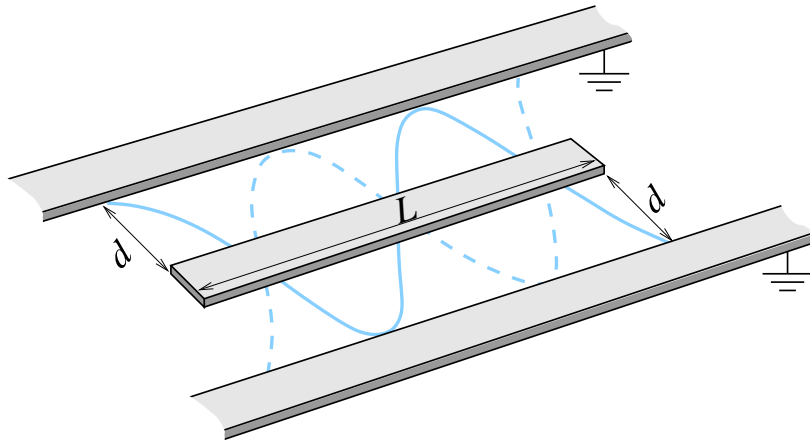
cycling transition  $|s\rangle \rightarrow |f\rangle \Rightarrow R_f \simeq \frac{1}{2}\Gamma_f$

$\Rightarrow$  Collect fluor. with **D** ( $\eta < 1$ ) for time  $T$ :  $S_f \simeq \frac{1}{2}\eta\Gamma_f T \gg 1$



# Atoms in Microwave CPW Resonator

## Superconducting CPW resonator



Strip-line length  $L \simeq 1$  cm and electrode distance  $d \sim 15 \mu\text{m}$

$u(\mathbf{r}) = \cos(m\pi z/L)$  or  $\sin(m\pi z/L)$  cavity mode function (1D)  
with  $m$  even or odd integer and  $z \in [-L/2, L/2]$ .

$m = 5$  ( $m + 1$  field antinodes)

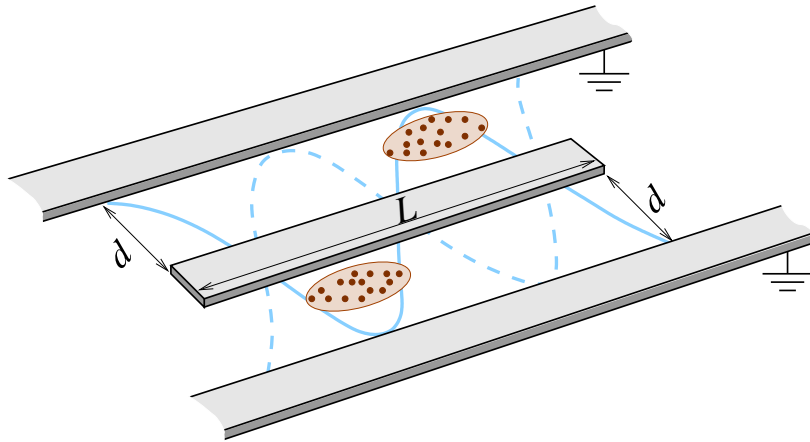
$$\lambda_c = \frac{2L}{m} \simeq 4 \text{ mm and } \omega_c = \frac{2\pi c}{\lambda_c \sqrt{\epsilon_r}} \simeq 2\pi \times 30 \text{ GHz}$$

$Q \simeq 10^6 \Rightarrow$  photon decay rate  $\kappa = \frac{\omega_c}{Q} \simeq 200 \text{ KHz}$

# The System



## Ensembles of cold ground-state atoms trapped near field antinodes



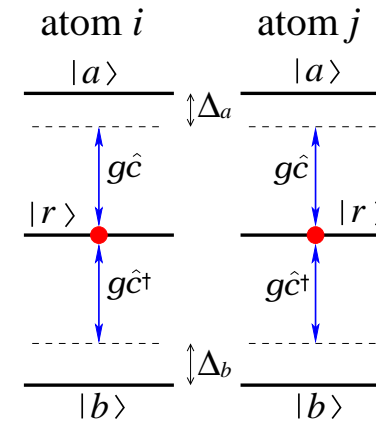
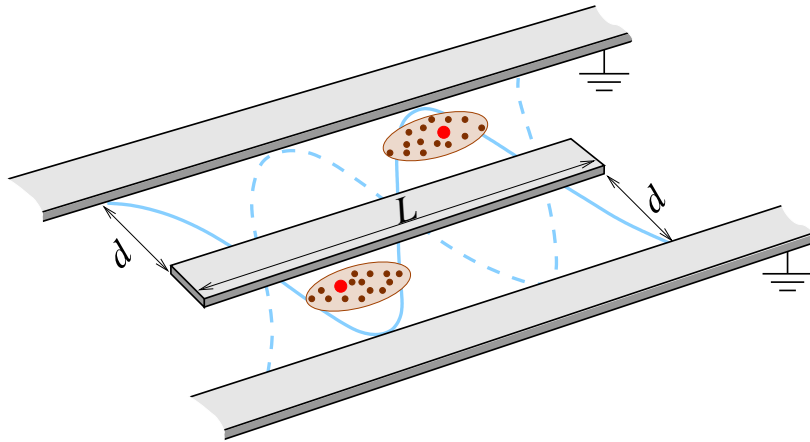
Atom density  $\rho_a \sim 2 \times 10^{13} \text{ cm}^{-3}$  trap volume  $V_a \sim d \times d \times \lambda_c/20$

$N = \rho_a V_a \sim 10^6$  atoms in each ensemble

# The System



## Excited Rydberg atoms $i$ & $j$ interact via common cavity mode



Hamiltonian for atom  $i$  + atom  $j$  + cavity field:

$$H = \hbar \sum_{l=i,j} [(\Delta_a \hat{\sigma}_{aa}^l - \Delta_b \hat{\sigma}_{bb}^l) + (g_{br}^l \hat{c}^\dagger \hat{\sigma}_{br}^l + g_{ar}^l \hat{c} \hat{\sigma}_{ar}^l + \text{H.c.})]$$

$\Delta_a = \omega_{ar} - \omega_c$  and  $\Delta_b = \omega_{rb} - \omega_c$  detunings

$\hat{\sigma}_{\mu\nu}^l = |\mu_l\rangle\langle\nu_l|$  atomic transition operator  $\hat{c}, \hat{c}^\dagger$  cavity field operators

$g_{\mu\nu}^l = -\frac{\wp_{\mu\nu}}{\hbar} \epsilon_c u(\mathbf{r}_l)$  atom-field coupling ( $g_{br}^l \approx g_{ar}^l \equiv g_r$ )

$\epsilon_c = \sqrt{\frac{\hbar\omega_c}{\epsilon_0 2\pi d^2 L}}$  field per photon in the cavity

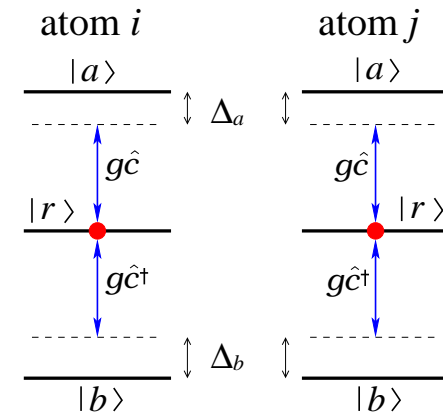
Atomic dipole matrix elements  $\wp \propto n^2 a_0 e \Rightarrow g_r \sim 2\pi \times 10 \text{ MHz}$  (for  $n \sim 50$ )

# Resonant Dipole-Dipole Interaction



$$\Delta_{a,b} \simeq g_r f \gg g_r \quad (f \gg 1)$$

- $|r_i\rangle |r_j\rangle |0_c\rangle \rightarrow |r_{i,j}\rangle |b_{j,i}\rangle |1_c\rangle$  (1Ph) nonresonant
- $|r_i\rangle |r_j\rangle |0_c\rangle \rightarrow |a_{i,j}\rangle |b_{j,i}\rangle |0_c\rangle$  (2Ph) **resonant**  
 (ac Stark compensated:  $\Delta_a - \Delta_b + s_a^{i,j} - s_r^i - s_r^j = 0$ )



# Resonant Dipole-Dipole Interaction

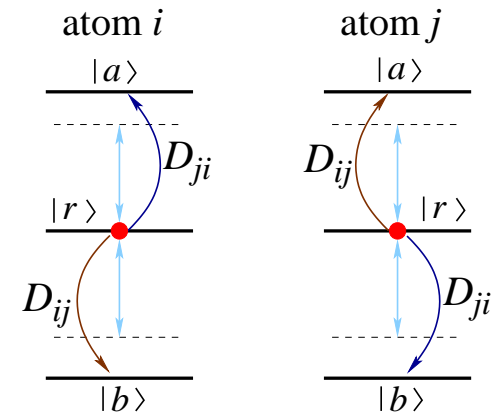


Effective cavity-mediated RDDI  
between atoms  $i$  and  $j$

**Effective Hamiltonian** (2nd-order in  $g_r$ ):

$$V_{ij}^{(2)} = \hbar D_{ij} (\hat{\sigma}_{br}^i \hat{\sigma}_{ar}^j + \hat{\sigma}_{ar}^i \hat{\sigma}_{br}^j) + \text{H.c}$$

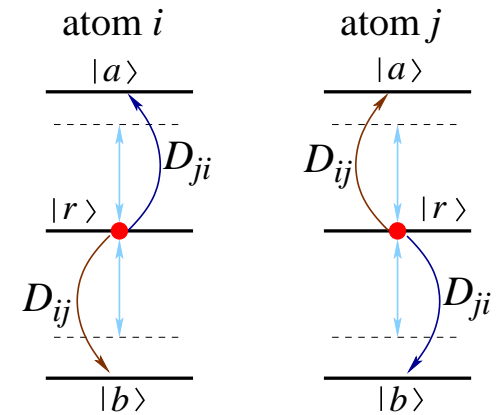
$$D_{ij} = \frac{g_{br}^i g_{ar}^j}{\Delta_b} = \frac{g_{ar}^i g_{br}^j}{\Delta_b} = \frac{g_r}{f} \equiv D \text{ — RDDI constant}$$



# Resonant Dipole-Dipole Interaction



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**Eigenstates of  $V_{ij}^{(2)}$**

$$|\psi_{ij}^0\rangle = \frac{1}{\sqrt{2}} (|b_i\rangle |a_j\rangle - |a_i\rangle |b_j\rangle) \quad \lambda_0 = 0$$

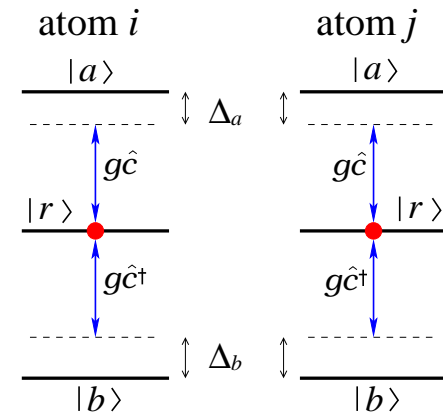
$$|\psi_{ij}^\pm\rangle = \frac{1}{\sqrt{2}} |r_i\rangle |r_j\rangle \pm \frac{1}{2} (|b_i\rangle |a_j\rangle + |a_i\rangle |b_j\rangle) \quad \lambda_\pm = \pm \hbar \sqrt{2} D_{ij}$$

# Van der Waals Interaction



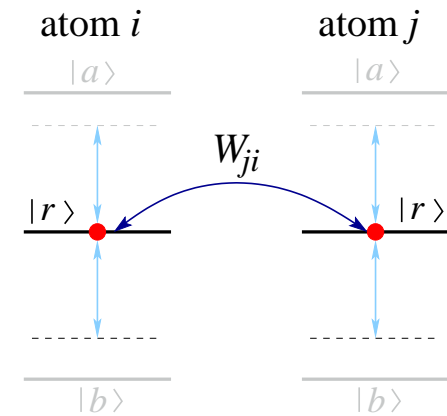
$$\Delta_a \simeq g_r(f - 1) \quad \& \quad \Delta_b \simeq g_r f \quad (f \gg 1)$$

- 1ph and 2ph transitions nonresonant



# Van der Waals Interaction

Effective cavity-mediated VdWI  
between atoms  $i$  and  $j$



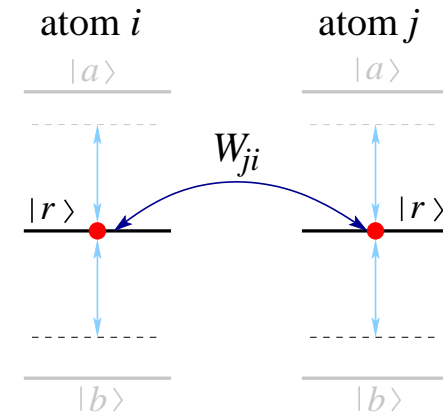
**Effective Hamiltonian** (4th-order in  $g_r$ ):

$$V_{ij}^{(4)} = \hbar \hat{\sigma}_{rr}^i W_{ij} \hat{\sigma}_{rr}^j$$

$$W_{ij} = \frac{2|g_{br}^i g_{br}^j|^2}{\Delta_b^3} - \frac{2|g_{br}^i g_{ar}^j|^2}{(\Delta_a - \Delta_b)\Delta_b^2} = \frac{4g_r}{f^3} \equiv W \text{ — WdVI constant}$$

# Van der Waals Interaction

Effective cavity-mediated VdWI  
between atoms  $i$  and  $j$



**Effective Hamiltonian** (4th-order in  $g_r$ ):

$$V_{ij}^{(4)} = \hbar \hat{\sigma}_{rr}^i W_{ij} \hat{\sigma}_{rr}^j$$

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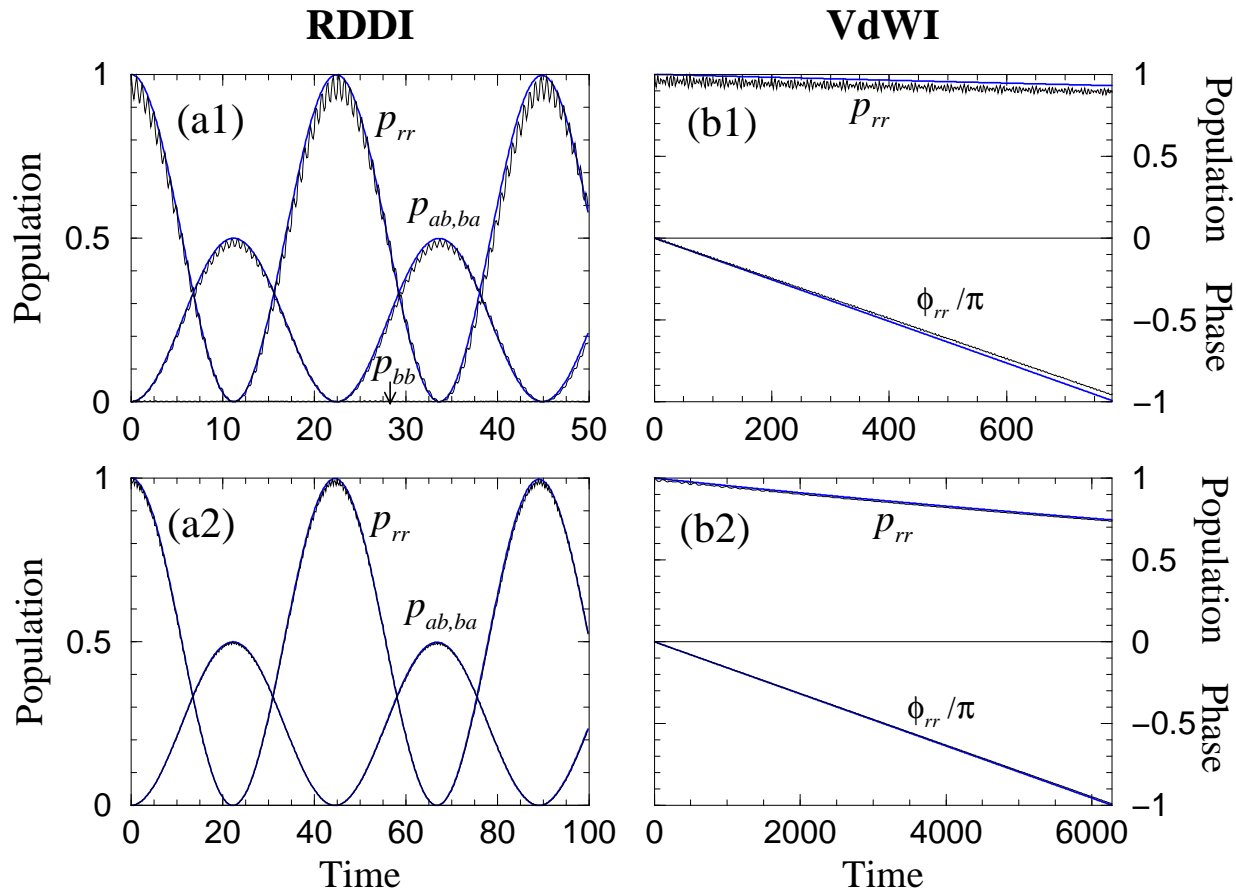
**Evolution due to**  $V_{ij}^{(4)}$

$$|r_i\rangle |r_j\rangle \rightarrow e^{i\phi_{rr}(t)} |r_i\rangle |r_j\rangle \quad \phi_{rr}(t) = Wt$$

# Numerical Simulations



Dynamics of Rydberg atoms  $i, j$  in CPW cavity,  $|\Phi(0)\rangle = |r_i\rangle |r_j\rangle |0_c\rangle$



(a1),(b1)  $f = 10$

(a2),(b2)  $f = 20$

$p_{\mu\nu}$  populations of  $|\mu_i\rangle |\nu_j\rangle$      $\phi_{rr}$  phase-shift of  $|r_i\rangle |r_j\rangle$

“—”: ME simulations for the full system using Hamiltonian  $H$

“—”: ME simulations for the effective Hamiltonians  $V_{ij}^{(2)}$  and  $V_{ij}^{(4)}$

# Applications of RDDI



## Intracavity Dipole Blockade [ $D = 2\pi \times 1 \text{ MHz}$ ( $f = 10$ )]

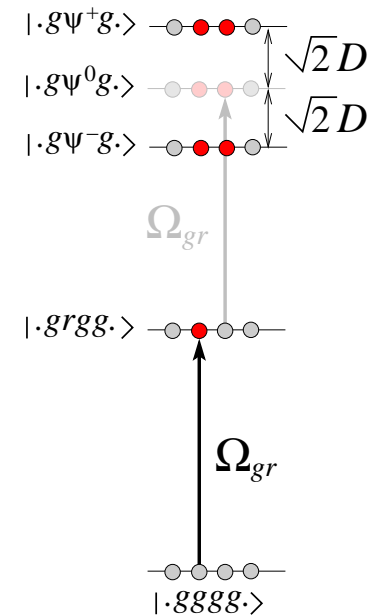
$$\langle rg | \hat{\sigma}_{gr} \Omega_{gr} | \psi^0 \rangle = 0 \ \& \ \Omega_{gr} < D \ \Rightarrow$$

Single Rydberg excitation of atomic ensemble by  $\Omega_{gr}$

(effective  $\pi$ -pulse:  $\sqrt{N} \Omega_{gr} T_1 = \frac{\pi}{2}$ )

$$\boxed{|s^{(0)}\rangle \rightarrow |r^{(1)}\rangle}$$

$$|s^{(0)}\rangle \equiv |g_1, g_2, \dots, g_N\rangle \quad |r^{(1)}\rangle \equiv \frac{1}{\sqrt{N}} \sum_i e^{i\mathbf{k}_{gr} \cdot \mathbf{r}_i} |g_1, \dots, r_i, \dots, g_N\rangle$$



# Applications of RDDI

## Intracavity Dipole Blockade [ $D = 2\pi \times 1 \text{ MHz}$ ( $f = 10$ )]

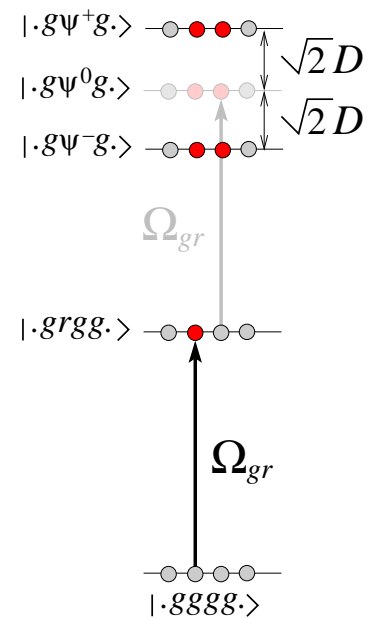
$$\langle rg | \hat{\sigma}_{gr} \Omega_{gr} | \psi^0 \rangle = 0 \ \& \ \Omega_{gr} < D \ \Rightarrow$$

Single Rydberg excitation of atomic ensemble by  $\Omega_{gr}$

(effective  $\pi$ -pulse:  $\sqrt{N} \Omega_{gr} T_1 = \frac{\pi}{2}$ )

$$|s^{(0)}\rangle \rightarrow |r^{(1)}\rangle$$

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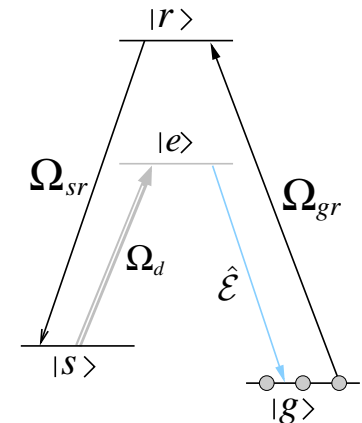
## Single Photon Generation

- Apply  $\Omega_{sr}$  ( $\pi$ -pulse:  $\Omega_{sr} T_2 = \frac{\pi}{2}$ )

$$|r^{(1)}\rangle \rightarrow |s^{(1)}\rangle$$

$$|s^{(1)}\rangle \equiv \frac{1}{\sqrt{N}} \sum_i e^{i\delta\mathbf{k} \cdot \mathbf{r}_i} |g_1, \dots, s_i, \dots, g_N\rangle$$

single collective Raman excitation of atomic ensemble  
(equivalent to EIT stored single photon)



- Turn on  $\Omega_d \rightarrow$  release single photon pulse  $\hat{\mathcal{E}}$

# Applications of RDDI



## Intracavity Dipole Blockade [ $D = 2\pi \times 1 \text{ MHz}$ ( $f = 10$ )]

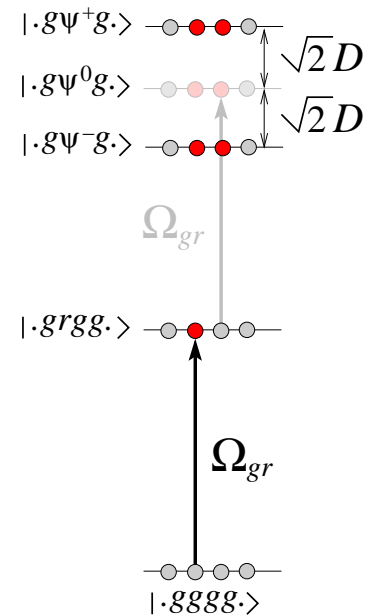
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Single Rydberg excitation of atomic ensemble by  $\Omega_{gr}$

(effective  $\pi$ -pulse:  $\sqrt{N} \Omega_{gr} T_1 = \frac{\pi}{2}$ )

$$|s^{(0)}\rangle \rightarrow |r^{(1)}\rangle$$

$$|s^{(0)}\rangle \equiv |g_1, g_2, \dots, g_N\rangle \quad |r^{(1)}\rangle \equiv \frac{1}{\sqrt{N}} \sum_i e^{i\mathbf{k}_{gr} \cdot \mathbf{r}_i} |g_1, \dots, r_i, \dots, g_N\rangle$$

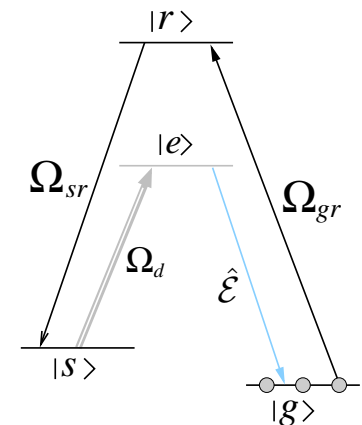


## Entangling two (or more) ensembles $A$ and $B$

- Apply  $\Omega_{gr}$  simultaneously to  $A$  and  $B$  ( $\sqrt{2N} \Omega_{gr} T_1 = \frac{\pi}{2}$ )

- Apply  $\Omega_{sr}$  ( $\pi$ -pulse:  $\Omega_{sr} T_2 = \frac{\pi}{2}$ )

$$\Rightarrow \frac{1}{\sqrt{2}} ( |s^{(1)}\rangle_A |s^{(0)}\rangle_B + |s^{(0)}\rangle_A |s^{(1)}\rangle_B )$$



# Applications of VdWI

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**Dispersive phase shift** [ $W = 2\pi \times 40$  KHz ( $f = 10$ )]

$$|r\rangle |r\rangle \rightarrow e^{i\phi(t)} |r\rangle |r\rangle \quad \phi(t) = Wt$$

# Applications of VdWI

**Dispersive phase shift** [ $W = 2\pi \times 40$  KHz ( $f = 10$ )]

$$|r\rangle |r\rangle \rightarrow e^{i\phi(t)} |r\rangle |r\rangle \quad \phi(t) = Wt$$

Ensemble qubits  $A, B, \dots$  in states

$$|\psi\rangle_q = \alpha_q |s^{(0)}\rangle_q + \beta_q |s^{(1)}\rangle_q$$

- Apply  $\Omega_{sr}$  to  $A$  &  $B$  ( $\Omega_{sr}T_1 = \frac{\pi}{2}$ ):

$$|s^{(1)}\rangle_{A,B} \rightarrow |r^{(1)}\rangle_{A,B}$$

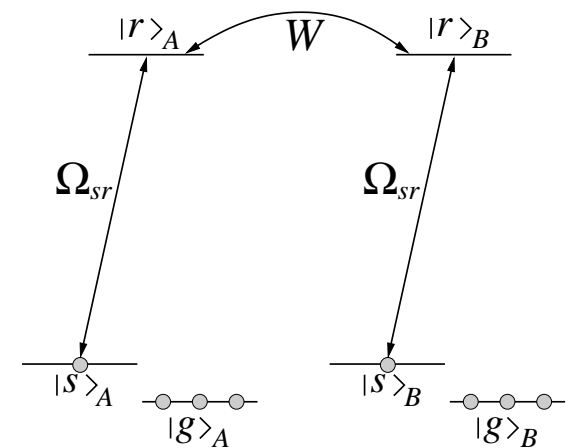
- Phase-shift  $\phi = WT_\pi = \pi$  (during  $T_\pi = \pi/W$ )

- Apply  $\Omega_{sr}$  again ( $\Omega_{sr}T_2 = \frac{\pi}{2}$ ):

$$|r^{(1)}\rangle_{A,B} \rightarrow |s^{(1)}\rangle_{A,B}$$

$\Rightarrow$  **Universal CPHASE gate between ensemble qubits  $A$  &  $B$**

$$|s^{(x)}\rangle_A |s^{(y)}\rangle_B \rightarrow (-1)^{xy} |s^{(x)}\rangle_A |s^{(y)}\rangle_B \quad (x, y \in [0, 1])$$



# Applications of VdWI

**Dispersive phase shift** [ $W = 2\pi \times 40$  KHz ( $f = 10$ )]

$$|r\rangle |r\rangle \rightarrow e^{i\phi(t)} |r\rangle |r\rangle \quad \phi(t) = Wt$$

Photonic qubits  $A, B, \dots$  in states

$$|\psi\rangle_q = \alpha_q |0\rangle_q + \beta_q |1\rangle_q$$

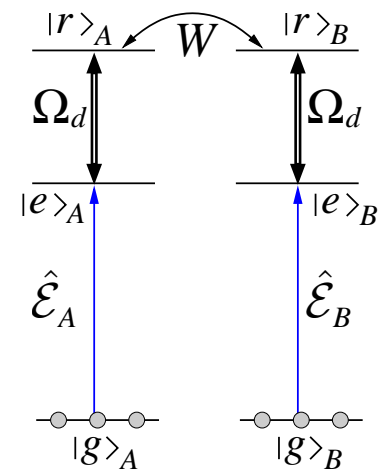
- EIT dark-state polaritons in ensembles  $A$  &  $B$

$$\Psi_{A,B} = \cos \theta \hat{\mathcal{E}}_{A,B} - \sin \theta \sqrt{N} \hat{\sigma}_{gr}^{A,B}$$

- Phase shift  $\phi = WT_{\text{int}} = \pi$  ( $T_{\text{int}} = \frac{L_a}{v_g}$ ,  $v_g = c \cos^2 \theta \ll c$ )

$\Rightarrow$  **Universal CPHASE gate between photonic qubits  $A$  &  $B$**

$$|x\rangle_A |y\rangle_B \rightarrow (-1)^{xy} |x\rangle_A |y\rangle_B \quad (x, y \in [0, 1])$$



# Summary

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Scalable and efficient quantum computation with photonic qubits requires:

- Deterministic sources of single-photons (qubit initialization)
- Reversible photon storage (static qubits)
- High-fidelity logic gates between flying/static qubits
- Reliable state detection

# Summary

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Scalable and efficient quantum computation with photonic qubits requires:

- Deterministic sources of single-photons (qubit initialization)
- Reversible photon storage (static qubits)
- High-fidelity logic gates between flying/static qubits
- Reliable state detection

EIT based (or related) techniques can implement these requirements

⇒ **Deterministic** all-optical quantum computation & communication  
may become possible