Interaction between plasmonic nanoparticles revisited with transformation optics

Alexandre Aubry, Dang Yuan Lei, Stefan A. Maier and JB Pendry

The Blackett Laboratory, Department of Physics,

Imperial College London, London SW72AZ
I. INTRODUCTION

![Image](image_url)

FIG. 1: (a) Two semi-infinite metallic slabs separated by a thin dielectric film support surface plasmons that couple to an array of dipoles $\Delta_x$, transporting energy to infinity. The array pitch is of $2\pi$. (b) The transformed material of (a) is a thin dielectric tube surrounded by silver. The array of dipoles, $\Delta_x$, is transformed into a single dipole, $\Delta_u$. (c) The transformed material of (b) is a pair of cylinders of diameter $D$, separated by a narrow gap $\delta$. The dipole source $\Delta_u$ is transformed into a uniform electric field $E_0'$.

Our canonical system is an array of dipoles placed between two semi-infinite metallic slabs (Fig.1(a)). Applying a conformal transformation that we will describe, the semi-infinite slabs of metal are transformed into a pair of metallic cylinders (Fig.1(c)). One the one hand, transformation optics provides a qualitative description of the surface plasmon modes propagating in such a structure. On the other hand, by solving the problem in the original frame, one can deduce the analytical solution in the transformed geometry under the electrostatic approximation. Thus, we first study the coupling of an array of dipoles with the surface plasmons supported by two semi-infinite slabs of metal. Then, by applying a conformal transformation to this system, we deduce the behavior of surface plasmons in a pair of cylinders and their coupling with the external field. An analytical expression of the absorption cross-section is derived. The absorption spectrum may exhibit several resonances which shift towards red when the gap between the two nanowires decreases. Conformal
transformation provides an elegant tool to describe the physical mechanism responsible for the surface plasmon resonances engendered in the nanoparticle pair. The electric field in the transformed geometry is also expressed analytically. The surface plasmons are shown to propagate along the surface of the cylinders. In the gap separating the two nanoparticle, the surface plasmons supported by each nanoparticle couple to each other: their wavelength as well as their group velocity decrease and an important field enhancement is then expected in the narrow gap separating the nanowires. At last, the question of radiative losses is raised when the structure size becomes comparable to the wave length. A theoretical correction is made on the electrostatic result to take into account radiation damping. Our analytical predictions are compared with numerical simulations.

II. THEORY UNDER THE ELECTROSTATIC APPROXIMATION

A. Conformal transformation

Our canonical system is an array of dipoles aligned along the $x$-axis, with pitch $2\pi$, located in a thin slab of insulator of thickness $d$ surrounded by two semi-infinite slabs of plasmonic material for $x < -d/2$ and $x > d/2$ (Fig.1(a)). Now apply the following conformal transformation,  

$$ w = r_o \exp(z) $$

where $z = x + iy$ and $w = u + iv$ are the usual complex number notations. The transformed material consists in a thin dielectric tube embedded in metal (Fig.1(b)). As regards the transformation of the source, the array of dipole is transformed into a single dipole $\Delta_u = r_o \Delta_x$ aligned along the $u$-axis. The pair of cylinders can then be derived applying the following conformal transformation (see Fig.1(c)),  

$$ z' = \frac{g^2}{w - r_o} $$

where $z' = x' + iy'$ is the usual complex number notation. The diameter of the two cylinders is given by,  

$$ D = \frac{g^2}{r_o \sinh(d/2)} $$

The distance $\delta$ separating the two cylinders can be expressed as,  

$$ \delta = \frac{g^2}{r_o} \tanh(d/4) $$
The shift $s$ between the axis $y'$ and the surface of the cylinder is given by

$$s = \frac{g^2}{r_o (1 + e^{d/2})}$$  \hfill (5)

We also define a key parameter,

$$\rho = \frac{\delta}{2D} = \sinh^2(d/4),$$  \hfill (6)

which is the ratio between the gap separating the two cylinders $\delta$ and the physical cross-section of the cylinders, $2D$. At last, the overall physical cross-section of the cylinders pair, $D_o = 2D + \delta$, can be expressed as

$$D_o = \frac{g^2}{r_o \tanh(d/4)} = \frac{g^2}{r_o} \sqrt{1 + \rho}. \hfill (7)$$

Transformation of the dipole $\Delta u$ is equally interesting - see Fig.1. We have chosen the origin of our inversion at the centre of the dipole. Thus the two charges comprising the dipole, very close to the origin of the transformation, translate to near infinity in $z'$, which gives rise to a uniform electric field in the transformed geometry. If the original dipole had strength $\Delta u = r_o \Delta x$ then in the transformed frame, the electric field at the origin is given by

$$E'_0(z' = 0) = \frac{1}{2\pi\epsilon_0} \frac{r_o \Delta x}{g^2} \hfill (8)$$

Note that we made the choice of an electric field $E'_0$ polarized along $x'$. Actually, this polarization is by far more efficient to excite surface plasmon modes than a transverse polarization (along $y'$) (see e.g Ref.[20]). We shall assume that the dimensions of the cylinders pair is sufficiently small that the surface plasmon modes are well described in the near field approximation. In this case the dielectric properties of the nanostructure are the same as those of the slab from which it is derived. Also preserved under the transformation is the electrostatic potential associated with an excitation:

$$\phi(x, y) = \phi'(x', y') \hfill (9)$$

The mathematics of the conformal transformation closely links the physics at work in each of the very different geometries. Solving the relatively tractable slab problem (Fig.1(a)) solves the interacting nanowires problem (Fig.1(c)). First, we pause to give a qualitative account.

In Fig.1(a), an array of dipoles pumps energy into the surface plasmon modes supported by two semi-infinite metallic slabs. This implies a discrete modal structure for this system:
only certain spatial frequencies can be excited among the overall spectrum. The same modes are excited in the interacting nanowires by a uniform electric field $E'_0$ (Eq.8) which we shall take as due to an incident plane wave. The pair of cylinders will thus exhibit a resonant feature. The transformation of modes shown in Fig.1 tells us that surface plasmons propagate along the surface of the two nanowires. Their wavelength and their group velocity decrease when they approach the narrow gap separating the two nanoparticles, leading to an important field enhancement at this location. However, contrary to the kissing cylinders [20], the group velocity does not vanish, hence surface plasmons turn infinitely around the nanoparticles before being absorbed. This explains the resonant behavior: only surface plasmons modes displaying an integer number $n$ of spatial periods when propagating around the nanowire can be excited. After this brief qualitative account, we now present in details our analytical theory, by first solving the slab problem shown in Fig.1(a).

**B. Coupling of an array of dipoles to the surface plasmons supported by the metallic sheets**

The coupling of the array of dipoles to two semi-infinite metallic slabs is first addressed. The near-field approximation is made, hence we assume that the Laplace’s equation is obeyed. The dipoles $\Delta$ consist of two line charges. We wish to calculate the potential $\phi$ induced on the dielectric sheets by expanding the incident field $\phi_o$ of the dipole as a Fourier series in $y$:

$$
\phi_o(r) = -\frac{1}{2\pi\epsilon_0} \sum_{n=-\infty}^{n=+\infty} \Delta_u(r - 2n\pi) \frac{e^{i k y}}{|r - 2n\pi|^2} = \frac{1}{2\pi} \int dk \phi_o(k) e^{iky}
$$

(10)

$\phi_o(k)$ can be found by making a Fourier transform in a transverse plane at an arbitrary position $x$:

$$
\phi_o(k) = \int \phi_o(x, y) e^{-iky} dy = a(k) e^{-|k||x|},
$$

(11)

with $a(k) = -\frac{\text{sgn}(x) \Delta_x}{2\epsilon_0} \sum_{n=-\infty}^{n=+\infty} \delta[k - n]$

(12)

where $\delta[k]$ denotes the Dirac distribution. The incident field is a Dirac comb in the k-space, hence only surface plasmon modes associated to integer spatial frequencies $n$ can be excited. The next step of our calculation consists in deriving the field $\phi(k)$ induced by this dipole.
FIG. 2: Sketch of the electrostatic potential induced in a metallic slab by an infinite array of dipoles \( \Delta_x \) placed in the plane \( x = 0 \), for \( \epsilon < -1 \).

array in the metal plates located in the half planes \( x < -d/2 \) and \( x > d/2 \). Due to the odd parity of the incident potential \( \phi_0(k) \) and the symmetry of the system, the induced field \( \phi(k) \) is also of odd parity. As illustrated by Fig.2, this field can be expressed as follows:

\[
\phi(k) = \begin{cases} 
  b(k)e^{-|k|x} - b(k)e^{+|k|x}, & x > -d/2 \text{ and } x < d/2 \\
  c(k)e^{-|k|x}, & x > d_1 \\
  -c(k)e^{+|k|x}, & x < -d_2 
\end{cases}
\]  

(13)

The two unknowns \( b(k) \) and \( c(k) \) are then determined by the boundary conditions at the dielectric slab interfaces. They derive from the conservation of the parallel component of the electric field and of the normal component of the displacement field,

\[
a(k)e^{-|k|d} + b(k)e^{-|k|d/2} - b(k)e^{+|k|d/2} = c(k)e^{-|k|d/2} \\
a(k)e^{-|k|d} + b(k)e^{-|k|d/2} + b(k)e^{+|k|d/2} = \epsilon c(k)e^{-|k|d/2}
\]

Solving these two equations provides the following results,

\[
b(k) = -\frac{\Delta_x}{2\epsilon_0 \epsilon} \frac{\epsilon^{-1+1}}{\epsilon^{+1}} \sum_{n=-\infty}^{n=+\infty} \delta[k - n] \\
c(k) = -\frac{\Delta_x}{\epsilon_0(\epsilon + 1)} \epsilon^{+1} \sum_{n=-\infty}^{n=+\infty} \delta[k - n]
\]  

(14) (15)
An inverse Fourier transform of the induced potential derived in the k-space leads to the solution in the real space,

$$\phi(x, y) = \frac{1}{2\pi} \begin{cases} 
\int -c(k)e^{iky+|k|x}dk, & x < -d/2 \\
\int -2b(k) \sinh(|k|x)e^{iky}dk, & |x| < d/2 \\
\int c(k)e^{iky-|k|x}dk, & x > d/2 
\end{cases}$$  

By injecting the expressions of $a(k)$ (Eq.12), $b(k)$ (Eq.14) and $c(k)$ (Eq.15), we obtain

$$\phi(|x| < d/2) = \frac{\Delta}{2\pi\epsilon_0} \frac{\epsilon - 1}{\epsilon + 1} \sum_{n=1}^{+\infty} \frac{\sinh(nz) + \sinh(nz^*)}{e^{nd} - \frac{\epsilon - 1}{\epsilon + 1}}$$  

$$\phi(x > d/2) = -\frac{\Delta}{2\pi\epsilon_0} \frac{\epsilon + 1}{\epsilon + 1} \sum_{n=1}^{+\infty} \beta_n \frac{e^{nd}(e^{-nz} + e^{-nz^*})}{e^{nd} - \frac{\epsilon - 1}{\epsilon + 1}}$$

$$\phi(x < -d/2) = \frac{\Delta}{2\pi\epsilon_0} \frac{\epsilon + 1}{\epsilon + 1} \sum_{n=1}^{+\infty} \beta_n \frac{e^{nd}(e^{nz} + e^{nz^*})}{e^{nd} - \frac{\epsilon - 1}{\epsilon + 1}}$$

with $\beta_0 = 1/2$ and $\beta_{n>0} = 1$. Note that contrary to the case of kissing cylinders for which the induced potential was obtained by directly picking out the pole due to the surface plasmon modes [20], the system here consists in an infinite sum of discrete modes which gives rise to a resonance when $\exp(nd) = \text{Re}[(\epsilon - 1)/(\epsilon + 1)]$.

C. Electric field in the slab frame and dipolar moment in the transformed frame

From the expression of the induced potential $\phi$ for $|x| < d/2$ (Eq.17), we can deduce the electric field at the dipole

$$E_x(z = 0) = -\frac{\partial \phi}{\partial x}(z = 0) = -\frac{\Delta}{\pi\epsilon_0} \frac{\epsilon - 1}{\epsilon + 1} \sum_{n=1}^{+\infty} \frac{n}{e^{nd} - \frac{\epsilon - 1}{\epsilon + 1}}$$  

This electric field induced at the dipole is of particular interest, since it is directly related to the net dipole moment $p$ of the cylinder pair in the transformed geometry. Indeed, similarly to the relation linking the emitting dipole $\Delta$ to a uniform electric field $E'_0(z' = 0)$ in the transformed frame (Eq.8), the dipole moment $p$ can be deduced from $E(z = 0)$,

$$p = 2\pi\epsilon_0 \frac{g^2}{r_0} E(z = 0)$$  

Injecting the expression of $E_x(z = 0)$ (Eq.20) into the last equation, replacing $\Delta_x$ by its expression (Eq.8) and $g^2/r_o$ using Eq.7, the induced dipole moment can be expressed as the
product of a polarizability $\gamma$ with the incident electric field $E'_0$ in the transformed frame,

$$p = \gamma E'_0,$$

with $\gamma = -4\pi\epsilon_0 \frac{\rho}{\rho + 1} D_o^2 \left\{ \frac{\epsilon}{\epsilon + 1} \sum_{n=1}^{+\infty} \frac{n}{(\sqrt{\rho} + \sqrt{1 + \rho})^{4n} - \frac{\epsilon - 1}{\epsilon + 1}} \right\}.$$

(22)

This expression of the dipole moment exhibits the contribution of each surface plasmon mode supported by the interacting nanowires.

D. Absorption cross-section

Dipoles and fields exchange roles in the two frames, but the product is unchanged. Therefore, energy dissipation is the same in each geometry [20]. The dipole energy pumped into the surface plasmons in the metal slab(s) (Fig.1) maps directly onto the power absorbed by the cylinder pair from the incident electric field $E'_0$ in the transformed frame:

$$P = -\frac{\omega}{2} \text{Im} \left\{ \Delta^* . E(z = 0) \right\} = -\frac{\omega}{2} \text{Im} \left\{ E'_0^* . p \right\}$$

(23)

If we inject the expression of $p$ (Eq.22) into the last equation and renormalize it by the incoming flux $P_{in} = \epsilon_0 c_0 |E'_0|^2/2$, the absorption cross-section $\sigma_a = P/P_{in}$ of the cylinders pair can be deduced

$$\sigma_a = 4\pi k_0 \frac{\rho}{\rho + 1} D_o^2 \text{Im} \left\{ \sum_{n=1}^{+\infty} \frac{n}{\frac{\epsilon - 1}{\epsilon + 1} (\sqrt{\rho} + \sqrt{1 + \rho})^{4n} - 1} \right\}.$$

(24)

$k_0 = \omega/c_0$ is the wave number in vacuum. Note that, rigorously, this expression corresponds to the extinction cross-section. However, as radiation losses are neglected under the quasi-static approximation, this quantity is here strictly equivalent to the absorption cross-section. $\sigma_a$ scales as the square of the overall size $D_o$ of the cylinder pair, which is typical of a 2D configuration. As pointed out previously when investigating the dipole moment (Eq.22), the overall absorption cross-section is the sum of each contribution due to the surface plasmon modes supported by the cylinder pair. Each mode may give rise to a resonance at a frequency satisfying the following relation

$$\left( \sqrt{\rho} + \sqrt{1 + \rho} \right)^{4n} = \text{Re} \left\{ \frac{\epsilon - 1}{\epsilon + 1} \right\}.$$

(25)

Note that this condition of resonance only depends on the ratio $\rho$ between the gap separating the nanoparticles and their diameters (Eq.6).
Fig. 3 illustrates this resonant feature by displaying $\sigma_a$ as a fraction of the physical cross-section, for $D_o = 20$ nm. For this figure as well as in the following of the study, the metal is assumed to be silver with a surface plasma frequency $\omega_{sp} = 3.67$ eV and permittivity taken from Johnson and Christy [24]. As shown in Fig.3, the absorption spectrum is strongly dependent on the gap separating the two nanoparticles and let show three distinct regimes.

- **Weak coupling regime ($\rho > 0.5$, i.e for a gap larger than the cylinder diameter):** all resonances can only occur at the vicinity of the surface plasma frequency $\omega_{sp}$ for which Eq.25 diverges. In that case, the coupling between the two nanoparticles is weak and the system exhibits the same absorption spectrum as an individual cylinder.

- **Strong coupling regime ($\rho < 0.5$):** when the two nanoparticles are approached by less than one diameter, resonances for small $n$ start to arise at a smaller frequency than $\omega_{sp}$. These resonances are red shifted when the gap decreases and the absorption spectrum displays several resonances in the visible spectrum in addition to the classical resonance at $\omega_{sp}$. As we will see, each resonance denoted by index $n$ corresponds to a surface plasmon mode showing $n$ spatial periods when propagating around the cylinder surface.
• Kissing cylinders regime ($\rho \rightarrow 0$): this regime has been extensively studied in Ref. [20].

The number of excited modes becomes infinite, leading to a continuous and broadband absorption spectrum.

When a resonance arises, the cylinder pair constitutes a powerful light harvesting device for an incident wave polarized along $x'$ (see Fig. 3). Even for such a small particle size ($D_o = 20$ nm), the absorption cross-section can be superior to the physical cross-section. For constant ratio \( \rho, \sigma_a/D_o \) scales linearly with $D_o$. Thus higher cross-sections could be obtained for larger cylinders but in this case our near field analytic theory may not be valid as we will see in Sec. III.

On the contrary, the device exhibits a weak absorption cross-section if the incident wave is polarized along $y'$. This fact has already been pointed out in previous numerical studies [3, 4]. For small kissing cylinders, only the resonance associated with the individual cylinder can be excited with an external field polarized along $y'$, as it has been already demonstrated for kissing cylinders [20].

E. Electric field in the transformed geometry

A pair of kissing cylinders is a nanostructure capable of an efficient harvesting of light at certain resonant frequencies. As we will see now, each of this resonance leads to a strong far-field to near-field conversion of energy: a considerable confinement and amplification of the electric field can be found in the narrow gap separating the two nanoparticles.

Under the conformal transformation, the potential is preserved (Eq. 9). The electric field \( \mathbf{E}'(x', y') \) in the transformed geometry can then be easily deduced from the potential,

\[
\begin{align*}
E'_{x'} &= -\frac{\partial \phi'}{\partial z'} \frac{\partial z^*}{\partial x'} + \frac{\partial \phi'}{\partial z^*} \frac{\partial z'}{\partial x'} = -\frac{\partial \phi'}{\partial z'} + \frac{\partial \phi'}{\partial z^*} \\
E'_{y'} &= -\frac{\partial \phi'}{\partial z'} \frac{\partial z^*}{\partial y'} + \frac{\partial \phi'}{\partial z^*} \frac{\partial z'}{\partial y'} = -i \frac{\partial \phi'}{\partial z'} + i \frac{\partial \phi'}{\partial z^*}
\end{align*}
\]

Using the expression of the potential $\phi$ given in Eqs. 17-19, the electric field $\mathbf{E}'$ can be expressed as a function of $\mathbf{E}_0'$ (Eq. 8), $D_o$ (Eq. 7) and $\rho$ (Eq. 6):
- For $|z' - D/2 + s| > D/2$ and $|z' + D/2 + \delta + s| > D/2$ (i.e outside the cylinders):

$$E_{x'}' = \frac{E_0}{\epsilon + 1} D^2_o \frac{\rho}{1 + \rho} \sum_{n=1}^{+\infty} \frac{n(\sqrt{\rho + \sqrt{1 + \rho}})^{4n}}{(\sqrt{\rho + \sqrt{1 + \rho}})^{4n} - \epsilon^{n+1}}$$

$$\times \left[ \frac{1}{z'^2} \left( 1 + \sqrt{\rho - D_o} \right)^{n-1} + \frac{1}{z'^2} \left( 1 + \sqrt{\rho + D_o} \right)^{n-1} \right]$$

$$+ \frac{1}{z'^{n+2}} \left( 1 + \sqrt{\rho - D_o} \right)^{n-1} + \frac{1}{z'^{n+2}} \left( 1 + \sqrt{\rho + D_o} \right)^{n-1}$$

$$\right]$$ (26)

$$E_{y'}' = i \frac{E_0}{\epsilon + 1} D^2_o \frac{\rho}{1 + \rho} \sum_{n=1}^{+\infty} \frac{n(\sqrt{\rho + \sqrt{1 + \rho}})^{4n}}{(\sqrt{\rho + \sqrt{1 + \rho}})^{4n} - \epsilon^{n+1}}$$

$$\times \left[ \frac{1}{z'^2} \left( 1 + \sqrt{\rho - D_o} \right)^{n-1} + \frac{1}{z'^2} \left( 1 + \sqrt{\rho + D_o} \right)^{n-1} \right]$$

$$- \frac{1}{z'^{n+2}} \left( 1 + \sqrt{\rho - D_o} \right)^{n-1} - \frac{1}{z'^{n+2}} \left( 1 + \sqrt{\rho + D_o} \right)^{n-1}$$

$$\right]$$ (27)

- For $|z' - D/2 + s| < D/2$ (i.e in the cylinder on the right of Fig.1(c)):

$$E_{x'}' = \frac{E_0}{\epsilon + 1} D^2_o \frac{\rho}{1 + \rho} \sum_{n=1}^{+\infty} \frac{n(\sqrt{\rho + \sqrt{1 + \rho}})^{4n}}{(\sqrt{\rho + \sqrt{1 + \rho}})^{4n} - \epsilon^{n+1}}$$

$$\times \left[ \frac{1}{z'^2} \left( 1 + \sqrt{\rho - D_o} \right)^{n-1} + \frac{1}{z'^2} \left( 1 + \sqrt{\rho + D_o} \right)^{n-1} \right]$$

$$\right]$$ (28)

$$E_{y'}' = i \frac{E_0}{\epsilon + 1} D^2_o \frac{\rho}{1 + \rho} \sum_{n=1}^{+\infty} \frac{n(\sqrt{\rho + \sqrt{1 + \rho}})^{4n}}{(\sqrt{\rho + \sqrt{1 + \rho}})^{4n} - \epsilon^{n+1}}$$

$$\times \left[ \frac{1}{z'^2} \left( 1 + \sqrt{\rho - D_o} \right)^{n-1} - \frac{1}{z'^2} \left( 1 + \sqrt{\rho + D_o} \right)^{n-1} \right]$$

$$\right]$$ (29)

- For $|z' + D/2 + \delta + s| > D/2$ (i.e in the cylinder on the left of Fig.1(c)):

$$E_{x'}' = \frac{E_0}{\epsilon + 1} D^2_o \frac{\rho}{1 + \rho} \sum_{n=1}^{+\infty} \frac{n(\sqrt{\rho + \sqrt{1 + \rho}})^{4n}}{(\sqrt{\rho + \sqrt{1 + \rho}})^{4n} - \epsilon^{n+1}}$$

$$\times \left[ \frac{1}{z'^2} \left( 1 + \sqrt{\rho - D_o} \right)^{n-1} + \frac{1}{z'^2} \left( 1 + \sqrt{\rho + D_o} \right)^{n-1} \right]$$

$$\right]$$ (30)

$$E_{y'}' = i \frac{E_0}{\epsilon + 1} D^2_o \frac{\rho}{1 + \rho} \sum_{n=1}^{+\infty} \frac{n(\sqrt{\rho + \sqrt{1 + \rho}})^{4n}}{(\sqrt{\rho + \sqrt{1 + \rho}})^{4n} - \epsilon^{n+1}}$$

$$\times \left[ \frac{1}{z'^2} \left( 1 + \sqrt{\rho - D_o} \right)^{n-1} - \frac{1}{z'^2} \left( 1 + \sqrt{\rho + D_o} \right)^{n-1} \right]$$

$$\right]$$ (31)
FIG. 4: Electric field for $\rho = 10^{-2}$ associated with the modes $n = 1, 2, 3$ (from left to right) at their corresponding resonant frequencies. (a) Amplitude of the imaginary part of $E'_{x'}$ normalized by the incoming field $E'_0$ (polarized along $x'$). (b) Amplitude of the imaginary part of $E'_{y'}$ normalized by the incoming field $E'_0$ (polarized along $x'$). For (a) and (b) plots, the color scale is restricted to [-10 10] but note that the field magnitude can be far larger in the narrow gap between the structures. (c) Amplitude of the imaginary part of $E'_{x'}/E'_0$ (blue) and $E'_{y'}/E'_0$ (red) along the cylinder surface as a function of the angle $\theta$ defined in the figure.

Note that in the near field approximation, which holds when the dimensions of the structure are less than the wavelength, the enhancement of electric field is independent of the size of the system. Fig.4 shows the result of our analytical calculation of the electric field associated to the first three modes taken at their resonant frequencies (Eq.25). The gap $\delta$ is fixed to $D/20$ ($\rho = 10^{-2}$). The metal is assumed to be silver with permittivity taken from Johnson and Christy [24].

Fig.4(a) and (b) represent the imaginary part of the field distribution along $x'$ and $y'$, respectively. These figures can be easily interpreted with conformal transformation. In the slab frame, the surface plasmon modes transport the energy of the dipoles along the surface.
metal slabs (see Fig.1). The same modes are excited in the transformed frame and propagate along the cylinder surface. As they approach the gap separating the two nanoparticles, the two surface plasmons supported by each nanoparticle couple to each other, their wavelength shortens and group velocity decreases in proportion. This leads to an enhancement of the field in the narrow gap. However, contrary to the kissing cylinders [20], the group velocity of surface plasmons does not vanish and energy cannot accumulate infinitely in the narrow gap. Instead, surface plasmons propagate indefinitely around the cylinders, leading to the resonant behavior pointed out previously.

Fig.4(c) represents the imaginary part of the field along the surface of the cylinders. The comparison between each mode allows to confirm our previous qualitative description: the index \( n \) associated to each mode corresponds to the number of spatial period covered by the surface plasmon when propagating around the cylinder. Fig.5(c) also highlights the drastic field enhancement that can be obtained in the gap between the two nanoparticles. Typically, for \( \rho = D/20 \), the field enhancement \( |E'|/E_0 \) can reach a value of 600. If this structure were deployed in a Raman scattering experiment [15, 16], sensitivity to molecules placed at the point would show an enhancement of \( 1.3 \times 10^{11} \) in sensitivity. Note that the field enhancement is less than one order of magnitude of the value obtained for kissing cylinders (\( \sim 10^4 \) [20]).

Fig.5 shows a more systematic investigation of the field enhancement that can be obtained with a pair of nanowires. The resonant feature, pointed out previously with the absorption cross-section in Fig.3, is also clearly visible for the field magnitude with a red shift of resonances when nanoparticles are close to each other. Each resonance leads to a drastic field enhancement that can be superior to \( 10^3 \) for \( \rho < 10^{-2} \). Fig.5 also shows that the field spreads spatially over a quite large part of the cylinder surface (\( |\theta| < 50 \) deg), unlike kissing cylinders where the field is extremely confined at the vicinity of the touching point.

Note that the local field amplification displayed by Fig.5 may be unrealistic in practice if the gap between goes below 1 nm. Indeed quantum mechanical effects, such as electron tunnelling or screening, have to be taken into account and may reduce the field enhancement relative to classical predictions [28]. The other limit of our analytical theory comes from the electrostatic approximation that assumes a structure with a dimension much smaller than the wavelength. The next section tackles with this issue. Radiation losses are taken into account analytically and our theoretical predictions are compared with numerical simulations.
FIG. 5: Field enhancement $|E'|/E'_0$ arising at the surface of the cylinders, plotted as a function of the angle $\theta$ and frequency, for different gaps between the two nanoparticles.

III. RADIATIVE LOSSES

We now turn to the question of validity of the near-field approximation.

A. Theory

B. Numerical simulations

IV. CONCLUSION


[29] C.H. Bohren and D.R. Huffman, Absorption and scattering of light by small particles. (John